BODY SIZE, PHYSIOLOGICAL TIME, AND LONGEVITY OF HOMEOTHERMATIC ANIMALS

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ABSTRACT

The concept of physiological time is extended from contributions of Adolph, Hill, and Stahl to include a wider range of events in the life histories of mammals and birds. The durations of physiological, developmental, and ecological cycles show nearly parallel exponential relationships to body size (mass). Maximum life span approximates a fixed multiple of shorter events, such as muscle-twitch time, circulation and filtration of blood plasma, respiratory and metabolic cycles, embryonic development, growth, sexual maturation, and even minimum population-doubling time.

Endothermy evolved independently in birds and mammals, which share the approximate $M^{1/4}$ allometry of physiological time, although with different absolute values of cycle times. Birds develop faster than similar-sized mammals, live longer, and their slower hearts actually beat more before expiring.

A time-scale proportional to $M^{1/4}$ may be difficult to synchronize with environmental cycles. Thus, the mouse experiences more events in a day’s fast or a winter cold than a mouse (a long cold winter being relatively longer for mouse than moose). This asynchrony of physiological time and astronomical time has resulted in the need for biological clocks.

Life span, like the other scales of physiological time, may be regarded, not as a direct product of natural selection, but rather as an allometric consequence of other characteristics subjected to natural selection.

INTRODUCTION

THE HISTORY of animal evolution is classically viewed as a phylogenetic tree branching into different forms, with examples of adaptive radiation such as Darwin’s finches at the tips of the branches. Within a single class of animals, however, the diversity
of body sizes is as impressive as the diversity of forms. Terrestrial mammals span six orders of magnitude in body mass, and living birds span nearly five. The natural selection for a particular body size includes more than external dimensions; it has profound effects on the entire biology of each species. Hill pointed out that the size of an animal dictates its "rate of living." Despite its importance, however, this relationship has too often been overlooked by a number of biologists.

Body mass, more than any other single descriptive feature, is the primary determinant of ecological opportunities, as well as of the physiological and morphological requirements of an animal. To preserve function at all body sizes, there has been a differential and usually non-linear scaling of each of the details of body form and life history (Hill, 1950; Stahl, 1962, 1965, 1967; Gould, 1966; McMahon, 1973, 1974, unpub.; Calder, 1974, 1981; Gunther, 1975; Schmidt-Nielsen, 1977). This scaling can be expressed and analyzed as power functions of body mass, such as the familiar equation describing the dependence of standard metabolic rate of mammals (\(H_{\text{smr}}\)) upon body mass (\(M\)) (Kleiber, 1961):

\[
H_{\text{smr}} = k M^{0.73}
\]  

(1)

Although body mass is the primary factor that determines physical requirements, other factors, such as age, leanness or height-diameter proportions, and relative brain size, may also be important. Incorporation of these other factors into multiple regressions for metabolism, lifespan, etc., may improve goodness of fit, explain more fully the variability in the data, or better assess the influence of body surface area or degree of cephalization (Tanner, 1949; Kleiber, 1950; von Schelling, 1954; Sacher, 1959, 1978; Sacher and Staffeldt, 1974). However, it is body mass, and not the mass of particular organs, that determines basic support costs and control objectives. Hence, we consider the singular power function of body mass to be the most useful descriptive tool for understanding the evolution of size.

In order to examine the evolution of body size and its requirements and consequences, the biologist must find patterns. Though we acknowledge that pattern-seeking can become quite subjective, the exponents (\(b\)) which relate physiological and morphological variables (\(Y\)) to body mass (\(M\)) in allometric equations of the form:

\[
Y = a M^b
\]  

(2)

seem to fall into quantitative clusters (Adolph, 1949; Stahl, 1967; Calder, 1974; McMahon, 1975). These clusters include: physical support (\(b > 1\)), control (\(b < 1\)), capacities of transport organs (\(b \approx 1\)), cyclic frequencies (\(b \approx -\frac{1}{4}\)), and volume-rates such as metabolic rate and cardiac output (\(b \approx \frac{1}{4}\)). In the following paragraphs we present these exponents as ranges of values derived by different authors (e.g., \(M^{0.84-0.92}\)).

**Physical Support**

To maintain skeletal strength, the cross-sectional area of the bones must increase more rapidly than their length. Hence, the skeleton becomes relatively more robust as body mass is increased (Kayser and Heusner, 1964; McMahon, 1973; Reynolds and Karlotski, 1977; Schmidt-Nielsen, 1977; Prange, Anderson, and Rahn, 1979; Anderson, Rahn, and Prange, 1979). A 3 g shrew has a skeleton that accounts for less than 4 per cent of its body mass, whereas the skeleton of man contributes 15 per cent of body mass, and the skeleton of a 3,000 kg elephant over 20 per cent. The scaling of skin mass departs nearly equally from linearity with body mass but in the opposite direction from skeletal mass. We have derived the following relationship between skin mass (kg) and body mass, based on four points (mouse, hamster, cat, cow): \(M_{\text{skin}} = 0.129 M^{0.90}\), corr. coef. = 0.999. This is similar to the relationship for the four rodents plus rabbit derived by Pace, Rahiman, and Smith, 1979: \(M_{\text{skin}} = 0.139 M^{0.94}\), corr. coef. = 0.993. Hence, the combined mass of skeleton and skin in mammals is close to a constant percentage of body mass (\(\approx 25\%\)), skin contributing a much larger portion in small animals.

**Control**

With increase in size the number of cellular units increases, but the number of functions served does not. Therefore, a control
circuit or regulating organ need not increase linearly with body size. (Thus, the headlight switch on a large bus may be only slightly heavier than that on a compact car.) For instance, mammalian brain mass is proportional to \( M^a \) (Brody, 1945; Sacher, 1959; Jerison, 1961). Thus, the brain of a 3 g mammal constitutes 6 to 9 per cent of its body mass, whereas the brain of an adult elephant contributes less than 1 per cent to its body mass. The cause of this two-thirds scaling of the brain is still unclear and should be investigated by neuroanatomists.

