Homework 6

1) An annual plant produces seeds at the end of the summer. A proportion $\sigma$ survive one winter, and a proportion $\alpha$ of these germinate the following spring. Of the remainder, a proportion $\sigma$ survive a second winter, and a proportion of these germinate the spring following this winter. None can germinate later than this. Germinated plants produce new seeds.

a) If $P_n$ is the total population – seeds and plants – show that

$$P_{n+2} = \alpha \sigma \gamma P_{n+1} + \beta (1 - \alpha) \sigma^2 \gamma P_n$$

b) What do $\gamma$, $\beta$ represent biologically?

c) Use the method we used for the Fibonacci series to find the dominant eigenvalue $\lambda$.

d) Show that $\lambda$ is greater than one if and only if

$$\gamma > \frac{1}{\alpha \sigma + \beta (1 - \alpha) \sigma^2}$$

e) Recall $R_0$ as defined in class is $\sum l_j F_j$. What is $R_0$ for this population?

f) Show that $\lambda > 1$ if and only if $R_0 > 1$.

2) The California spotted owl ranges from northern to southern California, in old growth forest. The owl matures at age 2 (that is, in its third year of life). Survivorship in the first year of life has been estimated to be $s_0 = 0.3$; yearling subadults and adults of all ages have approximately the same survival rates of $s_1 = 0.75$, and female fecundity is $b = 0.3$ (Noon and McKelvey 1996: *Annual Review of Ecology and Systematics*, **27**: 135-162). Construct a matrix for this population and find the dominant eigenvalue and stable eigenvector (normalized so that the sum in all age classes is one).
3) Assume you have a Leslie model, 

\[ F = \begin{pmatrix} s_0 F_1 & s_0 F_2 & s_0 F_3 \\ s_1 & 0 & 0 \\ 0 & s_2 & 0 \end{pmatrix} \]

a) Find an equation for the eigenvalues \( \lambda \), in terms of the \( l_j, F_j \).

b) Use the above to find an expression for 

\[ \frac{\partial \lambda}{\partial s_2} \]

c) Now check your answer in b) by considering the matrices (assume \( s_0 = 1 \))

\[ F(.8) = \begin{pmatrix} 0 & 4 & 2 \\ .4 & 0 & 0 \\ 0 & .8 & 0 \end{pmatrix} \quad F(.81) = \begin{pmatrix} 0 & 4 & 2 \\ .4 & 0 & 0 \\ 0 & .81 & 0 \end{pmatrix} \]