The first essential in chemistry is that you should perform practical work and conduct experiments . . .
- Jabir Ibn Hayyan, c.800 CE

Chapter 1: Numbers

Section 9: Data: Introduction

It’s likely that anyone who’s taken a physics, chemistry or biology course will think of an experiment which produces data as something like Figure 100, graphed as in Figure 101, and finally manipulated mathematically, as in Figure 102. But all of what we now think of as science is historically very recent; for over a thousand years after Mesopotamian or Greek scholars, Europeans trying to understand the universe were opposed to experiments, wrote polemics against using numbers, and had no idea of data analysis (see p113).

In this section, and in the notes, we’ll look at how the Western view of science was constructed, and we’ll start with one example: how scientists came to believe that the atmosphere is like an ocean of air, with us at the bottom of that ocean. And like water, air has weight, which exerts pressure.

The idea of atmospheric pressure gave a clear answer to a Renaissance paradox: how do pumps work? In Galileo’s Dialogues Concerning Two New Sciences (1638), one of the characters mentions the problem:

This pump worked perfectly so long as the water in the cistern stood above a certain level; but below this level the pump failed to work. When I first noticed this phenomenon I thought the machine was out of order; but the workman whom I called in to repair it told me the defect was not in the pump but in the water which had fallen too low to be raised through such a height; and he added that it was not possible, either by a pump or by any other machine working on the principle of attraction, to lift water a hair’s breadth above eighteen cubits; whether the pump be large or small this is the extreme limit of the lift.

A modern explanation is that lifting the handle of a pump evacuates air from the tube of the pump, and atmospheric pressure forces water up the tube. Since atmospheric pressure is finite, it can only lift the water so far.

But the term ‘evacuate’ implied a vacuum had been created. Since Aristotle said a vacuum was impossible, the preferred explanation was that the eighteen cubit limit was due to a force resisting the
formation of a vacuum. The history of these discussions is long and complex; see C. Webster, The Discovery of Boyle’s Law, and the Concept of the Elasticity of Air in the Seventeenth Century, Archive for History of Exact Sciences Vol. 2, No. 6 (31.12.1965), pp. 441-502.

Eighteen cubits is over thirty feet, and there are many kinds of pumps; as Webster (above) remarks, “…such evidence was confused and unreliable, since real or imaginary pumps of other designs were in principle capable of lifting more than eighteen cubits of water.”

What was needed was evidence that was not confusing and was reliable; this was provided by Evangelista Torricelli (1608-1647). He knew Archimedes’ work on water pressure: the pressure on a submerged object was proportional to the weight of the water above it, which would be the density of the water times the depth of the water (the proportionality factor was the acceleration of gravity, a quantity completely unknown at this time). Torricelli also knew mercury was denser than water, therefore if he used mercury rather than water, the height the liquid mercury could be drawn would be much less than thirty feet.

In addition, he realized he could get rid of pumps entirely: Figure 103 illustrates his idea. When the tube is tipped over into the mercury, the level of mercury dropped to about 76 centimeters. Torricelli claimed that the weight of the atmosphere, pushing on the mercury in the dish, pushes mercury up the tube. It was a great demonstration (actually carried out by his student, Vincenzo Viviani). No complicated pumps needed, no thirty feet of water. Easily understood; reliable because anyone could repeat the experiment. You don’t have to depend on ancient authority or even Toricelli’s authority.

Interpretations still varied. If the mercury had filled the entire tube, but had now dropped by 24 cm, the top of the tube must contain a vacuum. The mathematician/scientist/philosopher Blaise Pascal argued, as did others, that mercury was not forced up the tube by atmospheric pressure, but drawn up by nature’s resistance to the formation of a vacuum. Standoff.

But Torricelli had one more demonstration in mind. Take a surface submerged in water. Archimedes taught that the higher above the surface one went, the less pressure there would be. If the height of mercury in the tube represented the pressure of the atmosphere at the surface of the earth, the higher above the surface of the earth one went, the lower the mercury in the tube would be. Archimedes expressed this as a proportion: the change in mercury level could be computed relative to the height above the surface of the earth. This computation is important to the theory, because it eliminates counter-
arguments like ‘maybe something else caused the change.’ Maybe, but why would the change match Archimedes so well? This kind of experimental technique was new in its time; see p118.