The mass of mammalian endocrine glands also bears a less than linear proportionality to body mass: mass of pituitary \( \propto M^{0.56-0.76} \); mass of thyroid and adrenal glands each \( \propto M^{0.80-0.92} \); and mass of pancreas \( \propto M^{0.91} \). The organs that control the composition of circulating blood have similar mass exponents: mass kidneys \( \propto M^{0.89} \) and mass liver \( \propto M^{0.87} \) (Adolph, 1949; Stahl, 1965).

**Capacities of Organs**

Qualitatively, the basic vertebrate visceral plan was preserved during evolutionary size changes. Because heart, lung, gut, blood volume, etc. were not dispensable, one of them could not usurp the space of another. Hence, those organs which function volumetrically scale essentially linearly to body size. The heart mass scales to \( M^{0.98} \), blood volume to \( M^{0.99-1.02} \), cardiac stroke volume to \( M^{1.06} \), spleen to \( M^{1.02} \), lung mass to \( M^{0.99} \), and respiratory vital capacity and tidal volume to \( M^{1.03-1.04} \) (listed by or derived from Brody, 1945; Adolph, 1949; Stahl, 1965, 1967).

**Volume-Rates**

As described almost a half-century ago by Max Kleiber (Kleiber, 1932), the metabolic rates of eutherian mammals scale to \( M^a \). Parallel relationships (having similar exponents but different Y-intercepts, i.e., vertically displaced on log-log plots) have been described for passerine and nonpasserine birds and for marsupial mammals (Schmidt-Nielsen, 1977). The physiological meaning of the Y-intercepts is yet to be explained. The earlier metabolic work and attempts to explain the \( 3/4 \) exponent have been discussed elsewhere (Kleiber, 1961; McMahon, 1973).

Empirically, not only are the standard metabolic rates (measured as oxygen consumption rates) approximately proportional to \( M^a \), but other volume-rates (volume \cdot time\(^{-1}\)) of associated support functions, such as cardiac output, minute respiratory ventilation, renal glomerular filtration rate, and renal plasma flow, all scale in approximate proportion to \( M^{b} \) (Adolph, 1949; Edwards, 1975).

**Frequencies**

The basal rates of cyclic events are inversely related to approximately the \( 1/4 \) power of \( M \). The cardiac rate of mammals \( \propto M^{-0.25} \) and of birds \( \propto M^{-0.23} \). This scaling must extend to the endogenous depolarization-repolarization characteristics of the sinoatrial node as well as to the mechanical efficiency of the cardiac muscle at the natural frequency for the mass and tension characteristics of the heart muscle. The relation is consistent with a model of elastic similarity of scaling, discussed below (McMahon, 1975).

Similar exponents describe respiratory frequency: \( M^{-0.26} \) for mammals (Stahl, 1967), \( M^{-0.28} \) for passerine birds and \( M^{-0.31} \) for nonpasserine birds (Calder, 1968).

Adolph (1949) treated frequencies as reciprocals of time (time = 1/frequency): breath duration \( \propto M^{0.28} \), heart beat duration \( \propto M^{0.27} \), and gut beat duration \( \propto M^{0.31} \) for mammals. Many other physiological periods or times have since been found to scale near \( M^a \).

**INTERRELATIONSHIPS**

The basic maintenance of an animal's internal composition is a logistical problem of obtaining the right amounts (in regard to body mass) at the right times (in regard to mass-dependent time scales), for all of the many simultaneous and interrelated functions of the living state. Not all species have the same requirements. In meeting these requirements the evolution of body size entails quantitative proportional adjustment of the duration of many cycles, ranging from msec to years on the absolute time scale. Regardless of their length, these cycles scale approximately as \( M^a \). In view of this apparent universality, we might expect that body size and the intrinsic time scale which it requires
would be dominating characteristics of the life history. Such appears to be the case (Blueweiss et al., 1978; Western, 1979).

Is there any explanation for why animals’ use of time should vary in proportion to the fourth root of their body mass? Dimensional analysis can be used to derive the relationship between time and body mass with regard to muscle stress. If vertebrates are designed in such a manner that support structures retain the same relative strength regardless of body size, then an evolutionary increase in length must be accompanied by a disproportionately greater increase in diameter. McMahon (1973) found that the characteristic lengths (l) of bones, muscles, etc. are proportional to $M^{0.6}$ whereas their diameters (d) scale in proportion to $M^{0.6}$. He derived these proportionalities for scaling by elastic similarity in birds and mammals. According to this model, each cross-sectional area (d$^2$) is proportional to $M^{0.6}$ and the mass of each muscle, limb, or entire animal is $\propto (l \cdot d^2)$. Hence, muscle forces (mass $\times$ acceleration) can be described as $(l \cdot d^2)(l \cdot r^2)$. The maximum stress (i.e., force divided by cross-sectional area) generated in homologous muscles is roughly constant (Hill, 1950). Therefore:

$$\text{Mass} \times \text{acceleration} \propto \frac{l \cdot d^2}{\rho} \propto \frac{M^0}{\rho} = \text{constant}; t \propto M^{0.6} \quad (3)$$

Hence, if l is proportional to $M^{0.6}$ and if muscle stress is constant, time (t) scales directly as length or as $M^{0.6}$. The above model, derived for static stresses, assumes skeletal failures occur by buckling. Prange (1977) has derived a model based on dynamic forces in which he finds $d \propto l^{0.6-1.25}$. This model would therefore predict $t \propto l \propto M^{0.35-0.33}$. Recent data from Alexander, Jayes, Maloiy, and Wathuta (1979) on the scaling of limb bones, supports this “non-elastic” scaling.

In addition, and perhaps even more fundamental, the time animals require to perceive and respond to their surroundings seems to scale to $M^{0.6}$. An animal’s neurons form its communication system both within the animal and between the animal and its environment. The diameters of homologous neurons of all mammals, and therefore their conduction velocities, are nearly constant.

