Problem 1: Let \( E_{nu} = \bigcup_{n=1}^{\infty} \{ \lfloor \frac{n!}{\pi^2} \rfloor \} \) be a set of points.

Since \( E_{nu} \) is a union of finite sets, it is measurable.

Since \( E_{nu} \) is a union of finite sets, it is a subset of a countable union of finite sets.

It follows that

\[
\lim_{N \to \infty} \mu(\bigcap_{n=N}^{\infty} E_{nu}) = 0
\]

By the Cantor set property.

Let \( E \) be a set such that

\[
\mu(E) \leq \sum_{n=1}^{\infty} \mu(E_{nu}) \leq 2^{-n}
\]

(by above limit).

Hence, if \( E = \bigcup_{n=1}^{\infty} E_{nu} \),

\[
\mu(E) \leq \sum_{n=1}^{\infty} \mu(E_{nu}) \leq 2^{-n}
\]

Now if \( x \in E \), then \( x \in \bigcap_{n=N}^{\infty} E_{nu} \).

For \( x \in E \), then \( x \in \bigcap_{n=N}^{\infty} E_{nu} \).

Hence, \( |P(n) - f(x)| < \frac{1}{n} \) for all \( n \geq N \), est. unif. cont.
Case 1: \( S_1 = 1 \)

Case 2: \( S_1 = 0 \)

In this case, \( S_2 \) is 1.

Conversely, consider \( E \) as \( E \). For \( E \), given \( E \) and \( E \) are mutually exclusive, we have

For any \( E \), \( E \), and \( E \), we get

Problem 5.5:

It's been a while since I solved something like this. I think that \( f(x) = \sin(x) \) is the desired example.
Hence,
\[ \int_{E} \chi_{A_n} \, d\mu = \mu(E) \]
for \( \forall A \in \mathcal{A} \).

If \( \mu(E) \geq \lambda \), then
\[ \sup_{E} \int_{E} \chi_{A_n} \, d\mu = \sup_{E} \int_{E} l_{12} \, d\mu \]
In fact,
\[ \sup_{E} \int_{E} l_{12} \, d\mu = \sup_{E} \int_{E} l_{1} \, d\mu \]
by MCT.

\[ \sup_{E} \int_{E} l_{12} \, d\mu \geq E \]

For any \( E \), \( E \subseteq [-N, N] \) and \( N \) large enough,
\[ \mu(E) \geq 0 \]
Choose an approximation of Littlewood-Lebesgue (up to a set of measure zero).

Here \( \sup_{E} \int_{E} l_{12} \, d\mu \rightarrow \int_{E} \mu(E) \to 0 \).

Hence \( \forall N \in \mathbb{N}, \int_{E} l_{12} \, d\mu = \mu(E) \to 0 \) if \( \mu(E) \to 0 \).

Problem 5 (additional work):
\[ \int_{E} f \, d\mu = f \text{ in } L^2 \text{ if and only if } \int_{E} f \, d\mu = f \text{ in } L^2 \text{ for any } \chi_{E} \in L^2 \text{ is } \mu \text{-measurable.} \]

Note that \( \int_{E} 1 = 1 \) so that
\[ \left( \int_{E} l_{12} \right)^{1/2} = 1. \]

By Hölder's inequality,
\[ \left( \int_{E} \|g\| \right)^{1/2} \leq \|f\|_{L^2} \cdot \|g\|_{L^2} = \|f\|_{L^2} \cdot \|g\|_{L^2}. \]

Now weak limit \( f = 0 \) so \( \int_{E} \|g\| \leq \|g\|_{L^2}. \)

If \( g \) is a constant \( \mu \)-a.e., \( \int_{E} \chi_{E} \, d\mu = 0 \) for \( \chi_{E} \) in \( L^2 \).
Let \( g = \chi_{E} \) where \( E \) has finite measure. Then
\[ \mu \text{-a.e., weak limit.} \]

Hence, \( g \) is also \( \mu \)-a.e.
Why is $\int f g = 0$ eventually?

By Littlewood, $E$ is essentially the limit distant one of intervals.

For each such interval, perhaps the endpoint slightly to move them dyadic.

This new approximates $E$, problems

nicely eventually, in the following sense.

If $E = [\frac{1}{2}, 1]$ for example,

we've split at stage $n$

into intervals of length $2^{-n}$

and dyadic endpoints.

So $E \rightarrow \left[ \frac{1}{2}, \frac{1}{2} \right] \cup \left[ \frac{1}{4}, \frac{3}{4} \right] n = 1$

positive negative

$\rightarrow \left[ \frac{1}{4}, \frac{1}{2} \right] \cup \left[ \frac{1}{4}, \frac{3}{4} \right] \cup \left[ \frac{3}{4}, 1 \right] n = 2.$

negative positive negative

Notice that $\left[ \frac{1}{2}, \frac{3}{4} \right]$ has measure less than $2^{-n}$.

So as intervals will clearly not be 0, it doesn't cancel exactly.

Instead, choose $\frac{1}{2}$ to a dyadic interval close by,

you should fix the above issue (in the limit).