An apparatus was carried up an actual mountain by, of all people, Pascal’s brother-in-law, and the change in the height of mercury was exactly what was expected. Pascal was now convinced: air has weight and it is that weight which drives mercury up the tube.

Alas, the change in height of mercury turned out to match the Archimedean prediction too exactly, raising questions whether the experiment had actually been done. Nonetheless, Torricelli’s work inspired others across Europe to theorize and experiment with air. Isaac Beeckman in Holland compared the air surround the earth to a large sponge; Renee Descartes compared it to the fleece of wool. More significantly, Marin Mersenne in France actually experimented on air, finding that it could be compressed to 1/1000 of its original volume, and then expanded again. See C. Webster, The Discovery of Boyle’s Law, and the Concept of the Elasticity of Air in the Seventeenth Century, Archive for the History of Exact Sciences, 2(6) 1965.

This was the situation when Robert Boyle and his assistant Robert Hooke began work. Although the two published over forty experiments on air pressure, we’ll look at only one:

Divers ways have been proposed to show both the Pressure of the Air, as the Atmosphere is a heavy Body, and that Air, especially when compressed by outward force, has a Spring that enables it to sustain or resist equal to that of as much of the atmosphere, as can come to bear against it, and also to show, that such Air as we live in, and is not condensed by any human or adventitious force, has not only a resisting Spring, but an active Spring (if I may so speak) in some measure, as when it distends a flaccid or breaks a fullblown bladder [. . .].

Robert Boyle, New Experiments, Physico-mechanicall, touching the Spring of the Air, LONDON, Printed by Miles Flesher for Richard Davis, Bookseller in Oxford, MDCLXXXII.

Boyle’s ‘spring of the air’, can be compared to an actual spring, an automobile shock absorber, Figure 104. This shock absorber uses a metal spring which contracts when pushed down, and returns to its original shape when left free.

The point of Boyle’s comment is that air behaves in the same way (see Figure 105). We’ll look at how Boyle and Hooke’s took the issue beyond analogies with sponges and fleece, to prove the Springe of Air. (Hooke later found the general law governing the behavior of springs: Hooke’s Law. We’ll explore this later.)
Figure 106 shows something like what Boyle might have used; a bent tube, sealed off on the left, open to the atmosphere on the right. Also, “That the tube being to (sic) tall that we could not conveniently make use of it in a Chamber, we were fain to use it on a pair of Stairs, which yet were very lightsome, the tube being for preservations sake by strings so suspended, that it did scarce touch the box […] .” Robert Boyle, *New Experiments, Physico-mechanical, touching the Spring of the Air*, cited above. His tube was a good deal larger than the one in Figure 106.

But: if you just pour mercury into the tube, it compresses the air on the left. Boyle jiggled the tube to equalize the pressure on both sides. He then poured mercury in on the open side, and noted the height of the air on the left, as well as the mercury on the right, columns $A$ and $B$ in the table below.

These heights should be proportional to the volumes of each; for the mercury, that would be the pressure exerted on the compressed air. So the numbers recorded would really be volume and pressure. With one fudge: the tube was open to the air, so atmospheric pressure needed to be accounted for, in column $D$.

![Figure 106: Boyle’s Experiment](image)

Modern version of the experimental apparatus. The tube contains mercury; it is open on the right side, but sealed on the left. Adding mercury on the right compresses the air on the left.

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**Table of the Condenation of the Air**

**AA.** The number of equal spaces in the shorter leg, that contained the same parcel of Air diversely extended.

**B.** The height of the Mercurial Cylinder in the longer leg, that compris’d the Air into those dimensions.

**C.** The height of a Mercurial Cylinder that counterbalanc’d the preſsure of the Atmosphere.

**D.** The Aggregate of the two last Columns $B$ and $C$, exhibiting the preſsures sustained by the included Air.

**E.** What that preſsure should be according to the Hypothesis, that fipopos the preſsures and expansions to be in reciprocal proportion.
When Boyle showed these numbers to his friends, several people, including Hooke, thought that pressure and volume seemed to be ‘in reciprocal relationship’; in modern terminology, \( A = \text{Constant} / D \).

