Viscous approximation to transonic gas dynamics: flow past profiles and charged–particle systems

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Abstract.

A boundary value problem in a domain Ω is considered for a system of equations of Fluid-Poisson type, i.e. a viscous approximation to a potential equation for the velocity coupled with an ordinary differential equation along the streamlines for the density and a Poisson equation for the electric field.

A particular case of this system is a viscous approximation of transonic flow models. The general case is a model for semiconductors.

We present an overview of the problem and, in addition, we show an improvement of the lower bound for the density that controls the rate of approach to cavitation density by a quantity of the order of the viscosity parameter to the power that corresponds to the inverse of the enthalpy function.

This is a necessary step in the existing programs in order to show existence of a solution for the transonic flow problem.

§1. Introduction

The present paper deals with steady-state fluid level model that is an approximation to inviscid potential flow that changes type. Viscosity and friction are added and existence of 2 dimensional solutions has been established by the authors (see [GM]), along with some uniform bounds in the viscosity parameters, for geometries and boundary conditions corresponding to flow in channels that include gas flow past a profile and charged particle transport in the modeling of semiconductor devices.

These models also appear in higher hierarchies of macroscopic approximation of particle–charged systems in the modeling of electron–ion plasmas and semiconductor devices where the transport is induced by the superposition of an internal and an externally applied electric field.

The resulting macroscopic approximation yields a fluid level equation coupled with a Poisson equation for the corresponding electric potential. See, for instance, Anile and Muscato [AM], Azoff [Az], Baccarani and Woderman [BW], Blotekjaer [Bo], Bringer and Schön [BS], Jerome [J], Markowich, Ringhofer and Schmeiser [MR], Poupaud [P3], on justifications for these models. In addition, the Appendix contains a small survey about mathematical developments on charged–particle transport in the modeling of semiconductor devices. Here, we "model a model" by tailoring the steady state electro-hydrodynamic model into a solvable problem. By solvable we mean we have established in [GM] an existence theorem for a solution. The implication of the theorem is that model can be computed.

The "fixed up" model has potential gas flow as a special case hopefully give insight into certain semiconductor regimes, just as potential transonic flow did for the full fluid equations with small viscous effects. Indeed, if no electric field is present, the system reduces to a two-dimensional steady irrotational compressible viscous flow model in a "channel". Classical references on this model can be found in Courant-Friedrichs [CF], Morawetz [M1], Serrin [Se] and Synge [Sy].

Transonic flow equations

We recall that the transonic flow model is given by

$$\operatorname{div}(\rho\nabla\varphi) = 0, \qquad \frac{1}{2}|\nabla\varphi|^2 + i(\rho) = K \tag{0.1}$$

where φ is the potential flow function, $\nabla\varphi$ the associated velocity field, and $i(\rho)$ represents the enthalpy function and is usually a power law for the density ρ that satisfies $i(\rho), i'(\rho) > 0$. The constant K, the Bernoulli's constant, needs to be positive.

Existence of physical meaningful solutions to system (0.1), i.e. entropic weak solutions, remains an unsolved problem for any domain or boundary values.

Hence, in an attempt to construct entropic solutions to system (0.1), viscous approximation models are usually considered, with the hope that they can be solved and have enough uniform bounds in the viscosity parameter in order to obtain compactness results that would yield the existence of entropic solutions to the inviscid system (0.1) by vanishing viscosity methods.

A good approximation to transonic flow models (that is, steady potential flow) is given by

$$\operatorname{div}(\rho\nabla\varphi) = 0 \tag{0.2}$$

$$\frac{1}{2}|\nabla\varphi|^2 - (K - i(\rho)) = \nu\, g(|\nabla\varphi|)\, \Delta\varphi. \tag{0.3}$$

For $\nu = 0$, (0.3) is just Bernoulli's law. See as references for this viscous formulation and its justification, Courant and Friedrichs [CF], Serrin [Sn], Synge [Sy], Morawetz [M1]. Synge [Sy] showed exponential solutions for a linearized thermodynamical system of a viscous fluid that conducts heat, near constant equilibrium states, when specific entropy and specific volume are taken as basic thermodynamical variables. In particular, he stressed the difference in the constituitive form of the viscous terms when comparing hydrodynamics with thermodynamics.

Numerical experiments have been widely developed for system (0.1). For a survey on numerical simulations for transonic flow and approximations see Jameson [Ja] and references therein.

