M361 Assignment 4

Due in class Thursday, September 25.

1. Evaluate
$$\lim_{z\to i} \frac{z^2+iz+2}{z^2+1}$$
(You may use any results discussed in lecture, i.e. you do not need to use the ϵ-δ definition).

2. (a) Find functions $f, g : \mathbb{C}\backslash\{0\} \to \mathbb{C}$ such that $\lim_{z\to 0}(f(z)+g(z))$ exists but $\lim_{z\to 0} f(z)$ does not exist.
   (b) Find functions $f, g : \mathbb{C}\backslash\{0\} \to \mathbb{C}$ such that $\lim_{z\to 0}(f(z)g(z))$ exists but $\lim_{z\to 0} f(z)$ does not exist.

3. Give an example of a continuos function $f : S \to \mathbb{C}$ such that S bounded but f is unbounded.

4. (a) Show that
$$\lim_{w\to 0} \frac{\overline{w}^2}{w} = 0.$$
Hint: You may use the fact that $\lim_{z\to z_0} f(z) = 0$ *if and only if* $\lim_{z\to z_0} |f(z)| = 0$.
   (b) Using part (a), show that the origin is the only point where the function $f : \mathbb{C} \to \mathbb{C}, f(z) = \overline{z}^2$ is differentiable.

Exercises from the textbook:
p. 62-63 #2(a)(b), 3, 8.