Boyle himself was not particularly interested in numerical relationships: ironic, as the mathematical relation is known as Boyle’s Law. This may have been Hooke’s contribution; his background and interests were very different from Boyle’s; see p121.

In any case, Boyle added an extra column, \( E \), comparing the measured data in \( D \) with the theoretical \( 1/A \). The columns in the table require us to look at one row at a time; a graph gives us an overview of the difference between experiment and prediction (Figure 107), though graphing data wasn’t used until the 1800’s (see p118).

Although the numbers seemed to be ‘in reciprocal relationship’, it also seems there’s a substantial difference between the data and its reciprocal. This is actually no surprise: Column \( E \) doesn’t take the constant into account. How would we do that? If \( A = \text{Constant} / D \) as suggested, then a graph of \( A \) versus \( 1/D \) should be a line, the constant would be the slope of that line. Figure 108 shows this graph, which really does looks like a (slightly wiggly) line.

But how to find the slope? Again, we have techniques that hadn’t been invented in Boyle’s time. Astronomers had just begun to think about how to choose the best example from a series of different observations of a planet’s position (using the then-new technique of averaging data). To find the best line through a set of two-dimensional data hadn’t been invented in Boyle’s time. We’ll cover this in Section 10; for now, we’ll accept that it gives the ‘best’ straight-line version of the wiggly line, in the form \( y = mx + b \).

Figure 109 shows the two curves together; the green line has intercept \(-2.45353 \times 10^{-5}\) and slope \(0.00071009\). The graph suggests that the red curve is close to being a straight line, except more like \( A = \text{Constant} / D + \text{ExtraConstant} \) instead of \( A = \text{Constant} / D \). Why is the extra constant there at all?

The rulers were marked off in units of \( 1/16 \), it would be easy to make an error reading the true height of the mercury; the error would be somewhere in the \( 1/16 \) gap, so at most an error of \( \frac{1}{2} \times \frac{1}{16} = .03125 \), about 128 times larger than observed. Another issue is “heights should be proportional to the volumes of each”; volume is proportional to height only when the tube is a perfect cylinder; glassblowers in the 1700’s were nowhere near perfect.

For the time, the data is very good.
We’ve talked history and philosophy, but the subject of Section 9 is really the numbers. We’ll look at:

i) In column $A$ of Boyle’s data (p109), measurements were only taken every two units. The process of selecting just a few of the many possibly units is called sampling. How does sampling data affect the results we get?

ii) In column $B$, heights of mercury were recorded to one 16th of an inch. A modern experiment might talk of “recording to two decimal places”; in either case this process is called quantization. We saw variation in these numbers producing errors; how does this work?

iii) The data Boyle collected was restricted to only a limited number of pressures – or equivalently, heights of mercury: We were hindered from prosecuting the trial at that time by the casual breaking of the tube. But because an accurate Experiment of this nature would be of great importance to the Doctrine of the Springe of the Air . . . [insert about ten subordinate clauses] the several Observations that were thus successively made, and as they were made set down, afforded us the ensuing Table (p109). Robert Boyle, New Experiments, Physico-mechanicall, touching the Spring of the Air, cited above.

This is called range restriction. At very high pressures, Boyle’ Law no longer works; range restriction can cause problems.

iv) Boyle was dis-interested in using his data; his Law was discovered by others. This was intentional:

Boyle’s books were among the first scientific writings to embody the principles laid down by Bacon that the story should be told without embellishment or flights of rhetoric, but as a straightforward account of what had actually been done in the experiments, what had been observed as a result, and what the theoretical implications were.


Francis Bacon’s scientific program was a reaction against Aristotle’s program of explanations; Bacon believed the business of the experimenter was to provide facts; from accumulated facts would grow control over Nature, and bring about a scientific utopia (see p110). For Bacon, and Boyle, experiments produce facts, and facts speak for themselves.

Boyle was unusual for his time in letting the facts speak at great length; he described all he details of his experiments. He knew that very few had the means to repeat those experiments; he wanted his readers to believe the results were as he said.