In a recent work, see [GM], we posed and solved a boundary value problem to a class of potential fluid-Poisson systems, which includes system (0.2)-(0.3). There we show the existence of smooth strong solutions that have uniform bounds in the viscosity parameter. More precisely, we show that there is a one parameter family of solutions (φ^ν, ρ^ν) which are infinitely differentiable in the flow domain, with ρ^ν and $|\nabla\varphi^\nu|$ uniformly bounded in ν, and strictly positive for ν fixed.

This is a first and fundamental step in order to achieve a convergence in ν result that would yield a weak solution for the inviscid problem, (0.1) in standard gas dynamics or the larger problem for inviscid hydrodynamic–Poisson systems.

In the case of 2–dimensional transonic flow, constructing weak solutions of system (0.1) from solutions of (0.2)-(0.3) has been outlined by Morawetz in [M1], [M2] using methods of compensated compactness presented by Murat [Mu], Tartar [Tt] and Di Perna [DP], G.Q.Chen [CG], for the initial value problem for the one-dimensional time dependent compressible fluid system of two equations, as in 1–dimensional isentropic gas dynamics with a power pressure law. Recently, 1-dimensional isentropic gas dynamics has been solved in unbounded domains by means of artificial viscosity and perturbations, for any positive initial density; see Lions, Perthame, Tadmor [LP] and Lions, Perthame and Souganidis [LS].

All these methods require uniform estimates in the parameters of the approximation. Clearly, if we pursue a weak solution (φ^0, ρ^0) of problem (0.1) which is a limit of some subsequence of solutions (φ^ν, ρ^ν)

of (0.2)–(0.3), then it is to be expected, if there is strong convergence, that $i(\rho^0)$ and $\frac{1}{2}|\nabla\varphi^0|^2$ are numbers between 0 and K, and hence $i(\rho^\nu)$ and $\frac{1}{2}|\nabla\varphi^\nu|^2$ should also be between 0 and K up to, at most, an $\mathcal{O}(\nu^\beta)$-correction.

In fact, we have shown in [GM] that if the speed $|\nabla\varphi^\nu|$ is prescribed in a section of the boundary adjacent to two streamlines boundaries, then, normaly $0 < k_\nu < i(\rho^\nu) < K$ but

$$0 < \frac{1}{2}|\nabla\varphi^\nu|^2 \leq \sup \frac{1}{2}|\nabla\varphi^\nu|^2 \leq \widetilde{K}, \tag{0.4}$$

where $\widetilde{K} = (K + C\nu^{1/2})M$, with M a constant that depends only on the flow domain and C depends on the boundary data and the flow domain.

Later, Gamba [G5] showed that any smooth solution (ρ^ν, φ^ν) of (0.2)–(0.3), that satisfies estimate (0.4) (potential isentropic gas flow case), also satisfies

$$0 < \frac{1}{2}|\nabla\varphi^\nu(x)|^2 \leq K + \mathcal{C}\ \frac{\nu}{\left(\mathrm{dist}\{x, \partial\Omega\}\right)^2}, \tag{0.5}$$

for any x in the interior of the 2 or 3 dimensional flow domain Ω, and a growth condition on the enthalpy function $i(\rho)$, to be specified below. In particular, if the enthalpy function is the one associated with a γ–pressure law with $1 < \gamma$, then the necessary growth condition is satisfied for $1 < \gamma < 2$ in the 2–dimensional case and $1 < \gamma < \frac{3}{2}$ in the 3–dimensional one. The constant $\mathcal{C}$ depends on $\Omega, \widetilde{K}$ and the growth conditions for the functions $i(\rho)$ and $g(|\nabla\varphi(x)|)$ from (0.3).

In addition, for the 2–dimensional case (where [GM] showed existence of solutions for a boundary value problem associated with (0.2)–(0.3) that satisfied estimate (0.4)) we extend estimate (0.5) to some boundary points x in $\partial\Omega \setminus \partial_3\Omega$, where $\partial_3\Omega$ denotes the section of the boundary of the flow domain Ω where the speed was prescribed (named outer boundary). The parameter ν is replaced by ν^β for these boundary estimates and the exponent β depends on the location of x in $\partial\Omega \setminus \partial_3\Omega$, and $\mathcal{C}$ denotes a number that depends on the local curvature of the boundary at the point x and the data of the boundary problem and the coarse bound $\widetilde{K}$. In fact $\mathcal{C}$ is bounded by a function of the Jacobian transformation that corresponds to the conformal map that takes Ω into a rectangle.

