4) (10 points) Let $f(x) = (1 + x)^{\frac{2}{3}}$; let $a = 0$.
   a) Find $t(x)$.
   b) Use $t(x)$ to approximate $(.97)^{\frac{2}{3}}$.

\[
\begin{align*}
   f'(a) &= 1 \\
   f'(x_1) &= \frac{2}{3} (1 + x_1)^{-\frac{1}{3}} \\
   f'(0) &= \frac{2}{3} \\
   t(x_1) &= 1 + \frac{2}{3} x_1
\end{align*}
\]

\[
(1 + x)^{\frac{2}{3}} = .97^{\frac{2}{3}} \\
1 + x = .97 \\
x = -.03
\]

\[
t(x) = 1 + \frac{2}{3} (-.03) = 1 + 2(-.01) = 1 -.02 = .98
\]
5) (40 points) A point $P(x, y)$ is on the hyperbola $xy = 1$. What $x, y$ make $P$ closest to the origin, $(0, 0)$?

a) Write the objective function in terms of the variables.

b) Write the constraints.

c) Use b) to eliminate one variable from a).

d) Find $x, y$.

c) $l^2 = x^2 + y^2$

d) $xy = 1 \quad x \neq 0, y \neq 0$

c) $y = \frac{1}{x}$; $l^2 = x^2 + \frac{1}{x^2}$

d) $2l \frac{dl}{dx} = 2x - \frac{2}{x^2} = \frac{2x^2 - 2}{x^2}$.

$\frac{dl}{dx} = 0$ when $2x^2 - 2 = 0$ $x^2 = 1$ $x = \pm 1$.

$y = \frac{1}{x}$

$x = 1$ $y = \frac{1}{1} = 1$ \(\{(1, 1)\}\)

$x = -1$ $y = \frac{1}{-1} = -1$ \(\{(-1, -1)\}\)

*Show this is a minimum.*

Since it is an open interval, use $\liminf$ is optimal if only one $y$.

| $1^{st}$ closest test | $0$ | $1$ |

$\liminf$ so given
B) (35 points) Calculus is finally over & you drop your calculator over the edge of the Grand Canyon. 6,400 feet below, you see it hit the ground. How fast is it going?

Remember: no memorized formulas from other classes; keep units throughout the problem (except for \( g \); you can put those units in at the end).

\[
\begin{align*}
\mathbf{h}(t) &= -\frac{1}{2}gt^2 + v_0t + l_0 \\
v(t) &= -gt + v_0
\end{align*}
\]

\[\text{Condition: } v_0 = 0\]

from \(6,400 \text{ ft} \): \(l_0 = 6,400 \text{ ft}\)

\[
\begin{align*}
h \text{ + ground: } \mathbf{h}(t) &= 0 \\
-\frac{1}{2}gt^2 + 6,400 \text{ ft} &= 0
\end{align*}
\]

\[
\begin{align*}
\frac{1}{2}gt^2 &= 6,400 \text{ ft} \\
t^2 &= \frac{6,400 \text{ ft}}{g} \\
t^2 &= \left(\frac{6,400 \text{ ft}}{32 \text{ ft/sec}^2}\right) \\
t^2 &= \left(\frac{1,000 \text{ ft}}{\text{sec}^2}\right) \\
t &= \left(\frac{32 \text{ ft}}{\text{sec}^2}\right)^{\text{sec}} = 20 \text{ sec}
\end{align*}
\]

\[
\begin{align*}
v &= -gt = -\frac{6,400 \text{ ft}}{20 \text{ sec}} \\
&= -320 \text{ ft/sec}
\end{align*}
\]
1) (30 points) Haloperidol is an antipsychotic; the drug decays exponentially in the blood. The maximum recommended blood concentration is 16mg/L; the minimum effective concentration is 4mg/L. If you administer the maximum dose, and need to administer another dose 22 hrs later, what is the relaxation time of Haloperidol? (You can leave the logs in your answer, but be sure to track units).

\[ H(t) = H_0 e^{-kt} \]

\[ H_0 = 16 \text{ mg/L} \quad H(22 \text{ hrs}) = 4 \text{ mg/L} \]

\[ 16 \text{ mg/L} e^{-kt} = 4 \text{ mg/L} \]

\[ e^{-kt} = \frac{1}{4} \]

\[ -k(22 \text{ hrs}) = -\ln 4 \]

\[ \frac{22 \text{ hrs}}{\ln 4} = \frac{1}{k} \]
Let $f(x) = (1 + x)^2$, $a = 0$

a) Find $f(a)$, $f'(x)$, $f'(a)$, $t(x)$

b) Use $t(x)$ to compute $\sqrt{49}$

c) The actual value is .7. How large is the error?

\begin{align*}
\text{a) } f(x) &= f(0) = (1 + 0)^2 = 1 \\
&= f(0) = \frac{1}{2} (1 + 0)^{-\frac{1}{2}} = \frac{1}{2} \\
&= f'(0) = \frac{1}{2} (1 + 0)^{-\frac{1}{2}} = \frac{1}{2} \\
&= t(x) = f(0) + f'(0)(x-0) = 1 + \frac{1}{2}x
\end{align*}

\begin{align*}
\text{b) } \sqrt{1 + x} &= \sqrt{49} \\
&= 1.149 - 1 = .149 - 1 = \frac{4}{25} - 1 = \frac{25 - 25}{25} = .95
\end{align*}

\begin{align*}
\text{c) error } &= |f(x) - t(x)| = 1.7 - .9595 = 1.0495
\end{align*}