For modern scientists, this has been problematic; recently, labs have
been unable to repeat some important experiments and get the same results; we’ll discuss this later. There’s a second issue: mathematical manipulation of data is still manipulation; can it be trusted? This is again an issue arising recently: is the mathematics being applied correctly?

v) There’s a hidden issue: Boyle chose to record only pressures, but temperature is important (see p123). The data we leave out can lead to errors; we’ll see this in other fields; again later.
Notes for Chapter 1 Section 9: Data: Introduction

p106 With the introduction of experiments, we’re entering what is called The Scientific Revolution. There are a number of problems with that phrase; to being with, science of a kind has been done in most cultures – African, Arabic, Chinese, European, Indian, Mayan, Native American, Polynesian, . . . . On top of that, scholars question whether there was one, or many, or no scientific revolutions; for a range of contemporary surveys, see for example Richard S. Westfall, The Construction of Modern Science, Cambridge University Press 1978; Steven Shapin, The Scientific Revolution, University Of Chicago Press 1996; and Peter Dear, Revolutionizing the Sciences, Princeton University Press 2009.

We won’t look at these issues. There’s a third issue, though: the word ‘science’ didn’t come into use until the 1850’s, two to three centuries after the period we’re discussing. The term previously used was natural philosophy, from Aristotle’s work:

For Aristotle, the identity of natural philosophy lay in its search for the intrinsic principles underlying natural phenomena, and this conception excludes a number of cognitive disciplines – (above all, mechanics, optics and astronomy) medicine and natural history – on the grounds that these are either not concerned with natural phenomena or do not pursue their enquiry in terms of underlying principles.


Aristotle as a philosopher dealt with the hard problems of his time: how is it that the world is changing? What are the causes of change and motion? Aristotle as a scientist was a naturalist; he made detailed studies of animals and plants; their physiology and behavior. How were they conceived? How was it they could grow into adult forms?

A typical example of an Aristotelian explanation is what Gaukroger refers to as ‘matter theory’ – stones fall because they are composed of the element earth, which seeks its natural place, the center of the universe (our planet). Hot air rises because it is partly composed of fire, whose natural tendency is to rise to the heavens. Many have seen in this a reflection of Aristotle’s biology; animals too move and grow according to their nature.

Although Thomas Aquinas in the 1200’s brought Christianity and Aristotelian philosophy into harmony, there remained issues. In Aristotle, the universe exists for all time, not created by God in six days. In Aristotle, knowledge could only come through the senses:
no bodily sensation after death, so the death of the soul, contradicting the immortality of souls (and, without immortality, no reward or punishment for our actions, undermining morality). Finally, if rocks are capable of motion according to their nature, they have a kind of life and spirit. This could lead to pantheism, that idea that God was actually in nature, not the creator of it.

All of this suggested that a natural philosophy more in tune with Christianity was needed. But Aristotle was becoming obsolete for other reasons, economic and technological.

Voyages of trade, exploration and colonization introduced new animals, plants and drugs. Voyages meant navigation; Portugal had a special institute and tools like the astrolabe, borrowed from Arab sailors.

Navigation meant geometry, even for Charles Darwin, on the Beagle. His bunkmate Midshipman Stokes had “main responsibility to look after and redraft the navigational charts which were the object of the voyage. To his chagrin, Darwin found his Cambridge education a poor substitute for Stokes’ practical expertise. ‘After looking at my 11 books of Euclid, & first part of Algebra (including binomial theorem?) I may then begin trigonometry after which I must begin Spherical?’” Janet Browne, *Voyaging*, Princeton University Press 1966.

An English translation of Euclid helped those who had no Oxford or Cambridge training in Latin; the astrologer John Dee wrote the preface. This was only one of many mathematics texts published; another was Edward Wright’s *Certaine errors in navigation* (1599):

> Wright wanted to do all he could to put reliable and verifiable information in the hands of England’s navigators and mariners. His meticulous accounts of observations set a new standard for accuracy and implicitly encouraged replication of results by recounting details about the instruments used and the precise locations where the observations were made. Wright’s early attention to the precise location and instruments used to make observations made him a trailblazer of verifiable, reproducible experimental knowledge.
There were also changes in the way knowledge was communicated. Universities had been founded in the 1200’s, to teach law (primarily Church law) and to teach literacy, preparing students for careers as administrators in the Church. But colleges of medicine were also created, and had to deal with practical problems like all the new drugs discovered through trade:

*The experiential approach to an understanding of the physical world was, to some extent at least, always promoted in the medical faculties. The Italian universities, Montpellier in France, and even the highly traditional Paris Medical Faculty expected medical students to study practical aspects of medicine by a kind of apprenticeship to a local practitioner, while undertaking their more theoretical studies in the university. From the sixteenth century medical schools became the prime sites for a number of facilities essential for the promotion of observational and empirical science: anatomy dissections, botanical gardens, and in some cases chemical laboratories.*


In addition to all this, the development of the telescope changed astronomy and astrology, even introducing heretical ideas like the Copernican system, in which the earth revolved around the sun (another Arabic idea: see F. Jamil Ragep, *Copernicus and His Islamic Predecessors: Some Historical Remarks* Hist. Sci. xlv 2007).

Ptolemy, Copernicus and Kepler devised mathematical systems to predict the positions of the known planets. They followed Greek ideas, that objects in the heavens move in perfect circles, with constant speed. Since they do not appear to do so, astronomers used ‘tricks’ to get the answers to come out right: circles moving on circles, off center circles, and, eventually, ellipses. This was criticized as “saving the appearances” – that is, adding more and more unjustified assumptions, just to get the right answer. True physicists, even as late as Galileo’s time, were supposed to begin from known truths, and then deduce from those how nature should behave. On the other hand, it was Church doctrine that planets do move in perfect circles, so a mathematician could claim that he wasn’t contradicting doctrine, merely ‘saving the appearances’. A kind of fraud; by Galileo’s time, a common joke was “What circle of Hell contains the mathematicians?” The answer, of course, is the circle of the fraudulent. Right next to Judas, and ever-so-slightly above Satan.
To us, this period was an odd mix of science and superstition, very far from Euclid, Copernicus and ideas of experiment and computation. Even Francis Bacon (who published the classic text on using experience to discover scientific truths, the *Novum Organum*), wrote "I [...] understand [magic] as the science which applies the knowledge of hidden forms to the production of wonderful operations; and by uniting (as they say) actives with passives, displays the wonderful works of nature." Bacon, *De Augmentis*. Some wanted to construct a 'science of prophecy'; Christopher Hill tells us:

Sir Walter Raleigh, Sir Francis Bacon, Sir Kenelm Digby and many other members of the future Royal Society, believed in sympathetic magic: [...] John Locke believed in it too. We cannot separate the early history of science from the history of magic. [...] Giordano Bruno, John Dee, Johannes Kepler, Tycho Brahe were all magi [magicians]. John Wilkins, future secretary of the Royal Society, in 1648 still quoted Dee and Fludd as authorities on 'mathematical magic.'


As another example, just before the Elizabethan period, the mathematician/astrologer John Dee was arrested on charges of "calculating", "conjuring" and "witchcraft," all of which, along with alchemy, were considered equally evil. Why? With Kepler’s laws, the orbits of the planets could be computed. This meant that knowing the day and hour of an individual’s birth, the position of the planets could be computed (retrospectively), and so their horoscope would be known (compare the early development of mathematics in Mesopotamian astronomy, p15). If the horoscope was for the king, or anyone in high office, such knowledge was dangerous, especially if the planets told of impending disasters. Elizabeth I herself, and her court, believed in magic – and again, with the right knowledge, spells might be cast against her. Knowledge of mysteries such as calculation was dangerous. In the English Civil War, pamphlets and almanacks proclaimed the fall of kings was near; people of the time said these prognostications likely caused the fall of king Charles I.

The period of *The Scientific Revolution* sees a mix of calculation, experiment, magic and theology. The idea that replaced Aristotle, and promised to unify these diverse subjects, was a revised form of Greek atomic theory known as either *corpuscularianism* or *mechanism*. The central idea was, again, matter theory, but this matter was composed of minute particles (the corpuscles) which had no qualities like Aristotles’ ‘natural tendencies’. Instead, the theory posited
There is no effect without a cause; no cause acts without motion; nothing acts on distant things except through itself or an organ or connection or transmission; nothing moves unless it is touched, whether directly or through an organ or through another body.

Pierre Gassendi, *Opera*, i, c1654.

That is, all effects – disease, light, motion, magic – all came from small particles bouncing off each other. In Greek times, there was an obvious objection: why would particles interact? They’d all fall, in straight lines, towards the center of the earth. None of the effects mentioned could ever happen.