In addition, [G5] gave an estimate similar to (0.5) for the potential fluid-Poisson system presented below. In this case the estimate reads

$$0 < \frac{1}{2}|\nabla\varphi^\nu(x)|^2 + \mathcal{R}(\varphi^\nu) - q\Phi^\nu \ \leq K + \mathcal{C}\ \frac{\nu^{1/2}}{\left(\mathrm{dist}\{x, \partial\Omega\}\right)^2}, \tag{0.6}$$

also for any x in the interior of the 2 or 3 dimensional flow domain Ω, and the same growth condition on the enthalpy function $i(\rho)$ as in the gas flow case. Here the constant $\mathcal{C}$ depends on $\Omega, \widetilde{K}$, the growth conditions for the functions $i(\rho)$ and $g(|\nabla\varphi(x)|)$ and the bounds on $\mathcal{R}$ and Φ_ν (these are proven to be ν independent bounds in the 2–dimensional existence theory).

We point out that *cavitation speed* in isentropic gas flow is the constant value $(2K)^{1/2}$. However, for the hydrodynamic fluid–Poisson system, *cavitation speed* is not constant any longer . It is the speed at vacuum state given by the model, i.e. $|\nabla\varphi^\nu(x)|$ reaches *cavitation speed* when it takes the value $(\,2(K\mathcal{R}(\varphi^\nu)-q\Phi^\nu)(x))^{1/2}$. Hence, the convergence analysis in the limiting vanishing parameter ν will also need estimate (0.6).

This estimate deteriorates as x is at $\nu^{\beta/2}$-distance from the outer boundary $\partial\Omega_3$, with $\beta = 1, \frac{1}{2}$, suggesting the possible formation of large boundary layers near $\partial\Omega_3$, as expected from viscous approximations in bounded domains.

Thus, boundary layers yielding values of the speed that are larger than *cavitation speed* can **only** be form at distances of order $\nu^{\beta/2}$ from the outer boundary.

It appears that the estimate (0.5) will still hold even if the section $\partial_3\Omega$ of the boundary is taken to infinity, as the conformal map that takes Ω into an infinite strip tends to the identity map at infinity. As a consequence the solutions would have speeds that remain smooth and uniformly bounded in ν as the outer boundary section $\partial_3\Omega$ is lost at infinite.

In the following first section we present the potential fluid-Poisson model and previous results for the boundary value problem, then we outline the results already obtained. Then we shall present an improvement for the lower bound of the density.

1. Presentation of the problem in the general case

We begin with the steady state conservation laws for mass and momentum and couple them to the Poisson equation for the electric field. The principal variables are charge density ρ, velocity $\vec{u}$, pressure P and electric field $\nabla\Phi$ where Φ is the electric potential. Thus, with $x \in \mathbb{R}^n, n = 2$ or 3, the conservation laws are

$$\operatorname{div} \rho\vec{u} = 0 \tag{1.1.1}$$

$$m\rho(\vec{u}\cdot\nabla)\vec{u} + \nabla P = q\rho\nabla\cdot\Phi + \vec{F} \tag{1.1.2}$$

and the Poisson equation is

$$\Delta\epsilon\Phi = q(\rho - C(x)). \tag{1.2}$$

The extra parameters m, q, ϵ are respectively electron effective mass (parabolic band approximation, see [Bo]) space charge constant and dielectric constant. The vector function $\vec{F}$ represents forces caused by viscosity or friction and we chose $\vec{F}$ to give us a solvable set of equations that is as consistent with the physics as has proved possible and is nowhere at wild variance with the physical problem. $C(x)$ is the doping profile function and represents the background charge. It is assumed (a) $C(x)$ is a non-negative step function.

In the approximation to the transonic problem $\Phi \equiv 0$ and (1.2) does not apear. The conservation of energy is replaced by taking the pressure P as a given function of $m\rho$, the mass density. This is consistent with the notion that the disturbances we are studying are weak. For example and for simplicity suppose we consider the fluid dynamical case with $\vec{F} = 0$ and now suppose $P = P(\rho, S)$ where S is entropy. The full conservation laws then admit shocks but a change in entropy across a shock would be third order in the strength of the shock. Thus we may take $P = P(\rho)$ provided third order errors may be neglected.

Potential flow and the choice of the viscous-friction force term $\vec{F}$.

