Group Work Subsequences, Again?

Assume $a_n$, $b_n$ are sequences with $a_n \to c$; $b_n \to c$. Make the twisty sequence

$$(c_n) = (a_1, b_2, a_3, b_4, \ldots)$$

Use $\epsilon$, $K$ to prove $c_n \to c$.

This works better with a formula:

$$c_{2k} = b_k \quad c_{2k+1} = a_{2k+1} \quad \text{for } k \geq 0.$$ 

Since $a_n \to c$, and $a_{2kn}$ is a subsequence, then $a_{2k+1} \to c$. So, given $\epsilon > 0$ there is a $K_0$ such that, for $k \geq K_0$, $|a_{2kn} - c| < \epsilon$.

Similarly, there is a $K_0$ such that, for $k \geq K_0$, $|b_{2k} - c| < \epsilon$.

Set $K = \sup \{2K_0, 2K_0 + 1\}$.

If $n > K$, we'll show $|c_n - c| < \epsilon$.

Case 1: $n = 2k$. Then $2k > K \geq 2K_0$ so $k \geq K_0$.

So $|c_n - c| = |b_{2k} - c| < \epsilon$.

Case 2: $n = 2k+1$. Then $2k+1 > K \geq 2K_0 + 1$ so $k \geq K_0$.

So $|c_n - c| = |a_{2k+1} - c| < \epsilon$.

So in either case, $n > K$ implies $|c_n - c| < \epsilon$. 