In the physics of the 1800’s, the question is different: we can predict the future of a collection of atoms if we know their starting position and starting velocity; together these are initial conditions. Gassendi’s solution to the problem of bodies all falling together was God’s providence: God provided the initial conditions, knowing how these would lead to all the matter and effects we see in the world.

In addition to reconciling Christian theology with natural philosophy, Gassendi’s idea had another advantage. He could claim that by studying natural philosophy, we are learning God’s intentions for the world. Newton titled his major work on motion, gravity, and the orbits of the planets *Philosophiae Naturalis Principia Mathematica*, or *Mathematical Principles of Natural Philosophy*. In the second edition, he added a section on God:

*His substance is unknown to us; we know God only through his attributes and the excellency of the natural order, and through the final causes of things. He is the God of providence *no variation in things arises from blind metaphysical necessity, which especially is always and everywhere the same [...] And thus much concerning God, to reason about whom, at least from phenomena is a concern of natural philosophy.*

Graphs had been used as early in the 1300’s, for scholarly and scientific purposes. The French mathematician Nicola Oresme

... conceived of the idea of using rectangular coordinates (latitude and longitude) and the resulting geometric figures (configurationes) to distinguish between uniform and nonuniform distributions of various quantities, such as the change of velocity in relation to time. ... In the discussion of motions the base line (longitudo) is the time, while the perpendiculars raised on the base line (latitudines) represent the velocity from instant to instant in the motion. ...


For a constant (uniform) velocity, distance = speed $\times$ time. The graph would be a horizontal line, and the speed would be the area under the graph. If the velocity increases uniformly, the graph is a slanted line; the area underneath is a trapezoid, whose area was known, and this again represents a distance (Figure 112). This result was also known to a group of scholars at Merton College, the ‘Oxford Calculators’.

Graphing actual data instead of philosophical concepts seems to have been the invention of William Playfair, in the early 1800’s; see Figure 113. Playfair even remarked that the graph allowed one to comprehend complex patterns more easily that a list of numbers – much the same reason we still use graphs today.

Torricelli used a form of investigation we’d call empiricism:

One of the distinctive features of modern science is a commitment to empiricism – a fundamental expectation that theoretical hypotheses will survive encounters with observations. Those that comport with the theory’s explanations and predictions confirm the theory. Anomalous observations that do not fit theoretical expectations disconfirm it. Moreover, experiments can be contrived to generate observations that might serve to confirm or disconfirm a theory.


This is how we think of science (see Avery on DNA, p39), but for over a thousand years, it wasn’t what European scholars did, again due to Aristotle.
Aristotle’s interest was not to discover new facts, but to explain why – why things happened and why they had to happen that way. As we discussed on p30, his explanations had to be of a certain type:

For Aristotle, who was to become the preeminent “ancient authority,” phenomena were, literally, data, “givens.” They were statements about how things behave in the world, and they were to be taken into account when discussing topics concerning nature. The immediate sources of phenomena were diverse: common opinion and the assertions of philosophers, as well as sense-perception. Given these statements, a system of syllogistic reasoning yielded, in principle, a theoretical description and explanation of them.

Peter Dear Totus in Verba: Rhetoric and Authority in the Early Royal Society, ISIS 1985, 76 145-161

There’s something off about this approach: had humans been concerned with ‘understanding’ rather than discovering, there wouldn’t be flint tools, pigments mixed for rock-painting, roots boiled to make them edible, firing of clay to make pots, planting of crops, smelting of metals, . . . . Apparently science, pursued as ‘understanding’, had little to do with how real people made real progress.

All that was to change. But what caused the change from deduction to empiricism? This is a technical question, and not a simple one; some natural philosophers used empiricism as early as the 1500’s. It’s not obvious how this influenced individuals thinking about physical science, but the early 1600’s was a time when scholars in general began to communicate and form societies. Others communicated by letters; the French priest Marin Mersenne maintained a web of contacts across Europe; his correspondents included Descartes, Fermat, Galileo, Hobbes, Huygens, Pascal and Torricelli. Knowledge of Torricelli’s work, for example, passed to England through Mersenne; see Stephen Shapin, Leviathan and the Air-Pump: Hobbes, Boyle, and the Experimental Life, Princeton University Press; Revised ed 2017.