If $\vec{F} \equiv 0$, we could look for an irrotational flow, curl $\vec{u} = 0$ so that $\vec{u} = \nabla\varphi$ where φ is the potential and reduce the system to conservation of mass, a Bernoulli law, and Poisson's equation. An irrotational flow is again consistent with the neglect of third order terms. What is more, it is mathematically useful because the system is reduced to fourth order (second order if $\Phi \equiv 0$ and weakly coupled if $\Phi \neq 0$). Henceforth we assume (1.3.1) $\vec{u} = \nabla\varphi$ and choose $\vec{F}$ so that we can make a similar reduction in order. Then we check the physical consistency of this choice of $\vec{F}$. Thus a preliminary choice would be

$$\vec{F} = \nabla\Psi$$

$$\Psi = \nu\Delta\varphi - K\varphi$$

where ν is a coefficient of viscosity and K a coefficient of friction. Then with $\vec{u} = \nabla\varphi$ the momentum equations (1.1.2) can be integrated to yield the Bernoulli's law:

$$\frac{1}{2}m|\nabla\varphi|^2 + i(\rho) = -q\Phi + \nu\Delta\varphi + K\varphi + \text{const}$$

Here $i(\rho)$ is the enthalpy, i.e. $i(\rho) = \int \frac{dP(\rho)}{\rho}$ where

$$\begin{cases} i(\rho) \geq 0 & \text{if } \rho \geq 0 \\ i(\rho) = 0 & \text{otherwise}, \end{cases} \tag{1.4}$$

and $i'(\rho) > 0$ if $\rho > 0$. Furthermore we assume (b) $P(\rho)$ is convex or $i''(\rho), i'''(\rho)$ are nonnegative.

We assume in this paper that the constant K is independent of ν and is adjusted by the data in order to be consistent with the inviscid problem (i.e. setting $\nu = 0$).

The first term in $\vec{F}$ is $\nu\rho\Delta\vec{u}$ which is the appropriate viscous force and the second is $\tau_p^{-1}m\rho\vec{u}$ which represents a friction term. The viscous term, with ν a constant coefficient of viscosity, is essentially the same as that of Serrin [Se] or Synge [S]. The friction term is, as in Baccarani and Woderman [BW] and Odeh, Gnudi and Rudan [OGR], the momentum density divided by a constant velocity relaxation time τ_p.

However we cannot solve the b.v.p. with this choice of $\vec{F}$. To solve the system the viscous term in Ψ, i.e. $\nu\Delta\varphi$, must be modified to go to infinity at zero speed like $|\nabla\varphi|^2$ and to zero at infinite speed like $|\nabla\varphi|^p$, $p \geq 3$. For similar reasons τ_p must become infinite as $\varphi \to \pm\infty$. Thus the force term $\vec{F}$ is given by

$$\vec{F} = \nabla\Psi, \quad \Psi = \nu\mathcal{G}\Delta\varphi - \mathcal{R}(\varphi) \tag{1.5}$$

where $\mathcal{G}$ and $\mathcal{R}$ have the desired properties.

The case of interest is low viscosity, ν small. It turns out that in our b.v.p. $|\nabla\varphi|$ and φ are uniformly bounded. In the range of speed and potential that occur $\mathcal{G}(|\nabla\varphi|) \sim 1$ (except near zero speed) and $R(\varphi)$ is linear. Thus the restrictions on viscous and friction coefficients have little effect on the physical interpretation of the result.

Therefore, using (1.5) for the force term, we obtain Bernoulli's law as follows

$$\frac{m}{2}|\nabla\varphi|^2 + i(\rho) = K + q\Phi - \mathcal{R}(\varphi) + \nu\mathcal{G}\,\Delta\varphi. \tag{1.6}$$

The conservation of mass (1.1.1) becomes for potential flow

$$\operatorname{div}\rho\nabla\varphi = 0. \tag{1.7.1}$$

For the 2-dimensional case, it is also possible to define a stream function ψ from (1.1.1), whose level curves describe the particle path at any point and are orthogonal to the corresponding level curves of the potential flow φ, that is with $\rho\vec{u} = (-\psi_y, \psi_x)$,

$$\operatorname{div}(\tau\nabla\psi) = 0 \tag{1.7.2}$$

where $\tau = \rho^{-1}$.

That motivated us to seek a boundary value problem for which we can prove existence and also establish the conditions that might eventually lead to the existence of a corresponding limiting solution as the parameter ν becomes very small with respect to the other constants involved in the model, i.e. the existence of a corresponding limiting inviscid solution as proposed by Morawetz [M1] in order to solve the transonic gas flow model. Such a program, see [M2] can not be carried out without better bounds than we get for the state variables ρ and $\vec{u} = \nabla\varphi$.

