Directions: Indicate all answers on the answer sheet provided. Please enter your name and student # where requested. An assertion is to be interpreted as a true-false question and answered “A” if true, “E” if false. Each question has only one correct answer.
Turn in legible scratchwork for possible partial credit. (Even for a True-False question, you can give a reason or counterexample.)

1. Suppose $y$ is a function of $x$ satisfying $y^3 - y - x = 6$, and that $y(0) = 2$. What is $y'(0)$?
   (Hint: Differentiate implicitly and solve for $y'$.)
   (A) 1/2   (B) 1/13   (C) 1/11   (D) 7/11   (E) None of these

2. If $f(x)$ is differentiable everywhere, and $f(1) = 3$, $f(2) = 1$, then by the Mean Value Theorem, there is a $c$ with $1 < c < 2$ and $f'(c) =
   (A) 2   (B) -2   (C) 1/2   (D) -1/2   (E) None of these

3. Suppose $f(x)$ is a function defined whenever $x \neq 0$. Suppose that $f'(x) < 0$ for all $x$ where $f$ is defined. Then we can conclude that $f(1) < f(-1)$.

4. Suppose $f(x) = 3\sqrt{x} - \frac{x}{2}$, for $x > 0$. Then at $x = 1$, $f$
   (Note: $f'(x) = \frac{3}{2\sqrt{x}} - \frac{1}{2}$.)
   (A) is decreasing   (B) is increasing   (C) has local max   (D) has local min
   (E) None of these

5. For the same $f(x)$ as in 4, on what interval is $f$ decreasing?
   (A) $(9, \infty)$   (B) $(0, 9)$   (C) $(\sqrt{3}, \infty)$   (D) $(0, \sqrt{3})$   (E) None of these

6. For the same $f(x)$ as in 4, at $x = 9$, $f$
   (A) is decreasing   (B) is increasing   (C) has local max   (D) has local min
   (E) None of these
7. Consider the function \( f(x) = x^3 - 3x - 1 \) defined for all real \( x \). \( f(x) \) has a **local maximum** at some point \( x = c \). What is \( f(c) \)? *(Note*** We are asking for \( f(c) \), not \( c \)!)*

(A) -1  (B) 0  (C) -3  (D) 1  (E) None of these

8. Now consider the function \( f(x) = x^3 - 3x - 1 \) defined on \([-10, 10]\). What is the **absolute maximum** of this function?

(A) 1  (B) 10  (C) 969  (D) Doesn’t exist  (E) None of these

9. Suppose now that \( g(x) = x^4 - x^3 \) defined for all real \( x \). Which of the following is true for this function?

(A) \( g(3/4) \) is a local max  (B) \( g(3/4) \) is a local min  (C) \( g(0) \) is a local max

(D) \( g(0) \) is a local min  (E) None of these

10. Suppose \( h(x) \) is a function defined for all real \( x \), and \( h'(0) = 0, h''(0) = 0, h'(x) > 0 \) for \( x < 0 \), \( h'(x) < 0 \) for \( x > 0 \). Which of the following is true for this function?

(A) \( h(0) \) is a local max  (B) \( h(0) \) is a local min  (C) \( (0, h(0)) \) is an inflection point

(D) \( h(0) \) is not a local extreme  (E) None of these

11. Suppose \( f(x) \) is defined for all real \( x \). Which of the following implies that \( f(0) \) is a local maximum?

(A) \( \{f'(0) = 0, f''(0) = 0\} \)  (B) \( \{f'(0) = 0, f(0) = 1\} \)  (C) \( \{f'(0) = 0, f''(0) > 0\} \)

(D) \( \{f'(0) = 0, f''(0) < 0\} \)  (E) None of these

12. Suppose \( g(x) \) is defined and differentiable for all real \( x \), and \( g'(x) = 0 \) only for \( x = 0 \). Suppose also that \( g(0) = 1 \), and \( \lim_{x\to\pm\infty} g(x) = 0 \). Which of the following is a correct conclusion?

