1) Let \( R \) be the region bounded by the curves \( y = x^2 \), \( y = x+2 \).

2) Sketch \( R \), and find \( a, b, c, d \).

\[
\begin{align*}
\text{a, b are the farthest left, right points in the region.} \\
\text{They occur when } y = x^2 = x+2 \\
\text{or } x^2 - x - 2 = 0 \text{ or } (x-2)(x+1) = 0 \\
\text{so } x = -1, x = 2. \quad a = -1, b = 2
\end{align*}
\]

\( c, d \) are the highest, lowest points in \( R \).

The lowest point is \( y = 0 \) so \( c = 0 \). The highest point occurs when \( x = 2 \). The \( y \) value at \( x = 2 \) is \( y = x^2 = 4 \). So \( d = 4 \).

3) Write \( R \) as a type 1 or type 2 region, and cartesian.

\text{Type 1: We already knew } a = -1, b = 2 \text{ so } -1 \leq x \leq 2. \text{ The type 1 region we need}

\text{is bottom curve } y = g \text{ (x). The picture shows that } y = x^2 \text{. Also a top curve, } y = 2 \text{ (x1 = x+2).}

\[
\begin{align*}
\text{So: } R = \{(x,y) | -1 \leq x \leq 2, x^2 \leq y \leq x+2\} \\
\int_{-1}^{2} \int_{x^2}^{x+2} f \, dy \, dx
\end{align*}
\]

\text{Note: We can't write } R \text{ as a single type 2 region.}
If we could, we'd have $y_L(x) \leq x \leq y_U(y)$.
So didn't need one function to describe the left curves.

\[ -y = x^2 \]
\[ -y = x^3 \]

Good thing then they let us choose whether to do type I or type II.

Example 2)

Let \( R = \{(x,y) \mid 1 \leq x \leq 2; \ x \leq y \leq x^2\} \)

a) Sketch the region.

\[ \int_{1}^{2} \int_{x}^{x^2} f(x,y) \, dy \, dx \]

This is easy -- \( R \) is a type I region.!!

Note again writing it as a single type I region

would be impossible.
Example 3

a) Let \( R = \{(x,y) \mid -1 \leq y \leq 1, 0 \leq x \leq 1-y^2\} \).

Sketch \( R \):

- \( y = 1 \) (top boundary)
- \( y = -1 \) (bottom boundary)
- \( x = 1-y^2 \) (right boundary)
- \( x = 0 \) (left boundary)

b) Write \( R \) as a type I region.

For \( x \leq x \leq b \), where \( a, b \) are furthest left, furthest right points in \( R \).

- \( x \) cannot go further left than \( 0 \), so \( a = 0 \).
- The rightmost \( x \) can go is the vertex of the parabola, when \( y = 0 \). So \( x = 1-0 = 1 \). So, \( 0 \leq x \leq 1 \).

Now I need \( y \leq y \leq y \) for top and bottom curves. They are clearly the parabola, since \( x = 1-y^2 \), \( y = 1-x \) and \( y = \pm \sqrt{1-x} \).

- Top: \( y = \sqrt{1-x} \)
- Bottom: \( y = -\sqrt{1-x} \)

So: \( R = \{(x,y) \mid 0 \leq x \leq 1, -\sqrt{1-x} \leq y \leq \sqrt{1-x}\} \).