In [GM] paper we focussed on the existence of solutions for the "viscous" potential fluid-Poisson system (1.6)–(1.7) and (1.2) in "smooth" four-sided domains and with boundary conditions relevant to the physical problem: two opposite tangential flow walls, an inflow condition by prescribing a constant potential flow function and prescribed speed in the remainding part of the boundary, which we shall call the outer boundary. These boundary conditions are to be described in detail later, nevertheless we add here that in the case of gas flow past a profile our outer boundary condition means that 'almost' Mach number is prescribed if ν is very small.

Also, as in the case of approximations to isentropic flow past a profile, we proved existence of a regular solution such that the density and speed are uniformly bounded in ν, away from zero and infinity, and the electric field is also uniformly bounded in ν, with bounds similar to those in (0.4) and (0.5).

A physically more correct one-dimensional system (1.1)–(1.2) was solved in [G1]. There, it is proved that the solution ρ, v and Φ is of uniformly bounded variation and that there exists a limiting inviscid solution ρ^0, v^0 and Φ^0 of bounded variation such that the convergence from the viscous solution to the inviscid one is pointwise and in $L^1(\Omega)$. In addition ρ^0 and v^0 might have admissible shocks, that is, discontinuities that keep the mass and momentum flux terms Lipschitz continuous functions that satisfy the "entropy condition" i.e. the density increases across the discontinuity in the direction of the particle path. In addition, the possible formation of boundary layers was analyzed in [G1] and [G3].

Remark Setting $\nu = 0$ in (1.6) we obtain a transonic type of equation for the inviscid model. Indeed, taking $i(\rho) = K + q\Phi - \mathcal{R}(\varphi) - \frac{m}{2}|\nabla\varphi|^2$, defining $c^2 = i'(\rho)\,\rho = \frac{dP}{d\rho}$ as the *local speed of sound* and combining with the flow equation (1.5), the potential flow function φ satisfies the P.D.E

$$(c^2 - m\varphi_x^2)\varphi_{xx} - 2m\varphi_x\varphi_y\varphi_{xy} + (c^2 - m\varphi_y^2)\varphi_{yy} = S(x, y, \varphi, \varphi_x, \varphi_y, \rho, \Phi, \nabla\Phi),$$

which is a **mixed-type** equation for φ, that is, an elliptic equation for speed values below $c\,m^{-1/2}$ and hyperbolic for speed values above $c\,m^{-1/2}$.

Further conditions

The function $\mathcal{R}$ is a Lipschitz function of φ an $\mathcal{R}(\varphi)$ is bounded above and below by constants $\mathcal{R}_U$ and $\mathcal{R}_L$ respectively.

. The function $\mathcal{G}$ of the "viscous" term $\nu\mathcal{G}\,\Delta\varphi$ is taken as $\mathcal{G} = G(|\nabla\varphi|)$ bounded below by a positive constant C_1, where G and G^{-1} are locally Lipschitz functions and satisfy

$$t^2\, G(t) \to C_1 > 0 \quad \text{as} \quad t \to 0, \tag{1.8.1}$$

and

$$t^{-1}\, G(t) \to C_2 > 0 \quad \text{as } t \to \infty, \text{ and } t^2 \frac{G'(t)}{G^2(t)} \text{ bounded.} \tag{1.8.2}$$

. The enthalpy function $i(\rho)$ satisfies $i(\rho), i'(\rho) > 0$ and the growth condition

$$\frac{1}{n-1} i(\rho) - \rho\, i'(\rho) \geq \frac{1}{n-1} k\, i(\rho) \quad \text{for some} \quad k, 0 < k < 1, \tag{1.9}$$

where $n = 2$ or 3 denotes the space dimension.

. For $\mathcal{R}_L < \mathcal{R}(\varphi) < \mathcal{R}_U$, the Bernoulli constant K satisfies the compatibility condition

$$K - \mathcal{R}_U + q\Phi_L(K) > 0 \tag{1.10}$$

where $\Phi_L(K) = \inf_{\partial_1\Omega\cup\partial_3\Omega} \gamma - \alpha \sup_{\overline{\Omega}} |F_x|^2 (\sup_{\overline{\Omega}} C(x) + i^{-1}(K - \mathcal{R}_L + q\Phi_U))$, for $\Phi_L(K) \leq \Phi^\nu \leq \Phi_U$ where $\Phi_L(K, \Phi_U)$ are ν-independent and depend on the domain Ω and the data of the problem.

