I'm going to look at the "involute of the circle"

\[ x^2 + y^2 = 1 \] which looks roughly like

Note the equation is unchanged if \(0\)
take \(x \rightarrow -x\) or \(y \rightarrow -y\). This tells me
that all four quadrants have the same area so
area = \(4 \times \text{area in } 1^{st} \text{ quadrant.} \)

Now we'll parameterize \( \theta \);
\[ x = \cos^3 \theta, \quad y = \sin^3 \theta . \]
Then \( \frac{dx}{d\theta} = \cos^2 \theta, \quad \frac{dy}{d\theta} = 3 \sin^2 \theta \cos \theta \)

\( x^2 + y^2 = 1 \) \( \Rightarrow \) \( \cos^6 \theta + \sin^6 \theta = 1 \)

\( \text{Cheer! The first quadrant is } 0 < \theta < \frac{\pi}{6}, \text{ so the area is} \)

\[ 4 \int_0^{\frac{\pi}{6}} \frac{1}{2} y(\theta) x'(\theta) d\theta \]

\( y(0) = \sin^3 \theta, \quad x(0) = \cos^3 \theta, \quad x' = 3 \cos^2 \theta \cos \theta \)
so \( \frac{dy}{d\theta} = -3 \sin^2 \theta \cos^2 \theta \sin \theta \)

old trick: \( \cos^2 \theta = 1 - \sin^2 \theta \) so \( \frac{dy}{d\theta} \)

an integrand of \( 3 \sin^4 \theta - 3 \sin^6 \theta \), will factor out \( \sin^2 \theta \)
and put it with the \( y \) at the end of the problem.

When we integrate, keep in mind that anything of
the form \( \left[ \cos^n \theta \sin^m \theta \right]^{\frac{\pi}{6}} \)

\( \sin \theta = 0 \) at \( \theta = 0 \). So, when we get a lot of muck,
most of it you can go to yr

\[ \int_0^{\pi/2} \sin^n \theta \, d\theta - \int_0^{\pi/2} \sin^{k-1} \theta \cos \theta \, d\theta \] do the high power

first, and use recursion

\[ \int \sin^k x \, dx = -\sin^{k-1} x \cos x + \frac{k-1}{k} \int \sin^{k-2} x \, dx \]

and for us, the whole \(-\sin^{k-1} x \cos x \to 0 \) at \(0, \pi/2\).

so

\[ \int_0^{\pi/2} \sin^n \theta \, d\theta - \int_0^{\pi/2} \sin^{n-1} \theta \, d\theta = \int_0^{\pi/2} \sin^n \theta \, d\theta - \frac{1}{2} \int_0^{\pi/2} \sin^{n-2} \theta \, d\theta \]

\[ = \frac{1}{6} \int_0^{\pi/2} \sin^{n+2} \theta \, d\theta = \frac{1}{6} \int_0^{\pi/2} \sin^{n-1} \theta \, d\theta \]

\[ = \frac{1}{6} \cdot \frac{1}{2} \cdot \left[ \int_0^{\pi/2} \sin^2 \theta \, d\theta \right] = \frac{1}{16} \left[ \theta \left. \right|_0^{\pi/2} \right] = \frac{1}{32} \pi \]

Now remember that factor of 12 = 4.3?

SO area = 4.3 \cdot \frac{1}{32} \pi = \frac{3}{8} \pi.

\( \theta \)

BTW without the parameterization we'd be doing

\[ x^{2/3} + y^{2/3} = 1; \quad y^{2/3} = 1 - x^{2/3}; \quad y = (1 - x^{2/3})^{3/2} \]

so I have to do

\[ 4 \int_0^1 (1 - x^{2/3})^{3/2} \, dx. \text{ Good luck } \theta \]