The philosophy of knowledge was also changing. Some of this was quite old – going back to Galen and even further to Hippocrates; see Wallace, Galileo’s Pisa Studies in Science and Philosophy, in Peter Machemer ed, The Cambridge Companion to GALILEO, Cambridge University Press, 1998. Without getting lost in the complexities of medieval scholastic philosophy, the change was a form of deductive reasoning called regressus (from the Latin, to return).

True knowledge was understanding causes. If you observe an eclipse, you understand the cause is a planet coming in front of the sun. The disc-like shadow results from the spherical shape of planets.
This doesn’t leave room for true knowledge about more complex phenomena, like illnesses; regressus helped with these. For example, when you observe a fever, the first step is to find a cause – say an imbalance of of hot/cold, wet/dry in the body. In the second step you use deduction to establish that the cause you found (guessed) really does result in the effect (thus regressus: return to the original).

The difficulty here is in guessing causes from effects. It’s similar to the problem of finding first premises in mathematics; Aristotle attributed this to a different cognitive state: insight, intuition, etc (p30). Throughout the 1500’s, scholars debated exactly what this extra function was (and whether it was needed). One term used was negotiatio intellectus – roughly, the work of the intellect:

Yet regressus also inserted an ambiguity into the understanding of this relationship, in the form of the negotiatio intellectus. Though an essential step in regressus, there was no consensus as to how the negotiatio was supposed to proceed. The discriminatory use of observations could be seen as a natural way to resolve this ambiguity. Thus, the regressus method suggested a novel methodology in natural science that admitted observations as epistemic grounds for accepting and rejecting theories.


Where does Torricelli’s work come in? He worked under Galileo, and may have learned the technique from him, then applied these ideas to atmospheric pressure, as we saw on p107.

We’ll spend serious time, later, on Galileo. For now, what is known with some certainty is that Galileo studied at the University of Pisa (a center of experimental botany and medicine); some of the scholars he is known to have worked with were using regressus. Galileo himself used regressus in his very early work (again, see Wallace, above, p96).

To emphasize the point, though: this is what may have helped lead to empiricism and a new way of doing science; it is not the final word; see the literature quoted above.

As an example of what careful research can achieve, it had been accepted that Galileo never did careful experiments. Part of the justification was that in the late 1890’s, the mathematician/physicist Antonio Favaro in Italy published the Edizione Nazionale of Galileo’s papers, and no evidence of experimental work was in them.

In the 1970’s, the historian Stillman Drake realized that Favoro had heavily edited the papers, and had not published miscellaneous
sketches and random notes at all. Returning to the original papers, Drake found

This unpublished material includes at least one group of notes which cannot satisfactorily be accounted for except as representing a series of experiments designed to test a fundamental assumption, which led to a new, important discovery. In these documents precise empirical data are given numerically, comparisons are made with calculated values derived from theory, a source of discrepancy from still another expected result is noted, a new experiment is designed to eliminate this, and further empirical data are recorded.


Hooke and Boyle were influenced by the cultural, political and religious conflicts of their time. Even before Elizabethan times,

Mathematics itself came in many guises both institutionally and extra-institutionally. Certainly geometry was taught at the universities, but also there were the mathematical sciences of astronomy, geography and sometimes mechanics. Outside the sanctioned institutions mathematics reigned quite lively in the realms of natural magic, astrology and hermetic practices, and the cabala, as well as in the more mundane, pragmatic spheres such as the principles of painting, construction of fortification and the design of machines.


This was the complex London in which Hooke worked. There was also the political-religious London of the Commonwealth and Protectorate. The Civil War was partly a religious and partly an economic war. The Anglican Church was the established church of England, and every loyal citizen had to be baptized into it – and pay tithes of 10% for upkeep of the church and the clergy. King Charles I was also returning to elaborate rituals, more characteristic of Catholicism than Protestantism. After Charles lost the war and his head, the Anglican church lost its status as the official State religion, as well as its tithes, and many of the vicars, bishops and other officials were turned out of their jobs. For centuries, bright young men had found good careers through the Church or the State (even Darwin, centuries later, considered being a clergyman). For Hooke, growing up in this period, there would be no clear path to employment. Fortunately,
some Cavaliers (adherents of the monarchy) still held University positions, understood the economic and scientific changes happening in England, and believed that the new order needed practical men, who could measure and compute:

Through John Wilkins’ efforts, a handpicked group of mathematically inclined and scientifically able men was assembled in Oxford in the early Commonwealth years. On the whole they were men of ‘cavalier’ persuasion – moderate supporters of the monarchy whose hopes for the future had been dashed by the violent termination of the reign of Charles I, and who now found themselves with no prospect of political or clerical preferment, constrained to make their living outside the established Church and the Government.