Nerve conduction velocities do vary, but not systematically with body size. For example, nerves of slow-moving skunks and sloths have low conduction velocities (Goffart, 1971; Van de Graaff, Frederick, Williamson, and Goslow, 1977) relative to man or baboon (Koeze, 1973; Mayer and Mawdsley, 1968), whereas those of the fast-moving cat are relatively high (Goslow, Cameron, and Stuart, 1977). However, over a broad size-range of mammals nervous messages travel along homologous nerves at nearly the same velocity. The amount of time necessary for a nervous impulse to reach its target is equal to the distance it must travel divided by its velocity. Since velocity along homologous nerves is roughly constant, the time required for an afferent message to arrive with information about the animal’s environment, or the time for an efferent message to reach the muscles or organs, is directly proportional to the distance those messages must travel. That distance is proportional to the characteristic length (l), or again $M^{0.6}$. Hormonal messages within the body likewise scale close to $M^{0.6}$, as the time (minutes) to circulate the entire blood volume in mammals is 0.35 $M^{0.21}$ (Stahl, 1967). Delcomyn (1980) has reviewed the evidence that rhythmic timing of cyclic movements, as in walking, swimming, and breathing, are intrinsic properties of the central nervous system. From this we must conclude that these intrinsic properties have been scaled allometrically as well.

These properties may have set the physiological time scales of birds and mammals in proportion to $M^{0.6}$. In fact, most cyclic events do transpire on time scales approximately proportional to $M^{0.6}$. Consequently, it may not be necessary to invoke special meaning or unique explanations to describe how one rate or another scales to body mass. One need not speculate that evolution has selectively dealt with any single biological cycle. It may be meaningless to consider the evolution of isolated traits. Gould and Lewontin (1979) argue against such an adaptationist approach. Hence, we propose no specific evolutionary explanation for the scaling of
physiological time. It seems more likely to us that the fundamental unit of physiological time, $M^0$, was an inevitable consequence of the geometry of changes in body size.

Brody (1945) used the term "physiological time" to denote a variable time scale among organisms of different size. A. V. Hill (1950) refined that concept, by suggesting that such things as power per unit weight, time for growth to maturity, and gestation period may be constant in all animals when compared per unit of physiological time; that is to say, 10 seconds for a large animal may be equivalent (physiologically) to 1 second for a small animal.

These ideas of physiological time have been confirmed during the past three decades. Adolph (1949) examined 34 morphological and physiological variables and related them allometrically to body size. The resulting "quantitative orderliness" suggested to him that organisms "may be pictured as systems of precise multiple interrelationships." To examine those interrelationships, Stahl (1962, 1963, 1965, 1967) combined allometric equations as ratios to form dimensional and dimensionless "criteria of similarity." For example, he calculated breath time (yrs) as $4.7 \times 10^{-5} M^{0.28}$ and pulse time (yrs) as $1.2 \times 10^{-5} M^{0.27}$. Therefore, dividing breath time by pulse times yields the dimensionless ratio of $3.9 M^{0.01}$. As the residual mass exponent is small, Stahl reasoned that there are about 3.9 heart beats per respiratory cycle in all mammals, regardless of size. By similar analysis, he estimated that all mammals have a basal energy use of about $8 \times 10^5 \text{ kj} \cdot \text{kg}^{-1}$ per lifetime. This idea of a finite total lifetime metabolism has been discussed further as "absolute metabolic scope" (terminology that can be confused with prior usage) (Boddington, 1978). Metabolic time on the scale of embryonic lifespan in birds has been analyzed by Rahn and Ar (1980).

Many additional biological periods have been examined and described as allometric functions of body mass since Stahl's work. McMahon (1975) has cited a number of these. Table 1 and Fig. 1 show these equations and many others which we have derived from available data for both birds and mammals. These cycle times span twelve orders of magnitude, from very rapid twitch contraction times to the periodic event with the very lowest frequency, lifespan. To facilitate comparisons, all the lines have been extrapolated from 1 g to $10^3$ kg to correspond roughly to the size range of terrestrial mammals (2 g to 7000 kg). This then shows the empirical basis of the "intrinsic time . . . converted to a standard clock time . . . by a metrical function depending upon constitutive parameters" which was treated only theoretically by Richardson and Rosen, 1979. The slopes are strikingly similar even though the lines themselves reflect function in: (1) individual organs (clearance of insulin, cardiac cycle); (2) systems of organs (metabolism of fat stores); (3) the entire animal (growth to maturity, lifespan); and (4) even populations of animals (minimum population doubling time).

For instance, the maximum rate of population growth is described by the variable $r$, the intrinsic rate of natural increase. The carrying capacity, $K$, represents the maximum steady-state population number the environment can support. These variables are incorporated in the simplest equation which describes density-dependent population growth, the logistic equation:

$$
\frac{dN}{dt} = rN \left( \frac{K - N}{K} \right)
$$

Species with high $r$ values undergo rapid population growth and are thought to be "colonizing" species. These "$r$-selected" animals differ from "$K$-selected" species, which have lower $r$ values (MacArthur and Wilson, 1967). One can calculate a theoretical minimum time required for a population to increase by a constant fraction of $K$ if equation (4) is integrated and solved for $t$ (after Crow and Kimura, 1970):

$$
t = \frac{1}{r} \ln \left( \frac{N_t}{N_0} \right)
$$

Fenchel (1974) demonstrated that $r$ varies as a regular exponential function of body size ($M^{0.23}$; corr. coef. = 0.98) in animals spanning 22 orders of magnitude of body mass. For mammals, $r$ (days$^{-1}$) = $6.3 \times 10^{-3} M^{0.26}$. Hence, substituting into equation (5):
TABLE 1

Equations relating cycle length (minutes) to body mass (kg)

(A) mammals; (B) birds. The mean mass exponent for all 27 equations is 0.247.

