Chapter 0: Introduction

Section 0.1: Why Calculus?

The story is, on Jody Foster’s first day at Yale she wrote a friend: ‘My Calculus book is three inches thick. I can’t survive three inches of calculus.’

My students are more direct: ‘I’m going to be a doctor. Why do I need calculus?’ Or, ‘I’m gonna be an engineer: we use computers for everything. Why do I need this class?’ Instead of a direct answer, let me tell a story about a doctor, and an engineer. And the ECG, a graph of the electrical currents in a heartbeat. (It’s an acronym for Electro Cardio Gram.)

The story begins in the 1790’s, when the wife of a doctor, Luigi Galvani, noticed that the muscles of some frog legs she’d hung out for dinner twitched when a spark of electricity touched them. Galvani followed up, experimenting for many years (see Figure 1), concluding that the nervous system operated through electricity. But in 1790 there were no instruments to measure electricity; scientists detected electrical impulses in the body using frog legs. In the 1830’s, Carlo Matteucci, a neurophysiologist, laid a frog leg on a beating heart, and the leg twitched in rhythm with the heartbeats, showing that heart muscles generate an electric current when they contract.

Opening the chest of a patient and laying a frog leg across the beating heart isn’t exactly a safe and easy diagnostic technique, but if the chest weren’t opened up, the electrical impulses from the heart would be too weak to measure. It took another seventy years before the Dutch physiologist Willem Einthoven developed a device to measure those weak currents, and then did the extra engineering to record them.

Figure 2 shows an early ECG machine. The electric currents from the heart cause a very thin wire to move; on the right, a light beam shines on the wire, and on the left, a motorized strip of photograph paper records the movements. And . . . the world’s first human ECG recording, Figure 3.
Technology advances: Einthoven’s moving strip of photograph paper was replaced by a pen moving up and down, writing on a moving strip of paper. The paper has lines and squares to make reading the recordings accurate. And the graph is sharper, showing more detail. (Figure 4).

Now the diagnosis: the patient is suffering from an erratic heart beat, and an MD can tell by measuring the distance between peaks. In Figure 4 each little box is .04 sec wide, so an old-fashioned MD would count the squares between the peaks, and compute how many beats per minute there were. In this (short) graph, an erratic heartbeat shows up as a varying number of beats per minute.

That’s what a 1950’s MD would do; even in 2014, in a visit to a hospital, my EKG was on a piece of paper a lot like Figure 4. But it doesn’t help in an emergency room situation; there isn’t time to sit down with a nice piece of paper and count squares. Fortunately, we’re not in the 1950’s: we have computers.

Technology advances: now it’s the engineer’s turn. To keep records of a patient’s heart condition, there’s no need to store dozens of sheets of paper in a file cabinet. Instead of paper, write the electrical signals directly into a computer. As in Table 1.

Once the data is in a computer—well, instead of drawing the graph on paper and counting squares, a microchip draws the graph on a LCD screen . . . and it counts the squares, it computes the beats-per-minute.

Today you can buy a cell-phone size ECG unit, as in Figure 5. An MD can get a quick sense of whether a heart is behaving normally, and then, if needed, run a full ECG (which takes 10 minutes or so, requires a nurse, all that square counting, and comes with a bill. Technology can be great.)

So . . . say you’re the engineer programming the chip. You’re working with the MD who also didn’t need calculus, and before you can get the chip to count squares, it has to somehow ‘see’ the peaks. How does that happen? If you do know calculus, you know peaks happen at critical points, points where the derivative is zero. The software to find critical points, using a microchip, was written by Jiapu Pan in Shanghai and Willis Tompkins in Wisconsin. The two used a technique called a derivative-based algorithm. Today their 1985 paper (Figure 6) is still one of the most widely read papers in biomedical engineering.

So that’s the end of our story: MD and Engineer do their jobs happily ever after, thanks to calculus.
Section 0.1: Numbers

In Section 0, I asked a medical question about an erratic heartbeat, and wound up with some numbers and a graph. Everyone expects that in a math book, but it is actually very weird.

The idea of 'number' is already odd. What is a number? It's clearly not a thing like a spoon, but if it's all mental, how did it get into our minds? Even then, how can numbers and graphs tell us anything useful about living, beating hearts? These sound like questions in philosophy (they are), and they go back to before Plato. I won't go there, not yet, anyway. I do want to talk about some recent ideas, and those ideas come from biology. Here's a typical study.

A scientist has made holes in a log and randomly places worms in those holes. A robin watches, and when the scientist leaves, the robin immediately goes to the log to munch on worms. (Figure 7). But it first selects the holes that have the most worms.

The study shows that these birds can "count" at least to the extent of perceiving 'more' and 'less.' Other studies show this ability is cross-species: robins, rats, newly hatched chicks, new-born babies. They won't pass an algebra test, but something in these tiny brains 'gets' number.

This is the first major point I want to make: this kind of research suggests that 'number' is like 'time' or 'space' – they seem mysterious because they are built into the way our eyes and brains perceive the world (see the Section ). Number does not tell us everything, but it is – let's call it a 'quality' – but it is a quality we can't perceive the world without.

