Spring 2024 – Graduate Course 'Methods of Applied Mathematics II'

M 383D (54020) and CSE 386D (62260)

General Information:

Meeting Hours & Lecture Room: T - TH 12:30-1:45pm, RLM 10.176

Instructor: Prof. Irene M. Gamba - Office: RLM 10.166, Phone: 471-7150
E-Mail: gamba@math.utexas.edu - Office hours: TBA and by appointment

Discussion Hours: TBA

Guidance textbook: Arbogast-Bona book or notes
Homework, Exams, and Grades: Homework will be assigned regularly. Students are encouraged to work in groups; however, each student must write up his or her own work.

Three mid-term exams will be given in class on the following tentative dates: The first one on **Thursday March 7th**, and the second one on **Tuesday April 23rd**.
There will not be a final exam.
The final grade will be based on the homework and the two exams.

Course Description: This is the second semester of a course on methods of applied mathematics. It is open to mathematics, science, engineering, and finance students. It is suitable to prepare graduate students for the Applied Mathematics I & II Preliminary Exam in mathematics and the Area A Preliminary Exam in the SCEM graduate program.

Semester I.

1. Preliminaries (topology and Lebesgue integration)
2. Banach Spaces
3. Hilbert Spaces
4. Spectral Theory
5. Distributions

Semester II.

6. **The Fourier Transform** (3 weeks)
   - The Schwartz space and tempered distributions.
   - The Fourier transform.
   - The Plancherel Theorem.
   - Fundamental solutions of PDE's.
7. **Sobolev spaces** (3 weeks)
   - Basic Definitions.
   - Extention Theorems.
   - Imbedding Theorems.

- The Trace Theorem.

8. **Variational Boundary Value Problems** (BVP) (3 weeks)
   - Weak solutions to elliptic BVP's.
   - Variational forms.
   - Lax-Milgram Theorem.
   - Galerkin approximations.
   - Green's functions.
9. **Differential Calculus in Banach Spaces and Calculus of Variations** (4 weeks)
   - The Frechet derivatives.
   - The Chain Rule and Mean Value Theorems.
   - Higher order derivatives and Taylor's Theorem.
   - Banach's Contraction Mapping Theorem and Newton's Method.
   - Inverse and Implicit Function Theorems, and applications to nonlinear functional equations.
   - Extremum problems, Lagrange multipliers, and problems with constraints.
   - The Euler-Lagrange equation.
   - Applications to classical mechanics and geometry.
10. **Some Applications** (if time permits)

Some additional references:

1. A. Adams, *Sobolev Spaces*, Academic Press, 1975.
2. -P. Aubin, Applied Functional Analysis, Wiley, 1979.
3. Caratheodory, *Calculus of Variations and Partial Differential Equations of the First Order*, 1982.
4. W. Cheney and H.A. Koch, *Notes on Applied Mathematics*, Department of Mathematics, University of Texas at Austin.
5. Debnath and P. Mikusinski, *Introduction to Hilbert Spaces with Applications*, Academic Press, 1990.
6. B. Folland, *Introduction to Partial Differential Equations*, Princeton, 1976.
7. M. Gelfand and S.V. Fomin, *Calculus of Variations*, Prentice-Hall, 1963; reprinted by Dover Publications.
8. Jost and X. Li-Jost, Calculus of Variations, Cambridge, 1998,
9. N. Kolmogorov and S.V. Fomin, *Introductory Real Analysis*, Dover Publications, 1970
10. Kreyszig, *Introductory Functional Analysis with Applications*, Wiley, 1978.
11. H. Lieb and M. Loss, *Analysis*, AMS, 1997.
12. T. Oden & L.F. Demkowicz, *Applied Functional Analysis*, CRC Press, 1996.
13. W.J. Olver, *Asymptotics and Special Functions*, Academic Press, 1974.
14. Reed & B. Simon, *Methods of Modern Physics, Vol. 1, Functional analysis*.
15. Rudin, *Functional Analysis*, McGraw Hill, 1991.
16. Rudin, *Real and Complex Analysis*, 3*rd* Ed., McGraw Hill, 1987.
17. Sagan, Introduction to the Calculus of Variations, Dover, 1969.
18. E. Showalter, *Hilbert Space Methods for Partial Differential Equations*, available at World Wide Web address http://ejde.math.txstate.edu//mono-toc.html. (Links to an external site.)
19. Stein and G. Weiss, *Introduction to Fourier Analysis on Euclidean Spaces***,** Princeton, 1971.
20. Yosida, *Functional Analysis***,** Springer-Verlag, 1980.