<table>
<thead>
<tr>
<th>Period (Cycle Length)</th>
<th>Funcional Scale</th>
<th>Minutes</th>
<th>C.I. *</th>
<th>Corr. †</th>
<th>Life Span Cycle Time</th>
<th>Reference† †</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. MAMMALIAN BIOLOGICAL PERIODS</td>
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<tr>
<td>Life span, in captivity</td>
<td>6.10 × 10^6 M^0 20</td>
<td>0.77</td>
<td>1</td>
<td></td>
<td>Sacher, 1959</td>
<td></td>
</tr>
<tr>
<td>98% growth time</td>
<td>6.35 × 10^6 M^0 26</td>
<td></td>
<td></td>
<td>9.61 M^0 06</td>
<td>Stahl, 1962</td>
<td></td>
</tr>
<tr>
<td>Min. time, population doubling</td>
<td>3.16 × 10^3 M^0 26</td>
<td>0.11-0.41</td>
<td>0.94</td>
<td>1.93 × 10^3 M^0 06</td>
<td>Fenchel, 1974</td>
<td></td>
</tr>
<tr>
<td>Time to reproductive maturity</td>
<td>2.93 × 10^4 M^0 18</td>
<td>0.64</td>
<td>2.08 × 10^3 M^0 02</td>
<td>Hafez et al., 1972</td>
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</tr>
<tr>
<td>50% growth time</td>
<td>1.85 × 10^3 M^0 25</td>
<td></td>
<td></td>
<td>3.30 × 10^3 M^0 05</td>
<td>Stahl, 1962</td>
<td></td>
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<tr>
<td>Gestation period</td>
<td>9.40 × 10^3 M^0 25</td>
<td>0.22-0.28</td>
<td>0.85</td>
<td>6.49 × 10^3 M^0 06</td>
<td>Sacher &amp; Staffeldt, 1974</td>
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<tr>
<td></td>
<td>9.54 × 10^3 M^0 26</td>
<td>0.72</td>
<td>6.39 × 10^3 M^0 06</td>
<td>Blueweiss et al., 1978</td>
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</tr>
<tr>
<td>Time to metabolize fat stores equal to 0.1% body mass</td>
<td>1.70 × 10^6 M^0 26</td>
<td>0.24-0.28</td>
<td>1.00</td>
<td>3.58 × 10^3 M^0 04</td>
<td>Kleiber, 1932</td>
<td></td>
</tr>
<tr>
<td>Half-life of drug (methotrexate)</td>
<td>5.8 × 10^1 M^0 19</td>
<td></td>
<td></td>
<td>1.05 × 10^3 M^0 01</td>
<td>Dedrick, Bischoff, &amp; Zaharko, 1977</td>
<td></td>
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<tr>
<td>Plasma clearance, inulin</td>
<td>6.51 M^0 27</td>
<td>0.23-0.31</td>
<td>0.98</td>
<td>9.37 × 10^3 M^0 05</td>
<td>Stahl, 1967; Edwards, 1975</td>
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<tr>
<td>Plasma clearance, para-aminohippurate</td>
<td>1.70 M^0 22</td>
<td>0.14-0.30</td>
<td>0.98</td>
<td>3.59 × 10^3 M^0 02</td>
<td>Stahl, 1967; Edwards, 1975</td>
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<tr>
<td>Time for circulation of blood volume</td>
<td>3.5 × 10^1 M^0 21</td>
<td></td>
<td></td>
<td>0.98</td>
<td>1.74 × 10^3 M^0 01</td>
<td>Stahl, 1967</td>
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<tr>
<td>Gut beat duration</td>
<td>4.75 × 10^2 M^0 31</td>
<td></td>
<td></td>
<td>1.28 × 10^3 M^0 11</td>
<td>Adolph, 1949</td>
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<tr>
<td>Respiratory cycle</td>
<td>1.87 × 10^2 M^0 26</td>
<td></td>
<td></td>
<td>3.26 × 10^3 M^0 06</td>
<td>Stahl, 1967</td>
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<tr>
<td>Cardiac cycle</td>
<td>4.15 × 10^3 M^0 25</td>
<td></td>
<td></td>
<td>1.47 × 10^3 M^0 05</td>
<td>Stahl, 1967</td>
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<tr>
<td></td>
<td>3.05 × 10^3 M^0 28</td>
<td>0.23-0.33</td>
<td>0.93</td>
<td>2.00 × 10^3 M^0 08</td>
<td>Calder, 1968</td>
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<tr>
<td>Twitch contraction time, soleus</td>
<td>1.06 × 10^3 M^0 39</td>
<td>0.29-0.49</td>
<td>0.99</td>
<td>5.75 × 10^3 M^0 19</td>
<td>Syrovoy &amp; Gutman, 1975</td>
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<tr>
<td>Twitch contraction time, extensor digitorum longus</td>
<td>3.14 × 10^4 M^0 21</td>
<td>0.12-0.31</td>
<td>0.99</td>
<td>1.94 × 10^10 M^0 01</td>
<td>Syrovoy &amp; Gutman, 1975</td>
<td></td>
</tr>
</tbody>
</table>