Hooke, though poor, was looked after by friends of his father. He worked his way through Oxford (likely as a servant to wealthier young men) and was noticed and taken up by Wilkins, who actually recommended Hooke to Boyle. Boyle employed Hooke to run his experiments, giving Hooke a base from which to explore natural philosophy.

Boyle’s path was rather different. His father married into money, accumulated more, mostly in the form of land in Ireland, became an Earl, and by the time of the Civil War was called the wealthiest man in England. A good deal of this land was redistributed after the fall of the monarchy, though Boyle was still quite well-off. He was, however, subject to different kinds of influences.

A problem arose when ancient belief in magic mixed with the end of an official religion. With that came the lifting of censorship of books, allowing many new religious sects to spring up: Ranters, Levellers, Anabaptists, Familists, Quakers, Diggers, Muggletonians . . . . As one example, belief in a world pervaded by spirits willing to assist magicians was consistent with beliefs that traditional religion was in error; everyone could contact the spirit of God:

...the Ranters embraced the concept of the Indwelling Spirit, but went further by claiming that anyone who had made a personal relationship with God was no longer bound by conventional society and that whatever was done in the Spirit was justifiable. This encouraged a sense of liberation from all legal and moral restraint. Organized forms of religion could be rejected, the concept of sinfulness dismissed and the Bible itself disregarded. Free love, drinking, smoking and swearing were regarded as viable routes to spiritual liberation.

Along with this, there was the sense that the nobility were no more
noble or deserving than commoners, that a worldly paradise of
equality among people was at hand – as was the Second Coming
of Christ. Besides the breakdown of what the wealthy considered de-
cent society, groups such as these had no use for Christianity, nobles,
kings or government; the end of the world was in sight, bringing a
heaven on earth, where everyone would be equal. While Boyle stood
to lose his estates and social position, more important for him was
Christianity and the Anglican Church. He saw heaven on earth in
a Baconian way, as the continued advancement of science and con-
trol over nature – not through magic, but though knowledge of the
natural world:

During the 1650s the reformers – Boyle, Walter Charleton, and others-
modified their philosophy in the face of the radical threat: in the place of
the now discredited occultism they adopted what Boyle called the corpus-
cular philosophy. This amounted to a Christianized Epicurean atomism
treated as a hypothesis to be tested by experiment. The corpuscularians
held with Epicurus that the world was made up of lifeless atoms colliding
in the vacuum of space. But the Puritan philosophers departed from Epi-
curus by denying that the world as we know it was the product of a long
succession of random atomic collisions. Rather they held that a providen-
tial God was responsible for all motion in the universe. He determined the
paths the atoms took and hence maintained the order of the universe. Not
only was this a workable scientific hypothesis capable of being refined and
elaborated by a Baconian program of experiment, it was also an attractive
candidate for adoption because it was applicable to social issues.

What united them all was the belief that rational explanations could be
arrived at for everything in the natural world, and that such form of
explanation were confirmation of the existence of an all-knowing God,
whose representatives on earth – the Anglican clergy– were the custodi-
ans and guides on behalf of those unable to rise to full understanding on
their own.

James R. Jacob and Margaret C. Jacob The Anglican Origins of Mod-
ern Science: The Metaphysical Foundations of the Whig Constitution,

These ideas pervaded even Newton’s work (p117), Newton also
added a more sophisticated ‘science of prophesy’.

p112 We’ll write Boyle’s Law in modern terms: let $P$ be the pressure
exerted on a gas and $V$ the volume of the gas. Then $PV = c$ where $c$
is a constant. This is true only when the temperature $T$ of the gas is
constant, which, during compression or expansion, isn’t the case.