I left for college in 1967: the sixties, when everything was changing. My dorm was co-ed, the first in the country. Vietnam war protests shut down the University and students smoked weed in the lounge. But also: an entering physics major, I picked up Feynman’s Lectures On Physics. Feynman wrote,

The special problem we tried to get at with these lectures was to maintain the interest of the very enthusiastic and rather smart students coming out of the high schools . . . . They have heard a lot about how interesting and exciting physics is – the theory of relativity, quantum mechanics, and other modern ideas. [Instead] they were made to study inclined planes, electrostatics, and so forth, and after two years it was quite stultifying.

Today, most students take an AP Calculus course in high school; they repeat the course in college, sometimes with the same book. They’ve heard about amazing discoveries in science, engineering, medicine and technology that change the world, but when they enter a calculus classroom, they’re magically transported back to 1967, as though the last fifty years of progress never happened.

This is not a calculus book; it’s a compendium of what gets left out of traditional books. Calculus touches on so much of the world: history, religion, philosophy, literature, psychology and especially the modern scientific and technical world.

Calculus is an amazing intellectual adventure. Let’s start.
Introduction: Why Calculus?

On Jody Foster’s first day at Yale she wrote a friend: ‘My calculus book is three inches thick. I can’t survive three inches of calculus.’ My students are more direct: ‘I’m going to be a doctor. Why do I need calculus?’ Or, ‘I’m gonna be an engineer: we use computers for everything. Why do I need this class?’ Instead of a direct answer, I’ll tell a story about a doctor, and an engineer, and the ECG, a graph of the electrical currents in a heartbeat, used to evaluate the health of a heart (for a quick summary, see p8).

The story begins in the 1790’s, when the Lucia Galvani, the wife of the Italian doctor Luigi, noticed that the muscles of some frog legs she’d hung out for dinner twitched when a spark of electricity touched them. Luigi followed up, experimenting for many years, concluding that the nervous system operated through electricity. But in 1790 there were no instruments to measure electricity; scientists detected electrical impulses in the body using frog legs. In the 1830’s, the neurophysiologist Carlo Matteucci laid a frog leg on a beating heart, and the leg twitched in rhythm with the heartbeats, showing that heart muscles also generate an electric current when they contract.

Opening the chest of a patient and laying a frog leg across the beating heart isn’t an easy or safe diagnostic technique, but the electrical impulses from the heart were too weak to measure with the technology of the time. It took another seventy years before the Dutch physiologist Willem Einthoven developed a device to measure those weak currents, and then did the extra engineering to record them.

Figure 2 shows an early ECG machine. The electric currents from the heart cause a very thin wire to move; on the right, a light beam shines on the wire, and on the left, a motorized strip of photograph paper records the movements. And . . . the world’s first human ECG recording, Figure 3.
Technology advances: Einthoven’s moving strip of photo-paper was replaced by a pen moving up and down, writing on a moving strip of paper. The paper has lines and squares to make reading the recordings accurate. And the graph is sharper, showing more detail (Figure 4).

Now the diagnosis: the patient is suffering from an erratic heart beat, and an MD can tell by measuring the distance between peaks. In Figure 4 each little box is .04 sec wide, so an old-fashioned MD would count the squares between the peaks, and compute how many beats per minute there were. In this (short) graph, an erratic heartbeat shows up as a varying number of beats per minute (for more details, see the note on p9).

That’s what a 1950’s MD would do; even in 2014, in a visit to a hospital, my ECG was on a piece of paper a lot like Figure 4. But it won’t help in an emergency room: there’s no time to count squares on a nice piece of paper. Fortunately, we’re not in the 1950’s: we have computers.

Technology advances: now it’s the engineer’s turn. To keep records of a patient’s heart condition, there’s no need to store dozens of sheets of paper in a file cabinet. Instead of paper, write the electrical signals directly into a computer.

Once the data is in a computer – well, instead of drawing the graph on paper and counting squares, a microchip draws the graph on a LCD screen . . . and it counts the squares, it computes the beats-per-minute.

Today you can buy a portable ECG unit (Figure 5). An MD can get a quick sense of whether a heart is behaving normally, and then, if needed, run a full ECG (which takes 10 minutes or so, requires a nurse, square counting, and $$. Technology advances.)

But before the chip can count squares, it has to somehow ‘see’ the peaks. Imagine an engineer programming the chip, working with an MD and neither took calculus. Calculus would have told them peaks happen at critical points, where a derivative is zero. Calculus tell us how to find critical points, but how does a computer do it? The software to find critical points, using a microchip, was written by Jiapu Pan in Shanghai and Willis Tompkins in Wisconsin, using a derivative-based algorithm. Figure 6 shows an example of the technique; it relies on successive mathematical operations to strip off the irrelevant parts of the beat.

This engineer and MD did need calculus, and their work is one of the most cited in biomedical engineering.
Notes

Notes for Chapter 0


p6 An ECG records the electrical activity of the heart. Here’s a quick view of how.

The heart is a series of chambers to hold blood, muscles to pump blood, and veins and arteries to channel the blood. Muscle contractions are associated with electrical currents (we’ll go into more detail later in the book) and the current is measured by placing electrodes on the body; the record of the current flow is the ECG; see below.

On the left, the different chambers of the heart, and the major nerves that orchestrate the sequence of contractions of the chambers (for example, when one chamber contracts, the following chamber has to be open to receive the blood pumped; see fibrillation, below. Timing these events is crucial). On the right, a printout gives a visual record of how the different parts of the heart contract, the size of the contraction, and the timing between contractions; compare Figure 3. Well over a hundred years of study, matching symptoms with EKG’s allows an MD to diagnose heart problems using this information (you
can practice this yourself; see http://www.healio.com/cardiology/learn-the-heart/ecg-review/ecg-interpretation-tutorial/approach-to-ecg-interpretation).

The ECG serves as a kind of microscope into the heart, with the advantage that the doctor doesn’t have to do surgery to inspect the internal workings of the heart. It’s a simple, cheap way of monitoring basic health.

The erratic heartbeat in Figure 4 is called fibrillation. As we said, the heart is a series of chambers and pumps, and a working heart chamber pumps only after it gets blood from the preceding chamber. In fibrillation, the pumping happens at random, and the blood can slosh around the heart, never getting to the body. Depending upon the chamber that has the fibrillation, this can lead to death within minutes.