Which brings me to the second point: rats, robins and baby chicks do not count out change at the local Starbucks; they don’t weigh themselves to see if they’ve gotten fat; they don’t check how fast they’re jogging or worry whether their cellphone is 4G. They don’t use numbers in the same way humans use numbers. Humans do these things for the same reason we drink coffee while we text a friend: we live in societies, and these behaviors are part of our society.

We’re OK that not every society in history has cappuccino for breakfast, but we’re not as tolerant about numbers. We use numbers to talk about almost everything (even "she’s a perfect ten"). But if a culture doesn’t use numbers in the same way we do, we call the culture backwards, and we wonder about their IQ.
Here’s an analogy. Evolution gives us the ability to recognize colors. But evolution doesn’t explain why Americans associate white with purity and red with danger, while Chinese associate red with good fortune (see Figure 8) and white with death. Color perception is built into our brains, but our culture that tells us what to do with colors.

So too, both our society and the way we interact with the world tell us what to do with numbers. Think about the concept of months. Americans name months after long deceased Roman emperors (June and August for Julius and Augustus Caesar). In contrast, the Nootka, a Northwest Coast culture on Vancouver Island, use month names like Eneecoresamilth, salmon fishing moon (this actually seems more sensible). The names show how this society interacts with the world.

We eliminate the emperors and instead use day-month-year designators like 14/7/15 (in Europe, anyway: in America it would be 7/14/15). But our religions tell a different story. Chinese New Year, Passover, Easter, Ramadan and Holi are all lunar celebrations: all are associated with the appearance of a full or new moon, and all occur on a different days or month as the years change.

Antiquated! Make make all holidays like Christmas: always on the 25th of December! But linking a holiday to a centralized calendar only works if everyone has access to that calendar. Holidays linked to the moon arose because everyone always has access to the moon. This was a sophisticated solution to scheduling public holidays, before instantaneous communications.

The scheduling of holidays by the moon work because the phases of the moon form a public standard. Many different cultures employ different kinds of standards. Polynesian men working together to make a boat would measure off distances using the length of their finger joints. We think of the ‘foot’ as a more sophisticated standard, but here’s how it was defined in one sixteenth century German town:

*Stand at the door of a church on a Sunday and bid 16 men to stop, tall ones and small ones, as they happen to pass out when the service is finished; then make them put their left feet one behind the other, and the length thus obtained shall be a right and lawful rood to measure and survey the land with, and the 16th part of it shall be the right and lawful foot.*
Even early empires constructed public standards: Figure 9 shows the Babylonian Sun God Shamash, holding a standard measuring rod and a coiled rope, both used in surveying land. In the Hebrew bible, Leviticus 19: 36, we read:

“You are to have honest balances, honest weights, an honest dry measure, and an honest liquid measure; I am Yahweh your God, who brought you out of the land of Egypt.”

Standards were serious issues; land would be taxed by area, requiring standard lengths. We would be in trouble if bakers all meant something different by a ‘pound’ of bread. In the above examples, each culture represented their standards as coming from the heavens. Israelites kept weights and measures in the Temple in Jerusalem; Athenians kept standards in the Parthenon on the Acropolis; finally, the weights and measures defining the metric system were kept in the French Legislature. Figure 10 shows British standards for lengths, in a public place where anyone can use them.

This is the second point: humans communicate with language and writing, and numbers are a way to communicate about a quality we perceive in the environment. We can’t communicate effectively without public standards. In modern society, we use numbers extensively, communicating ‘how much’ and ‘how many’ as well as ‘when’ and ‘how far’ and ‘how hot’, . . . . Modern society places high value on number-based standards.

It’s worth, then, considering another society, the Masai, a sub-Saharan culture. The Masai live off their cattle (see Figure 11); a typical greeting might be ‘I hope all your cattle are well.’ The cattle function in other ways in society: the number of cattle a family owns determines their wealth.

But Masai also tend their cattle; they give the cattle names, and sing to them. They also need to know which cows are giving milk, which are pregnant; which steer to use for breeding, and a great deal more. None of this information is numerical.

The ability to understand numbers is built into our minds; and different cultures have exploited this ability in many creative ways. But numbers are only one way to sense the world, only one quality of a vast universe.
I said that we have only recently begun to understand how the brain can know about the environment. What scientists learn from *ethology*, the study of animal behavior, is that many kinds of animals can perceive some aspects of number. That fact makes the study of number easier, because we can perform experiments on some kinds of animals that would be simply unethical on cats, chimps and humans. And a very good animal is the frog.

I’ll talk about this only very briefly, and at the level of the eye. I always thought that eyes took information and passed it directly on to the brain. Figure 12 shows that too simple a story. Anatomy and cell recordings show that the frog eye only passes certain kinds of information on to the brain. We say the eye *filters* visual information before passing it on for action.

The article ‘What the Frog’s Eyes Tells the Frog’s Brain’, explains how this works. It’s based on recordings of frog eye/brain activity while the frog is being shown various scenes. What the frog eye can see is differences in contrast (an insect standing out from the background), convexity (bug shape?), a moving edge (moving insect) and dimming of light (predator in back?). In short, the eyes pre-process the light, picking out only features that are important to the frog. The frog brain further processes that information, to recognize predator or prey.

