M361 Assignment 5

Due in class Thursday, October 2.

1. Suppose $f = u + iv : S \to \mathbb{C}$ is analytic (that is, f is analytic at every point in S). Show that u and v are harmonic functions on S, i.e.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0.$$

   Hint: you may assume the fact that if f is analytic then u and v have continuous partial derivatives, in which case the order of taking partial derivatives does not matter.

2. Recall from homework 2 the functions $S : \mathbb{C} \to \mathbb{C}, C : \mathbb{C} \to \mathbb{C}$

$$S(z) = \frac{e^{iz} - e^{-iz}}{2i}$$
$$C(z) = \frac{e^{iz} + e^{-iz}}{2}.$$

   (a) Why are S and C entire?

   (b) Show that $S'(z) = C(z)$ and $C'(z) = -S(z)$.

   Remark: From now on we will define $\sin z$ *for a complex number* z *to be* $S(z)$*. Similarly, we define* $\cos z$ *to be* $C(z)$*.*

Exercises from the textbook:
p. 71: #1(a)(b)(c)(d), #2(a)(b)(c), #3
p. 77: #1(a)(c)(d), #2(a)(b)(c), #4(a)(b)(c), #7.