B. AVIAN BIOLOGICAL PERIODS

<table>
<thead>
<tr>
<th>Period (Cycle Length)</th>
<th>Funcional Scale</th>
<th>Minutes</th>
<th>C.I. *</th>
<th>Corr. †</th>
<th>Life Span Cycle Time</th>
<th>Reference† †</th>
</tr>
</thead>
<tbody>
<tr>
<td>Life span, in captivity</td>
<td>1.49 × 10^7 M^0 19</td>
<td>0.70</td>
<td>1</td>
<td></td>
<td>Lindstedt &amp; Calder, 1976</td>
<td></td>
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<tr>
<td>Life span, wild</td>
<td>9.25 × 10^6 M^0 20</td>
<td>0.17-0.22</td>
<td>0.78</td>
<td>1.61 M^0 01</td>
<td>Lindstedt &amp; Calder, 1976</td>
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<tr>
<td>Incubation period</td>
<td>4.16 × 10^4 M^0 17</td>
<td></td>
<td></td>
<td>0.86</td>
<td>3.58 × 10^3 M^0 02</td>
<td>Rahn &amp; Ar, 1974</td>
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<tr>
<td>Respiratory cycle</td>
<td>5.37 × 10^2 M^0 33</td>
<td>0.30-0.36</td>
<td>0.93</td>
<td>2.77 × 10^2 M^0 14</td>
<td>Calder, 1968</td>
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<td>Cardiac cycle</td>
<td>6.42 × 10^3 M^0 23</td>
<td>0.17-0.29</td>
<td>0.85</td>
<td>2.32 × 10^3 M^0 04</td>
<td>Calder, 1968</td>
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<tr>
<td>Life span, wild</td>
<td>1.14 × 10^7 M^0 26</td>
<td>0.20-0.32</td>
<td>0.76</td>
<td>1.31 M^0 07</td>
<td>Lindstedt &amp; Calder, 1975</td>
<td></td>
</tr>
<tr>
<td>Time to metabolize fat stores equal to 0.1% body mass</td>
<td>1.06 × 10^2 M^0 28</td>
<td>0.23-0.32</td>
<td>0.98</td>
<td>1.41 × 10^3 M^0 09</td>
<td>Lasiewski &amp; Dawson, 1975</td>
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<tr>
<td>Respiratory cycle</td>
<td>4.38 × 10^2 M^0 28</td>
<td>0.16-0.41</td>
<td>0.69</td>
<td>3.40 × 10^8 M^0 09</td>
<td>Calder, 1968</td>
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<tr>
<td>Time to first breeding</td>
<td>2.32 × 10^6 M^0 22</td>
<td>0.11-0.32</td>
<td>0.91</td>
<td>6.42 M^0 03</td>
<td>Lack, 1968</td>
<td></td>
</tr>
</tbody>
</table>

* C.I., 95% confidence interval of slope.
† A few equations were derived by combining other regression equations; in those cases, a combined correlation coefficient has been estimated as a product of the individual correlation coefficients.
†† Source of equation or data used to calculate equation.
** The slopes of the lines relating gestation period to body mass are lower if examined within individual orders of mammals (see also Kihlström, 1972).
Fig. 1. The Relation between Body Mass and Lengths of Biological Periods or Cycles
For mammals (solid lines) and for birds (dashed lines). Individual equations (from Table 1) have been extrapolated to span the range of body sizes shown.

\[ t = 159 M^{0.26} \ln \frac{N_0 (K - N_i)}{N_i (K - N_i)} \]  

The time required for a population to increase in number from \( \frac{N_0}{K} \) to \( \frac{N_i}{K} \) is thus roughly proportional to the fourth root of body mass. In Fig. 1 we have used equation (6) to calculate the minimum time required for a population to double in number. Although \( r \) is difficult to estimate and the values given by Fenchel (1974) may be inadequate in some cases, they should not, however, be skewed toward either large or small body size.

A much shorter biological period is the half-life of drugs. Dedrick, Bischoff, and Zaharko (1977) derived a theoretical relationship which predicts plasma half-life \( (t_{1/2}) \) of drugs to vary with the fourth root of body mass. They determined \( t_{1/2} \) (minutes) of methotrexate for six mammalian species and found:

\[ t_{1/2} = 38 M^{0.19} \]

Weiss, Szegoleit, and Foster (1977) independently predicted that \( t_{1/2} \) should be proportional to \( M^{0.25-0.27} \) on a basis of biological similarity criteria.

The above are examples of many cycles that vary approximately as \( M^{0.25} \). The large number of biological time scales that adhere to this pattern appears to confirm Stahl (1962), who wrote:

In the analysis of growth and related matters it has become clear that two principles play a key role in biological analysis: conservation of volume (see above) and synchronism of times. . . . Corresponding processes are expected to occur in corresponding times. It appears that the time scale is uniformly that of \( M^{0.25} \) to \( M^{0.30} \) in mammals and probably in many other organisms (p. 209).

This observation is confirmed in Fig. 1, which suggests that identical biological cycles transpire about seven times longer in man, and twenty times longer in elephants, than in mice.

Although few of the slopes in Fig. 1 are exactly 0.25, they are all very nearly parallel. In fact, the mean of all the body mass exponents is 0.24. Unfortunately, the sources in lit. usually provide neither error estimates for many of these equations nor the data to calculate them. However, when available, the 95 per cent confidence intervals of most of the slopes include 0.25 (see table 1). Although variability in exponents may represent the variance in the biological periods themselves, it must also reflect the difficulty of quantifying them accurately. Certainly the longest cycles, such as growth to maturity or life span, cannot be determined with the same accuracy as heart rate; and even heart rate has been reported by different investigators as varying between \( M^{0.25} \) and \( M^{0.28} \) (Adolph, 1949; Calder, 1968). Little meaning should therefore be ascribed to slight differences in reported exponents. To examine the effects of these exponent devia-
tions over a broad size range, we have selected several equations from Table 1 and combined pairs of them (after Stahl) in Fig. 2. The resultant derived equations only estimate intercepts and exponents. Thus, they may differ slightly from direct measurements. Residual mass exponents from $M^{-0.04}$ to $M^{0.04}$ have relatively little effect on the value of each ratio, over a size range from shrew to elephant. According to these equations, 100 heart beats transpire in six seconds for a shrew and in 2.5 minutes for an elephant. As a consequence of those 100 cardiac cycles, the shrew’s total blood volume should circulate once, while the elephant’s should circulate one and one-half times. Thus, although their body sizes differ by a factor of one million, the ratio of blood circulation time to heart contraction time varies only slightly (see Fig. 2), and the apparent difference may only be “noise” in the measurements or in the power-law equations themselves. Since the 95 per cent confidence intervals of nearly all the equations include 0.25, the observed differences in exponents may not be statistically meaningful. Whether the variance in exponents is real or artificial, small residual mass exponents have relatively little effect over a wide range of bird and mammal body sizes. Hence, biological events or cycles do appear to transpire (1) as constant multiples of one another and (2) in a time proportional to body mass raised to about the $\frac{1}{4}$ power.