Charles Darwin’s brother once reminded him that Plato believed we are able to understand abstract ideas (like numbers) because our soul has experienced these abstractions before we came to exist on this earth. Darwin wrote in his notebooks that instead of a preexisting soul, we have pre-existing monkey ancestors. That is, perception of objects evolved to fulfill some purpose for organisms; we know numbers because in our evolutionary past, it was advantageous to perceive numbers.
Notes for Chapter One

p1 Figure 1 Galvani lab picture and details on Galvani from http://www.theiet.org/resources/library/archives/featured/galvani.cfm


p1 Figure 3 Picture of first human ECG. From Einthoven W. The galvanometric registration of the human electrocardiogram. Pfluger’s Arch.f.d. ges Physiol 1903;99:472D480.

p2 Figure 3 Picture of ECG indicating fibrillation, from http://millbasindoctor.com/atrial-fibrillation/

p2 Table 1 Table of raw ECG data from author’s personal collection.

p2 Figure 5 Picture of portable ECG unit from http://www.skymall.com/handheld-ecg-monitor/HDW107.html

p2 Figure 6 Flowchart of the Pan-Tompkins ECG algorithm from Pan, Jiapu and Tompkins, Willis: A Real-Time QRS Detection Algorithm, IEEE Transactions on Biomedical Engineering, 22(3) March 1985

p3 Figure 7 Image is from Simon Hunt, Jason Low and K. C. Burns, Adaptive numerical competency in a food-hoarding songbird, Proc. R. Soc. B (2008) 275, 2373D2379

p3 The study of the first philosophical problem, how can we know? is called epistemology: This problem pervades all of science and mathematics, and we’ll discuss it further into the book. The issues are Plato’s Theory of Forms, and Kant’s Forms of Perception.

The second philosophical problem is: how can mathematics can tell us anything useful about the world? The Nobel physicist Eugene Wigner wrote an influential paper, ‘The Unreasonable Effectiveness of Mathematics in the Natural Sciences,’ exploring this topic. A copy of the paper is at https://www.dartmouth.edu/~matc/MathDrama/reading/Wigner.html.

p3 Information on cross-species numeracy from Michael Tennesen, More Animals Seem to Have Some Ability to Count in Scientific American, Sep 2009, Vol. 301, Issue 3

p4 Figure 8 Picture of child with money envelope from https://www.sassandbelle.co.uk/blog/2015/02/19
Information from Nootka numbers from William J. Folan
*The Calendrical and Numerical Systems of the Nootka,* in *Native American Mathematics,* Michael Closs, ed.

There are issues with lunar calendars: someone has to decide the exact day the moon is full. In Islam, this is the duty of the Imam. In a small village in Africa, it would be the duty of the Chief Priest. Here Chinua Achebe, the Nobel prize-winning author, describing the process in his novel *Arrow of God:*

‘The moon he saw that day was as thin as an orphan fed grudgingly by a cruel foster-monther. He peered more closely to make sure he was not deceived by a feather of cloud.’

The quotation defining the foot is from Jacob Koebel, *Geometrei. Von künstlichem Feldmessen und absehen.*

The metric system seems the very peak of international scientific standards, but it was adopted for purposes of empire and commerce, not science. Opposition to its adoption was so intense that Bonaparte delayed it for decades. See J.L. Heilbron *The Measure of Enlightenment in The Quantifying Spirit in the 18th Century.* *The Metrology Handbook,* American Society for Quality Measurement, by Bennett, Keith Bucher, Jay L., ed.

Figure 9 Image of Sun God from https://en.wikipedia.org/wiki/Measuring rod

Figure 10 Picture of British length standards from *The Metrology Handbook,* American Society for Quality Measurement, by Bennett, Keith Bucher, Jay L., ed.

Figure 11 Picture of Masai elder from http://www.maasai-association.org/cowss.html

Information on Masai culture from http://www.bluegecko.org/kenya
The field of cognitive science examines the difference between what a Masai sees in her cattle and what a simple count reveals. The technical phrase is that each is a representation of the cattle. To illustrate the idea, Figure 14 shows how we’d see the pond. We imagine we’re seeing everything, but a bee or butterfly would see colors in the infrared or ultraviolet, and in that range, the plants would have markings telling the insect where pollen can be found. The insect sees what it needs to find food, and what the plant needs for pollination. Contrast this with Figure 15. The frog sees what it needs to avoid predators and to find food. Neither figure gives a complete idea of the pond; each is just one representation of the pond.

Figure 15 is a still from the movie ‘What the Frog Sees’ by Skip Battaglia. Video at http://www.visualnews.com/2012/10/16/the-frog-does-not-see-what-does-not-move-to-the-frog-to-move-is-to-exist/


A very good introduction to how the brain processes visual information is the book From Neuron to Brain, by John G. Nicholls and A. Robert Martin.

Figure 12 Retina diagram from http://hubel.med.harvard.edu/book/b9.htm

Figure 13 Pond from http://www.publicdomainpictures.net/view-image.php?large=1&image=24889
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