Lecture 2

FOURIER TRANSFORMS

AM and FM

We saw in the supplement on power spectra that the human range of hearing is concentrated in the range 400Hz to about 4000Hz. That's where we'd expect radio broadcasts to be, as well. But, check it out ... the AM radio band is 535kHz to 1705kHz, where kHz means 'kiloHertz' or 10^3·Hz. FM is 88 to 108 MHz, where MHz is megaHertz, or 10^6·Hz. – Humans can't even hear those frequencies – so what's the deal?

The first part of the deal is that low-frequency radiation is easily absorbed by the atmosphere, o it's not a very good medium for transmitting signals over distances. People living on mountains learned this long ago – they invent techniques like yodeling to transmit information. However, we're talking of using electromagnetic radiation, and that's the problem: electromagnetic signals have to be broadcast from antennae. Note, at right, a TV antennna. Why is it so tall? Because the theory of antenna design tells us that to broadcast a frequency ω, the antenna has to have height $\lambda/2$, where λ is the wavelength, the distance the wave travels in one cycle. That's c/ω where c is the speed of light, 186,000 miles/hr or $51\frac{2}{3}$ miles/sec. A 400Hz signal would require a 1/400 sec to complete one cycle, so it has wavelength $\left[51\frac{2}{3}miles/sec\right]/[400cycles/sec] = 0.12916667miles/cycle$ or 682 feet/cycle. An AM antenna at, say, 400 kHz, would have a wavelength less by a factor of a thousand.

That's where AM – amplitude modulation, and FM – frequency modulation, come in. Say you want to transmit a voice signal at 400Hz: $y_s = A_s\cos(2\pi f_s t)$ where f_s is our frequency. Multiply y_s by a *carrier wave* $y_c = \cos(2\pi f_c t)$, where f_c is called the *carrier frequency* and we can assume it is, say, around 1000kHz. Trig identity:

$$\begin{aligned} y_c y_s &= A_s\cos(2\pi f_s t)\cos(2\pi f_c t) \\ &= \frac{A_s}{2}\left[\cos(2\pi\{f_c + f_s\}t) + \cos(2\pi\{f_c - f_s\}t)\right] \end{aligned}$$

Thus, the product is a wave with two frequencies, $f_c \pm f_s$, which are going to be at, say $1,000,000 \pm 400$ Hz, well in the AM range.

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The process of multiplying one signal by another is called *modulation*, though in radio talk it's called *heterodyning*. The signals at $f_c \pm f_s$ are called *sidebands*. And all of this is pretty useless unless we can recover the original signal from the modulated signal.

Let $m(t)$ be the signal we want to broadcast, and $\cos(2\pi f_c t)$ be the carrier frequency. What goes out over the airwaves and comes in to the radio is $m(t)\cos(2\pi f_c t)$. Note, however, that

$$
[m(t)\cos(2\pi f_c t)]\cos(2\pi f_c t) = m(t)\cos^2(2\pi f_c t) = m(t)\left[\frac{1 + \cos(2\pi 2 f_c t)}{2}\right]
$$

$$
\frac{m(t)}{2} + \frac{m(t)}{2}\cos(2\pi 2 f_c t)
$$

The point here is that the original signal is now sitting at its original frequency, while another piece of it is sitting, modulated at twice the carrier frequency. this will produce sidebands at $2f_c \pm f_s$. But these sidebands will be very far away from any of the frequencies in $m(t)$. This means you can recover $m(t)$ simply by ignoring frequencies higher than any in m.

<u>Lab Problem</u> Try this scheme with the selection from Billie Holliday's song Good Morning Heartache, sampled at 44100Hz. What's the highest frequency in the song? Modulate it with a high frequency carrier wave $\cos(2\pi f_c t)$, and demodulate using the scheme suggested above. How does it sound?

There are a couple of problems with the above scheme. The first is it assumes that the carrier signal and the demodulating signal are in phase. What happens if they are out of phase? Try this with the demodulating signal $\cos(2\pi f_c t + \phi)$ where $\phi = \frac{\pi}{3};\ \frac{\pi}{2}$.

The phase of the original signal is intrinsically unknowable, so the above can become a serious problem. There's a nastier effect: while the broadcaster may control the carrier wave $\cos(2\pi f_c t)$ quite accurately, you may not be able to reconstruct that wave to the same accuracy. Assume that the broadcaster uses a carrier frequency f_s, but you use a demodulating frequency of $f_s + \Delta f$. Try reconstruction using values of Δf varying from one to five percent of f_s. How does the demodulated signal sound?