Remark: Equations (1.1) correspond to a dissipative approximation to a compressible flow model that satisfies the γ-law: $i(\rho) = \frac{\gamma}{\gamma-1}\rho^{\gamma-1}$, so condition (1.2) is satisfied with $k = 2-\gamma$ and $1 < \gamma < 2$, if $n = 2$ the space dimension and $k = 3 - 2\gamma$ and $1 < \gamma < \frac{3}{2}$ if $n = 3$.

However, since we prove that $|\nabla\varphi^\nu|$ is uniformly bounded above by a number M that depends only on the domain and the data of the boundary value problem for any $\nu \leq \nu_0$, with ν_0 depending also on the domain and the data of the problem, a posteriori we get an existence result and upper uniform bounds even if we take a viscosity that is linear away from stagnation points, that is by setting

$$G(|\nabla\varphi|) = 1 \quad \text{for } 0 < k_\nu < |\nabla\varphi| \leq M^{\frac{1}{2}}$$

and extending it smoothly to $[0, \infty]$ such that growth conditions (1.8.1) and (1.8.2) are satisfied. This is desirable for physical reasons.

Thus, we shall assume here that g is a monotone increasing function and

$$g(|\nabla\varphi|) = \left(1 + \frac{|\nabla\varphi|^2}{2}\right)^{\alpha/2}, \quad \alpha \geq 1 \tag{1.11}$$

for sufficiently large values of $\frac{|\nabla\varphi|^2}{2}$.

Finally, we point out that the compatibility condition (1.10) reduces to $K > 0$ for the approximation to transonic flow model, which is the standard assumption on the Bernoulli's constant. In the case of fluid-Poisson system under consideration, since $i^{-1}(\rho)$ has superlinear growth, then a K verifying (1.3) exists only if the data is chosen properly.

Then, by equation (1.7.1), introducing $\frac{\partial}{\partial\varphi}$ as a derivative along the streamline, yields the identity

$$\Delta\varphi = -\frac{\nabla\rho}{\rho}\cdot\nabla\varphi = -(\ln\rho)_\varphi|\nabla\varphi|^2. \tag{1.12}$$

Using (1.9) in (1.6), we obtain from (1.2) and (1.5) the complete system of equations for φ, ρ and Φ. We add the equation in ψ for convenience.

$$\operatorname{div}(\rho\nabla\varphi) = 0, \qquad \operatorname{div}(\tau\nabla\psi) = 0 \tag{1.13.1}$$

$$\frac{m}{2}|\nabla\varphi|^2 + i(\rho) + \mathcal{R}(\varphi) - q\Phi - K + \nu(\ln\rho)_\varphi|\nabla\varphi|^2\mathcal{G} = 0 \tag{1.13.2}$$

$$\Delta\Phi = \alpha(-C(x) + \rho), \qquad \alpha = \frac{q}{\varepsilon}. \tag{1.13.3}$$

Here $\tau = \rho^{-1}$ represents specific volume.

Note equation (1.13.2) states that the density is governed by the history of the flow on the streamline (it is analogous to upwind differentiation in some sense).

Indeed, heuristically, equation (1.13.2) is the result of the first order expansion of the density at values corresponding to "retarded" flow speed values, along the streamline, with an order of magnitude

$$\nu\frac{G(|\nabla\varphi|)\,|\nabla\varphi|}{(\gamma-1)\,i(\rho)}(\mathbf{x}) = \nu M^2\frac{G(|\nabla\varphi|)}{|\nabla\varphi|}(\mathbf{x}), \tag{1.14.1}$$

with $M(\mathbf{x}) = \frac{|\nabla\varphi|}{(\gamma-1)\,i(\rho)}(\mathbf{x})$ the local Mach number. Thus, equation (1.13.2) is the result of the first order formal Taylor expansion of the density ρ for the equation

$$\frac{|\nabla\varphi(\mathbf{x})|^2}{2} + i\left(\rho\left(\mathbf{x} + \nu\frac{G(|\nabla\varphi|)}{(\gamma-1)\,i(\rho)}\nabla\varphi(\mathbf{x})\right)\right) + \mathcal{R}(\varphi)(\mathbf{x}) - q\Phi(\mathbf{x}) = K \tag{1.14.2}$$

Remark: System (1.13) reduces to a two by two system, in the case of the gas flow problem, with a second and fist order equation. Conditions (1.8) modified the time–like step for the first order equation by a factor proportional to the Mach number and speed. These conditions are necessary for the existence problem as well as the problem of getting uniform bounds with respect to the viscous parameter, as has been shown in [GM]. However, we remark that a condition of the same kind as (1.8.1) has been used in computational transonics in the construction of *time–derivative preconditioner* algorithms for computational compressible viscous flows with very high Reynolds numbers in regions of low Mach flow. The preconditioner matrices are proportional to the Mach number and density as well (see Choi and Merkle [CM] and Nigro, Storti, Idelsohn and Tezduyar [IT]).