While allometric relationships are not biological laws, they do describe patterns as well as identify those animals which differ from the patterns. Thus far, our focus has been on the patterns themselves rather than on the deviants. Shrews are of interest because of their small sizes and metabolic intensities. Of the subfamilies of shrews, the Crocidurinae (white-toothed shrews) are characterized by slightly elevated metabolism (relative to weight-predicted values), whereas the Soricinae (red-toothed shrews) have markedly elevated metabolic rates (Vogel, 1976; Lindstedt, 1980). Vogel (1980) reports that, among similar-sized shrews, life spans and gestation periods are shorter in the Soricinae; hence the products of metabolic rate and other periods may be similar in both groups. Physiological time may transpire even more rapidly than predicted for the Soricinae, thus all their biological periods may be affected. Additional examples of such interactions with metabolic rate were cited by McNab (1980).

The resting oxygen consumption of the lethargic sloths is 35 to 50 per cent lower than that predicted for their body mass. Their nerve conduction velocities and heart and respiratory rates are also lower, and the

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**Fig. 2. Pairs of Equations from Table 1 Combined to Form Dimensionless Ratios of Periods**

As the residual mass exponents are small, the resultant ratios are nearly the same for small and large mammals alike.
plasma half-life of thyroxin is longer than in similar-sized “typical” mammals (Goffart, 1971). Maximum longevity of two-toed sloths is 47 per cent longer than that to be predicted from Fig. 1. These data suggest that the physiological time scales of sloths have been prolonged in comparison with eutherian norms for this body-size range.

**Physiological Time and Metabolic Rate**

Since the observation was first made that basal metabolism scales disproportionately with body mass, various explanations of that observation have been proposed. Rubner (1883) measured metabolism in dogs ranging in size from 3 to 30 kg. He found that the metabolic rates of all the dogs were essentially constant if expressed per unit body surface area. This “surface law” survived until measurements of metabolic rate were made over a broad enough size range to deviate significantly from the 2/3 surface exponent. An exponent of 2/3 is, of course, now apparent from data not only for birds and mammals, but for a wide variety of vertebrates and invertebrates (Hemmingsen, 1960). Metabolic rate is only one example of a large number of physiological variables that are expressions of volume divided by time. Others, such as glomerular filtration rate, minute volume, and cardiac output also scale near $M^{1/3}$.

Attempts to explain the $2/3$ exponent have focused on the exponent itself, perhaps as if nature had selected the volume-rate scaling. An alternative explanation may be that the scaling of biological volume-rates is the result of the inevitable consequences of changes in body size, physiological time scales as $M^{1/3}$. Volumes and organs of capacity have relatively little opportunity to scale except in direct proportion to body mass ($M^{1/0}$) among animals of the same design (see above). Hence, volume-rates must scale as $M^{1/3}$:

$$\frac{\text{volume}}{\text{time}} \propto \frac{M^{1/0}}{M^{1/3}} = M^{1/6} \quad (7)$$

From the foregoing it is obvious that evolution of a change in body size will affect every aspect of physiology and morphology. Hence, one need not hypothesize that specific biological rates or volume-rates, e.g., metabolism, have been selected as isolated phenomena, but that the physiological time scale makes $M^{1/6}$ volume-rates inevitable.

**Physiological Time, Ecology, and Evolution**

An organism’s physiology must respond to its environment, including the temporal cycles in temperature and light imposed by the earth’s daily rotation and the revolutions of moon around earth and of earth around sun, all obviously independent of the animal’s size ($M^0$). The simplest coupling of animal and environment would therefore be size-independent, but physiological time scales are size-dependent ($M^{2/3}$). It is difficult to envision the natural selection of such an incompatibility; and this difficulty suggests to us that physiological time allometry is not a direct outcome of natural selection, but rather an inevitable consequence of something else. As Hill (1950) observed, the “physiological time-scale . . . has to compromise with the constant time-scale of the external world.” The compromise seems to us to be manifested in the biological clocks which synchronize animal and environment.

While circadian and circennial rhythms are useful for coupling physiological and environmental time scales, they cannot completely solve the endurance problems of small animals. As indicated previously, conservation of body plan requires that the capacity of organs be in linear proportion to body mass, so that the energy reserves stored in the volume of the gut or as body fat constitute an essentially constant fraction of body mass ($M^{0}$), while the rate of expenditure from these reserves is a function of $M^{1/3}$. Energy amount divided by rate of utilization, $M^0 + M^{1/3}$, gives endurance time as $\propto M^{1/3}$. Availability of food, or of conditions suitable for safe and effective foraging, is usually on a daily or seasonal clock, not a $M^{1/3}$ clock. If an animal’s day is an alternation of feeding and fasting, then problems could arise in maintaining energy balance, in acquiring and storing enough during the activity or availability phase to make it through the fast. The animals most likely to have problems are the smallest homeotherms, hummingbirds and shrews.

Hummingbirds, the more easily observable of these two groups, are pressed to use
all of the daylight available except at the highest latitudes of hummingbird distribution, where summer days are the longest (Calder, 1975, 1976b). The limitations of time for attaining energy balance are observed in broad-tailed hummingbirds (Selasphorus platycercus) of the Colorado Rockies, where cool temperatures put a significant price on thermoregulation even in the summer. When feeding time is lost during rainstorms, which keep the females on their nests, there is a high likelihood of a negative energy balance for the 24-hour period. This deficit necessitates entry into torpor to conserve energy (Calder and Booser, 1973; Calder, 1975), as if calculations had integrated the remaining energy reserves, the remaining time, and the rates of consumption at normal temperature and in hypothermic torpor.

While the environmental day is equally long, physiologically, for the fast-running shrews as it is for hummingbirds, the largely underground or undercover habitats of shrews should ameliorate the disparity between environmental and physiological time scales. The behavior pattern of stock-piling food that is observed in captive shrews (Lindstedt, unpub.) probably represents a mechanism to permit several sleep-wake cycles per day (Crowcroft, 1954), even though most of the foraging must be done at night.