(A) The absolute min of \( g \) is 1.  (B) The absolute max of \( g \) is 1.

(C) \( g \) has a local max at \( x = 0 \), but it may not be the absolute max.

(D) \( g \) has an absolute max, and an absolute min.  (E) None of these
13. What is the absolute maximum of \( f(x) = \frac{x}{1+x^2} \) (defined for all \( x \))?  
(Note***: We want the value of \( f \), not the \( x \) where the max occurs.)  
(A) 0  (B) 1  (C) \( \frac{1}{2} \)  (D) 2  (E) None of these

14. What is the maximum possible area of a rectangle whose total perimeter is \( L \)?  
(Note*** We are asking for the area, not some length of a side.)  
(A) \( \frac{L^2}{16} \)  (B) \( \frac{L^2}{4} \)  (C) \( \frac{L^2}{8} \)  (D) \( L^2 \)  (E) None of these

15. Consider the function \( g(x) = x^3 - x^2 \) defined for all \( x \). On which of the following intervals is \( g(x) \) concave down?  
(A) \( (-\infty, \frac{2}{3}) \)  (B) \( \left( \frac{2}{3}, \infty \right) \)  (C) \( (0, \infty) \)  (D) \( (-\infty, \frac{1}{3}) \)  (E) None of these

16. Which of the following functions \( h(x) \) has an inflection point at \( (0,0) \)?  
(A) \( h(x) = x^2 \)  (B) \( h(x) = x^3 - x^2 \)  (C) \( h(x) = x^5 \)  (D) \( h(x) = x^5 + x^4 \)  (E) None of these

17. Consider the function \( f(x) = x \) on \( [0,1] \), and the partition \( P = \{0, \frac{1}{2}, 1\} \) of \( [0,1] \).  
What is the lower sum \( L_f(P) \) corresponding to this data?  
(Recall that this is a sum of the form \( \sum_{i=1}^{2} m_i(\Delta x)_i \) where \( m_i \) denotes a minimum value.)  
(A) \( \frac{1}{2} \)  (B) \( \frac{1}{4} \)  (C) \( \frac{3}{4} \)  (D) 1  (E) None of these

18. With the same function and partition as in the previous problem, what is the Riemann Sum obtained if we use the midpoint of each of the 2 subintervals of the partition as the points \( x_i^* \) \( (i = 1, 2) \) where \( f \) is evaluated?  
(Recall that this is a sum of the form \( \sum_{i=1}^{2} f(x_i^*)(\Delta x)_i \) where \( x_i^* \) are points in the sub-intervals; here we assume these are midpoints.)  
(A) \( \frac{1}{2} \)  (B) \( \frac{1}{4} \)  (C) \( \frac{3}{4} \)  (D) 1  (E) None of these

19. What is \( \int_1^4 \sqrt{x} \, dx \)?  
(A) 1  (B) \( \frac{14}{3} \)  (C) -\( \frac{1}{4} \)  (D) \( \frac{16}{3} \)  (E) None of these
20. What is \( \int_{0}^{\pi/4} \sec^2(x) \, dx \)?
(A) 1/3  (B) 0  (C) \( \frac{\sec^3(x)}{3} \bigg|_{0}^{\pi/4} \)  (D) 1  (E) None of these

21. Suppose \( f(x) = f_{1}^{x} \frac{dt}{1+t^4} \). What is \( f'(2) \)?
(A) \( \pi/5 \)  (B) -16/17  (C) 1/17  (D) 1  (E) None of these

22. Suppose \( f(x) = f_{1}^{x^2} \frac{dt}{1+t^4} \). What is \( f'(x) \)?
(A) \( \frac{2x}{1+x^8} \)  (B) \( \frac{2x}{1+x^4} \)  (C) \( \frac{1}{1+x^8} \)  (D) \( \frac{1}{1+x^6} \)  (E) None of these

Answers:
C B E B A C D C B A
D B C A D C B A B D
C A