The boundary value problem

We look at two possible boundary value problems in the case of 2–dimensional flow model. One of them (case I) is the one with data given in [GM], that is prescribing an inflow boundary, two adjacent tangential flow boundaries (i.e. two walls), and on the rest of it, we prescribe positive non-cavitating speed (i.e. the magnitude of velocity field).

The other boundary value problem (case II) is the one that corresponds to prescribing an outflow boundary condition (i.e the flow potential φ is a positve constant) on the section where the speed was prescribed in the above case.

In fact, we consider some special 2–D flow domains: Let Ω be as in figure 1 below

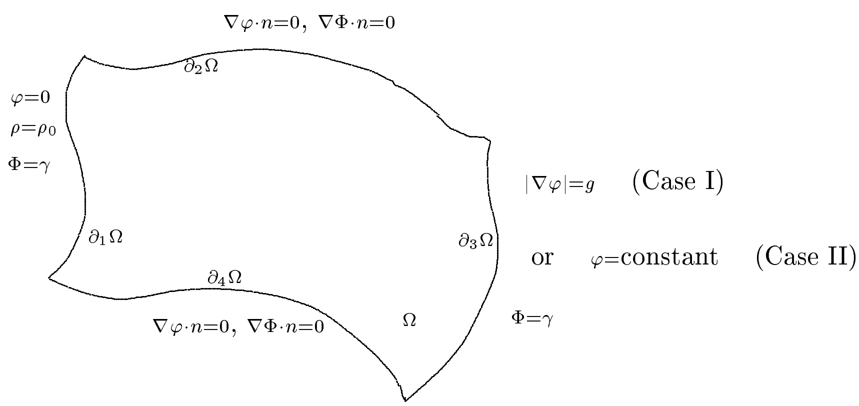

Fig. 1 The flow domain Ω and boundary data

That is , the boundary of Ω is the union of four smooth curves that meet each other at a right angle. Thus, there is a unique conformal transformation that takes Ω into a rectangle R that keep fixed three points (take any three of the angle points including the two that correspond to the inflow boundary meeting the tangential flow ones.) In addition the conformal map is smooth (C^3).

We denote the boundary sections as follows: $\partial\Omega_1$ the inflow boundary section, $\partial\Omega_2$ and $\partial\Omega_4$ the tangential flow boundary sections, and $\partial\Omega_3$ the remainder part of it.

Hence the two boundary value problems under consideration have both same data on the inflow and tangential flow boundary sections, namely, the potential flow function φ = constant on $\partial_1\Omega$, with $(\nabla\varphi \cdot$

$n|_{\partial_1\Omega})(\omega_1) < 0$ and ω_1 is a corner point where $\partial_1\Omega$ meets $\partial_2\Omega$; $\nabla\varphi \cdot n = 0$ on $\partial_\tau\Omega = \partial_2\Omega \cup \partial_4\Omega$. As usual n denotes the outer unit normal.

The density is prescribed at the inflow boundary, i.e. at $\partial_1\Omega$, so that $\rho = r(x)$ on $\partial_1\Omega$ and the electric potential Φ satisfies Dirichlet conditions, i.e. $\Phi = \gamma$ on $\partial_1\Omega$ and $\partial_3\Omega$, and $\nabla\Phi \cdot n = 0$ on tha tangential flow boundary $\partial_\tau\Omega$.

Thus the first boundary value problem prescribes $|\nabla\varphi| = g(x) > 0$, on $\partial_3\Omega$, and the other one just $\varphi =$ constant on $\partial_3\Omega$ (i.e an outflow boundary if this constant is larger that the one for the inflow boundary.)

