3. Note that $e^{2\pi k} = 1$

Whenever $2\pi k \neq \pi / 2 + 2\pi n$, $k \neq 0$.

Thus $f(2\pi k) = 0$ for all $k \neq 0$.

But, $2\pi k \to \infty$ so that

$\lim_{k \to \infty} f(2\pi k) = f(\infty) = 0$.

Now, the only meromorphic function in the extended complex plane is a rational function. Yet, $f$ is entire so $f$ must be a polynomial.

However, $f$ is a non-constant polynomial if and only if $\lim_{z \to \infty} |f(z)| = \infty$.

Since this limit is zero, it must be that $f$ is a constant function. We know that $f(z) = 0$ for some $z$, so $f(0) = 0$.

I think this is wrong because I do not use the hypothesis $f(z) = e^{2\pi i z}$.

Hence, I must be a Möbius transformation.

From here, I think we can algorithmically check that $P(Z) = \frac{1}{Z}$ is the only possibility.

C my reasoning is to go through the preimages of $g \circ f^{-1}: D \to D$, which include $CZ$, $\frac{Z - w}{1 - \overline{w}Z}$ with $|w| = 1$, w = 1. The only one I call get to work was $CZ$ with $c = 0$.

2. Similar to this, I'm not 100% sure this method works.

Let $z \to \infty$ so that $f(z) = 1$ and have $f(\infty) = 1$. So, $f$ is meromorphic in the extended plane and thus is a rational function.

We now look at the zeroes/poles of $f$. Evidently, if there is a zero at $z = \frac{1}{i}$ then there is a pole at $z = -z_i$.

(Clearly, this makes sense since we assume $f(0) = 1$, so there is no issue of $f(z) = 0$ concerning $z = i$).

The converse is also true. Hence, we can write

$$f(z) = \prod_{z_i \neq 0} \frac{z - z_i}{z_i} \cdot e^{\sum_{z_i} \frac{1}{z_i}}$$

where $\{z_i\}_{z_i \neq 0}$ are the zeroes of $f$. \[\]
The only thing I can think of (which seems rather silly) is the following:

1. Function $f(z) = f$.
2. Let $g(z) = \lim_{n \to \infty} f_n(z) = z e^{i \pi}$.

Just define it as the pointwise limit.

Now let $h(z) = \frac{1}{g(z)}$, which is analytic and agrees with $f(z)$ at $n = 2, 3, \ldots$

By the identity theorem in the extended plane, $\infty$ is an accumulation point.

This leads to:

$$f(z) = h(z)$$

implies $f = h$ everywhere. I may need further rigorous analysis for this, which I do not think is guaranteed.

**Addendum:** Just noticed this, so I thought I'd add it in. Let $g(z) = f(z) + \epsilon$

Then $g(z)$ has two fixed points.

Since $H$ is simply connected, by the Riemann mapping theorem we can construct $\tilde{g}(z) = H^{-1} \circ g \circ h(z)$

such that $\tilde{g}(z) = z$ at two fixed points. Then, $\tilde{g} = I_d$ by Schur's theorem.

Follows that $g(z) = z$, so I must be right.