The dimension of time has been neglected in more than one case of analysis of body size and ecology. For example, McNab (1963) correlated the home range of mammals with their body size, and by one means of calculation found home range proportional to $M^{0.63}$, an exponent statistically indistinguishable from that for metabolic rates ($\propto M^{0.75}$). This finding suggested that home range requirement was proportional to metabolic rate. Metabolic rate, however, has dimensions of power (energy per unit time). An area of home range can contain a certain amount of harvestable energy, but missing from the proportionality is a time function. Reanalyses (Buskirk in Calder, 1974; Harestad and Bunnell, 1979) showed that the home range of primary consumers was more closely related to $M^{1.0}$, not $M^{0.63}$. If $M^{1.0}$ is divided by metabolic rate, $M^{\kappa}$ (energy divided by power), we again get a physiological time scale, $\propto M^{\kappa}$.

Similarly, Fretwell (1972) proposed a model which limited minimum, maximum, and optimal body sizes of sparrows, as determined by the intersections of allometric curves for digested energy, captured energy, and metabolic rate (energy + time). When these have different dimensions, their crossings are merely a consequence of plotting scales and can have no biological significance. A model could have been constructed in a dimensionally correct manner by converting all lines to the same dimensions, either energy or power (through use of appropriate physiological time scales if such information were available: turnover time for gut contents, foraging time, etc.).

**Physiological Time and Longevity**

The longest period of an animal's physiology is its lifespan. Why is life-span scaling also approximately proportional to $M^{0.63}$? We examine three possible explanations which are not mutually exclusive:

1. Natural selection may have incorporated a safety factor of time beyond reproductive maturity that insures leaving progeny. (The animal that dies too soon has no "reproductive fitness").

2. Death comes when vital parts or their interaction no longer function adequately. ("The old ticker has just so many beats.")

3. Since there is a high correlation between longevity and brain mass, the larger the brain, the greater the control over homeostasis and consequently the longer life conditions can be precisely maintained.

**Safety Factor for Evolution of Life Span**

Natural selection acts on the reproductive fitness of an animal, and not on its life span directly. Once an individual is post-reproductive, it is out of the gene pool, and has no direct effect on evolution, except as it competes with or altruistically helps individuals still contributing to the gene pool. Thus, life span may be viewed as including a safety factor. The average animal must attain reproductive maturity with enough time for successful reproduction before biosenescence proceeds to a detrimental extent (Cutler, 1978). Thus, the expected life span would
exceed the reproductive period with some margin for error or accident, just as McMahon (1973) found that trees grew in height-diameter proportions with a standard safety factor four times the critical buckling dimensions. Vertebrate bones likewise have a tenfold safety factor over static load stresses (Schmidt-Nielsen, 1977).

We can examine the hypothesis of a “safety factor” for insuring reproduction by deriving, as a first approximation, an allometric equation for age at sexual maturity of \( n = 22 \) for mammals other than primates (using data in Hafez, Asdell, and Blandau, 1972, and typical species weights). The equation predicts (corr. coef. = 0.639, \( p < 0.001 \)) days to sexual maturity as

\[
t_{\text{sex mat.}} = 2.93 \times 10^5 M^{0.18}
\]

Sacher’s (1959) equation for maximum life span in captivity \( (6.10 \times 10^6 \times M^{0.20}) \) would confer a safety factor of \( 21 M^{-0.02} \) times sexual maturity (i.e., essentially independent of size). To be more realistic one should add gestation period plus time for a replacement number of newborn to attain independence, and then use mean life span in the wild (for which few data are available), rather than maximum longevity in captivity. Thus, the actual safety factor is considerably less than 21, but is, in all likelihood, independent of body mass.

Death due to Failure of Vital Organs

From Fig. 1 it appears that time scales are all approximately proportional to \( M^4 \). The parallelism of the various time-mass functions suggests, in effect, that independent of body mass, a life span consists of multiples of shorter time periods in the biology of the organism, \( 1.5 \times 10^9 \times M^{0.05} \) heartbeats, \( 0.33 \times 10^6 \times M^{0.06} \) breaths, \( 17 \times 10^6 \times M^{0.01} \) completed circulations of total blood volume, \( 36 \times 10^3 \times M^{0.04} \) g fat equivalent metabolized per kg body mass, \( 65 \times M^{0.06} \) times the gestation period, etc.

Of course, the times for circulatory, respiratory, and metabolic turnovers are basal figures; hence, allowing for activity, there are actually more heartbeats, breaths, and g fat catabolized. Maximum aerobic capacity, however, is apparently a fixed (size-independent) multiple of standard metabolism (Pro-thero, 1979; Taylor, et al. in press). Because of the size-independent metabolic expansion, the assumption that life span is a multiple of the shorter cycles of that life span is still valid when activity is considered. Thus a lifespan consists of about the same number of cardiac contractions whether the mammal is large or small. A parallel relationship obtains for birds, but the slower bird heart gets 38 per cent more basal heartbeats per life-span.

Consequently, in this explanation, one or more vital organs for body logistics eventually fail (see Harrison, 1978, for parallel at cellular level). Repair and resynthesis eventually reach limitations, and the heart, lungs, or kidney fail. Without proper \( CO_2/O_2 \) exchanges, provided by healthy cardiopulmonary systems, the body succumbs.

Statistical analysis of the causes of death in animals of a wide range in body size appears to be warranted. When one examines the proportional decline in physiological capacities with age in humans (Strehler, 1959), the steepest decline is in maximal breathing capacity. This decline, together with steep declines in vital capacity and cardiac index, could be regarded as a voluntary decline attributable to decreased activity associated with sedentary employment, amusement, and retirement. On the other hand, renal plasma and glomerular filtration rates show the greatest involuntary declines. Perhaps significant in this regard is the fact that kidney tissue has the highest in-vitro metabolic rate (per mg dry mass, von Bertalanffy and Pirozynski, 1951; Malzahn, 1974). If the concept of lifetime energy expenditure (Sacher, 1959, 1977; cf. metabolic curve, Fig. 1 of this study) has a bearing on the limits to maximum life span of the animal, could it not have a bearing on the functional life span of specific organs? Therefore we might expect the intensely active kidney to burn out first. The effects of aging on the kidney have been reviewed recently (Epstein, 1979), but tissue metabolic rates were not considered.

