1) A spotlight is 100 feet away from a curtain; an actor is walking along the curtain at 1 foot per second. At the instant that she is 25 ft along the curtain, how fast does the spotlight have to rotate so as to keep her in the spotlight?

1)(5 points) What quantities are changing? Write your answer in words and with variables.
2)(10 points) Draw a diagram relating the variables in 1), and label it with the variables.
3)(10 points) Find a formula relating the variables in 1).
4)(5 points) Differentiate both sides of the formula.
5)(10 points) Find the numerical values of the quantities in 4)

2) The evil Dr. Drakken has just blasted off in his rocket, going straight up into the air at 600 mph. Our superheroine, Kimmie, is sitting in her Jaguar XKE. She immediately speeds up, driving towards the launch site at 70 mph. The instant she is 30 miles from the site, Drakken is 40 miles up. She looks up at the rocket: how fast is the angle between her and the rocket changing, at that instant?

1)(5 points) What quantities are changing? Write your answer in words and with variables.
2)(10 points) Draw a diagram relating the variables in 1), and label it with the variables.
3)(10 points) Find a formula relating the variables in 1).
4)(5 points) Differentiate both sides of the formula.
5)(10 points) Find the numerical values of the quantities in 4)
14.4) \( a = \) distance of actor "clown" curtain.

more precisely, drop a perpendicular line from
spotlight to curtain. \( a \) is the distance from
the actor to that perpendicular.

\( \theta = \) angle spotlight makes with perpendicular.

1.2) \[
\begin{align*}
\tan \theta &= \frac{a}{100 \text{ ft}} \\
\sec^2 \theta \frac{d\theta}{dt} &= \frac{1}{100 \text{ ft}} \frac{da}{dt}
\end{align*}
\]

1.5) \[
\frac{da}{dt} = 1 \text{ ft/sec}
\]

\[
\sec^2 \theta = 1 + \tan^2 \theta = 1 + \left( \frac{a}{100 \text{ ft}} \right)^2
\]

\( a = 25 \text{ ft} \) so \( \sec^2 \theta = 1 + \left( \frac{25}{100 \text{ ft}} \right)^2 = 1 + \frac{1}{16} = \frac{17}{16} \)

solve for \( \frac{da}{dt} \)

\[
\frac{1}{100 \text{ ft} + \sec \theta} \cdot \frac{1 \text{ ft}}{\sec} = \frac{1}{1700} \text{ sec}
\]

\( \frac{1}{1700} \text{ sec} 

\text{note units!!} \)
21) Chongny: 600 m/hr.

\( h = \text{height of Drakken's rocket} \) = 70 m/hr.

\( k = \text{distance of Kermis Jaguar from launch site} \)

\( \theta = \text{angle between KP and rocket} \)

\[ \begin{align*}
\text{Diagram:} \\
\frac{h}{\theta} &= \frac{\text{adj}}{\text{opp}} = \frac{h}{k} \\
\frac{dh}{dt} &= \text{constant} \text{ so } \frac{\Delta h}{\Delta t} = \frac{600}{1} = 600 \text{ m/hr} \\
\frac{dk}{dt} &= \text{decreases so } \frac{\Delta k}{\Delta t} = -10 \text{ m/hr} \\
\text{sec}^2 \theta \frac{d\theta}{dt} &= \frac{k}{dr} - h \frac{dk}{dt} \\
&= \frac{k^2}{k^2} \\
&= 1 + \text{tan}^2 \theta = 1 + \left( \frac{h}{k} \right)^2 = 1 + \left( \frac{40 \text{ m}}{20 \text{ m}} \right)^2 = 1 + \frac{16}{4} = 25
\end{align*} \]

25) \( h = \text{height = 40 miles} \)

\( k = \text{distance to site = 30 miles} \)

\( \frac{dh}{dt} = \text{increasing so } \frac{\Delta h}{\Delta t} = 600 \text{ m/hr} \)

\( \frac{dk}{dt} = \text{decreasing so } \frac{\Delta k}{\Delta t} = -10 \text{ m/hr} \)