Life Span and Brain Mass

There are three correlations from which it may be concluded that life span is prolonged as a function of greater brain mass (Sacher,

First, the correlation between life span and brain mass is tighter (accounting for 79% of variance in life span) than the correlation between life span and body mass, which accounts for 60 per cent of the variance in life span (Sacher, 1959). Thus, brain mass by itself is a better predictor of life span than is body mass.

Second, our species has a maximum life span about four times what the mammalian equation predicts from body mass, but our brains are about six times as heavy as predicted from body mass of a typical eutherian mammal (Sacher, 1959; Fischer, 1968). Hence our maximum life span is only twice that predicted from brain mass (Sacher, 1959). However, a larger brain, together with manual dexterity, has provided tools, strategies, and cultural practices which have improved the longevity of Homo sapiens compared to that of our wild ancestors. Cutler (1978) suggested that maximum life span potential may be related to ability to learn and teach, abilities enhanced by greater brain size.

The third argument is one of reason rather than quantitative empiricism. The brain is the master regulator or controller of physiological processes through neural and hormonal mediation. The more precise the regulation, the better the "milieu interieur" for the preservation of life. A larger brain has more cells, more capacity to regulate, and therefore to preserve.

Although a good correlation certainly suggests a causal relationship, it does not establish that such a relationship exists. For example, spleen mass and adrenal mass are also better predictors (higher correlation coefficients) of lifespan than is body mass (Calder, 1976a; Economos, 1980). We must, therefore, look carefully at the relative strengths of correlations.

We argue that the reason for the better prediction of life span from brain mass is that brain mass has less variability than body mass. The body mass of any individual or species can vary widely, depending upon its energy balance situation. If food intake exceeds metabolic demands, fat is stored and body mass increases; if feeding does not meet those demands, stores are depleted, and emaciation can result.

In contrast, the brain is limited in size by a bony exoskeleton, the sutures of which are fused in the adult. Thus, the brain is a less variable index of body size than is body mass. Table 2 shows that within species, or when the mammals are treated together, the coefficient of variation for brain mass is less than half that for body mass. It becomes obvious why brain mass can give a better correlation with life span than can body mass.

Finally, we must consider the relative influences of central control capacity and the requirements and characteristics of body size in limiting life span. Brain mass scales quite reliably as $M^{0.67}$ for mammals (Sacher, 1959; Jerison, 1961) and $M^{0.60}$ for birds (Alexander, 1971). Life span is proportional to $M^{0.20}$ for mammals (Sacher, 1959) and $M^{0.18-0.26}$ for birds (Lindstedt and Calder,

<table>
<thead>
<tr>
<th>Species</th>
<th>n</th>
<th>x</th>
<th>SD</th>
<th>CV*</th>
<th>x</th>
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<td>-</td>
<td>-</td>
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<td>-</td>
<td>-</td>
<td>0.12-0.15</td>
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<td>61.5</td>
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<tr>
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<td>646</td>
<td>75.8</td>
<td>0.12</td>
<td>452</td>
<td>148.3</td>
<td>0.33</td>
</tr>
</tbody>
</table>

n, sample size.
* Coefficient of variation.

a From Yahlokov, 1966.
b Calculated from data in Quiring, 1950.
SD, standard deviation.
A ratio of life span to brain mass expressed as common allometric powers of body mass yields:

\[
\text{life span/brain mass} \propto \frac{M^{0.20}}{M^{0.67}} = M^{-0.47} \tag{9}
\]

Therefore, the amount of gain in maximum life span per unit increase in brain mass actually decreases as the animal's size increases. If an increase in brain size directly increases life span, why does the proportionate effect decrease with size? Although the residual mass exponent would be reduced to \(\approx -0.25\), the case for brain-dependence would not be improved qualitatively if brain surface, rather than mass, were to be employed.

Additional insight may be gained from the following comparison. On a basis of equal body mass, birds have smaller brains but longer life spans than mammals. The ratios of maximum life span in captivity (years) to brain mass are as follows:

- mammals: \(\frac{11.6}{11.3} M^{0.20} = 1.03 M^{-0.47} \tag{10}\)
- birds: \(\frac{28.3}{8.2} M^{0.19} = 3.45 M^{0.41} \tag{11}\)

Accordingly, a larger brain mass per se has not endowed mammals with greater longevity.

As regards homeostatic regulation, birds can tolerate larger ranges in body temperature, plasma osmotic concentrations, and blood pH than most mammals (Braun and Dantzer, 1972; Calder and Schmidt-Nielsen, 1966, 1968; Calder and King, 1974). Consequently, we should reconsider the hypothesis that longer life necessitates finer homeostatic regulation.

Correlations have been improved when life span is expressed as a multiple regression on body and brain masses, index of cephalization, etc. (Sacher, 1959, 1977), but the fact remains that this relationship is no proof that large brain and (or) large body and (or) such indices are either cause or explanation for longevity.

We doubt that it is brain size, as such, that sets a limit to animal longevity. We interpret the allometry of longevity as a further manifestation of Stahl's "synchronism" of times (Stahl 1962, 1963). Excellent correlations of life span and brain mass probably reflect the low variance in brain masses rather than express a causal relationship. We consequently suggest that answers to gerontological problems will probably lie in other theories of aging than those based upon relative cephalization.

If there is a lesson to be drawn from comparative gerontology, it is not a suggestion of how to increase the life span, which is itself a second- or third-stage consequence of metabolic logistics. Life ends when these vital functions fail. The goal of gerontology should not be to prolong life in quantity of years, but to prolong the living of a full life of unimpaired function during the allotted \(M^a\) span of years.

We live in deeds, not years; in thoughts, not breaths;
In feelings, not in figures on a dial.
We should count time by heart-throbs.
He most lives
Who thinks most—feels the noblest—acts the best.
—Philip James Bailey, A Country Town

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