\( \text{sec}^2 \theta = 1 + \text{tan}^2 \theta = 1 + \left( \frac{h}{k} \right)^2 = 1 + \left( \frac{40 \text{ m}}{20 \text{ m}} \right)^2 = 1 + \frac{16}{4} = 25 \)

\[ \begin{align*}
\frac{\text{sec}^2 \theta}{dt} &= \frac{k}{dr} - h \frac{dk}{dt} \\
&= \frac{k^2}{k^2} \\
&= 1 + \text{tan}^2 \theta = 1 + \left( \frac{h}{k} \right)^2 = 1 + \left( \frac{40 \text{ m}}{20 \text{ m}} \right)^2 = 1 + \frac{16}{4} = 25
\end{align*} \]

\[ \begin{align*}
\frac{25}{a} \frac{\text{d} \theta}{\text{dt}} &= \frac{\text{sec}^2 \theta}{(30 \text{ m})^2} \\
&= \frac{(180)(100) \text{ m}^2}{\text{hr}} + 2 \cdot (100) \text{ m}^2 \text{ hr} \\
&= \frac{208}{9} \text{ hr}^2 = \frac{1}{9} \text{ hr} \\
\frac{25}{a} \frac{\text{d} \theta}{\text{dt}} &= \frac{208}{9} \text{ hr} \quad \frac{\text{d} \theta}{\text{dt}} = \frac{208}{9} \text{ hr} \frac{1}{25} \text{ hr} = \frac{208}{25} \text{ hr}
\end{align*} \]
4. (30 points) You’ve finally escaped from Gulag 408. You are in a balloon going straight up (after all, this is a calculus gulag). A minute after you leave ground, you are 40 ft up and continuing to rise at 100 ft per minute.

At the same time (a minute after the balloon leaves ground), security guards are standing still thirty feet from where the balloon left the ground. How fast is the angle of elevation between you and the security guards changing?
a) (10 points) What quantities are changing? Write your answer in words and with variables.
b) (5 points) Draw a diagram relating the variables in a), and label it with the variables.
c) (5 points) Find a formula relating the variables in a).
d) (5 points) Differentiate both sides of the formula.
e) (15 points) Find the numerical values of the quantities in d). Be sure to keep units in your computation.

\[ h = \text{height of balloon} \quad \theta = \text{angle of elevation} \]

\[ \tan \theta = \frac{h}{30} \]

\[ \sec^2 \theta \frac{d\theta}{dt} = \frac{1}{30} \frac{dh}{dt} \]

\[ \frac{dh}{dt} = 100 \text{ ft/min} \]

\[ \sec^2 \theta = 1 + \tan^2 \theta = 1 + \left( \frac{h}{30} \right)^2 = \left( \frac{100}{30} \right)^2 = \frac{25}{9} \]

\[ \frac{dh}{dt} = 30 \text{ ft/min} \]

\[ \sec^2 \theta \frac{d\theta}{dt} = \frac{1}{30} \frac{dh}{dt} \]

\[ \frac{25}{9} \frac{d\theta}{dt} = \frac{1}{30} \cdot 100 \text{ ft/min} \]

\[ \frac{d\theta}{dt} = \frac{9}{25} \cdot \frac{1}{100} \text{ ft/min} \]

\[ = \frac{30}{25} \text{ min} \]

\[ = \frac{6}{5} \text{ min} \]

\[ \boxed{\text{(optional carrying unit)}} \]
4) (40 points) You knew it all along—a flying saucer is lifting straight up into the air, at a steady 50 ft/min, carrying Kathy’s brain to the galaxy Elvis. Cheering students stand watching, 100 ft away from the liftoff site. How fast is the distance between the students and the saucer changing, when the saucer is 100 ft in the air??

a) (10 points) What quantities are changing? Write your answer in words and with variables.

b) (5 points) Draw a diagram relating the variables in a), and label it with the variables.

c) (5 points) Find a formula relating the variables in a).

d) (5 points) Differentiate both sides of the formula.

e) (15 points) Find the numerical values of the quantities in d). Be sure to keep units in your computation.

\[
a) \quad h = \text{height of saucer} \quad s = \text{dist between saucer and students}.
\]

\[
b) \quad \sqrt{s^2 + h^2} \quad c) \quad s^2 = h^2 + (100 \text{ ft})^2
\]

\[
d) \quad 2s \frac{ds}{dt} = 2h \frac{dh}{dt}.
\]

\[
e) \quad h = 100 \text{ ft} \quad s = ? \quad s^2 = h^2 + (100 \text{ ft})^2 = (100 \text{ ft})^2 + (100 \text{ ft})^2 \quad s = 100 \sqrt{2} \text{ ft}
\]

\[
\frac{dh}{dt} = 50 \text{ ft/min}
\]

\[
\frac{ds}{dt} = \frac{h}{s} \frac{dh}{dt} = \frac{100 \text{ ft}}{\sqrt{2} \cdot 100 \text{ ft}} \cdot 50 \text{ ft/min}.
\]

\[
= \frac{50}{\sqrt{2}} \text{ ft/min}
\]