Existence and uniform bounds

In [G2] and [GM] we solved the first boundary value problem for a more general system of equations than (1.10), namely,

$$\text{div}\,(\rho\nabla\varphi) = 0 \tag{1.15.1}$$

$$-\nu(\ln\rho)_\varphi = f(|\nabla\varphi|^2, \theta, Q_B(\rho, \varphi, \Phi)) \tag{1.15.2}$$

$$\Delta\Phi = \alpha(\rho - C(x)) \tag{1.15.3}$$

where Q_B denotes the squared speed given by Bernoulli's equation (that is, equation (1.5) for $\nu = 0$) and $\theta = \arctan\frac{\varphi_y}{\varphi_x}$, the directional angle of $\nabla\varphi$ taken to be zero at some point on the inflow boundary.

We showed, [GM], that solutions ρ, φ and Φ in $C^{1,\alpha}$, $C^{2,\alpha}$ and $W^{2,p}$ respectively, exist for sufficiently small $\nu > 0$ and under conditions (1.8).

In addition $0 < i(\rho) \le L^*$ for all ν where L^* depends only on the Bernoulli constant K, the bounds for the function $\mathcal{R}$, the domain Ω and the data of our boundary value problem (the latter controls $\sup\Phi^\nu$.)

Then, Q_B takes the value $Q_B = K - i(\rho) - R(\varphi) + q\Phi$ which is the square of the speed given by Bernoulli's Law as long as $Q_B > 0$ (see [GM]).

Thus, any solution of system (1.15) with

$$f(|\nabla\varphi|^2, Q_B) = \left(\frac{m}{2}|\nabla\varphi|^2 - Q_B\right)\,\left(|\nabla\varphi|^2\, G(|\nabla\varphi|)\right)^{-1} \tag{1.16}$$

yields a solution of system (1.13).

In addition, uniform bounds for φ and $|\nabla\varphi|$ hold for the particular case $\nu \ll m$ in an appropriate length scale.

That is, if $i(\rho)$ satisfies condition (1.9) and $\mathcal{G}$ satisfies (1.8), there is a $\nu_0 = \nu_0(\mathcal{G}, \Omega)$ such that

$$0 < k_\nu < |\nabla\varphi^\nu| \le K^*, \quad 0 < l_\nu < \rho^\nu < L^*, \quad \text{for} \quad \nu \le \nu_0 \tag{1.17}$$

and $|\nabla\Phi^\nu|$, $|\Phi^\nu| \le M^*$ all in $\overline{\Omega}$ with K^*, L^* and M^* independent of ν.

In fact the upper bound $\widetilde{K}$ for the speed is given in terms of cavitation speed and the domain Ω. That is, if $F\colon \Omega \to R$ is the conformal map that takes Ω into a rectangle R,

$$K^* = \{\sup_{\overline{\Omega}} |K - \mathcal{R}(\varphi) + q\Phi| + C\,\nu^{1/2}\} \frac{\sup_{\overline{\Omega}} |F_x|}{\inf_{\overline{\Omega}} |F_x|}, \qquad \nu \le \nu_0 \tag{1.18}$$

where $|F_x|$ is the Jacobian of the real valued transformation associated with F, $\nu_0 = \nu_0(k, \mathcal{G}, \|F\|_{C^{1,1}(\overline{\Omega})})$ and $C = C(k, \mathcal{G}, \|F\|_{C^{1,1}(\overline{\Omega})}, K, q, \|\mathcal{R}\|_{C^{0,1}(\overline{\Omega})}$, boundary data).

Then, as we stated above estimate (1.17) is not sharp, as it gives that the speed corresponding to the viscous flow is bounded away from a factor of *cavitation speed* by a term of order $\mathcal{O}(\nu^{1/2})$.

Indeed, the factor is $\sup_{\overline{\Omega}} |F_x| \cdot \{\inf_{\overline{\Omega}} |F_x|\}^{-1} = \exp\{\,\mathrm{osc}(\log|F_x|)\}$ and it is related to the geometry of the domain, a sort of measure of how far the flow domain is from a rectangle, since

$$\mathrm{osc}(\log|F_x|) \le \sup_{\Omega} |\nabla(\log|F_x|)| \mathrm{diam}(\overline{\Omega}) \le \sup_{\overline{\Omega}} |D_{ij}F| (\inf|F_x|)^{-1} \mathrm{diam}(\overline{\Omega}).$$

Unless the domain Ω is originally a rectangle (or ν-close to a rectangle) the value of K^* is rather coarse.

The proof of the sharper estimate requires the existence of an approximation φ^ν solution to (1.13) under conditions (1.8), (1.9) and (1.10), such that $|\nabla\varphi^\nu| < K^*$, where K^* is a ν-uniform constant that depends on the data and the flow domain Ω (see [GM] for the 2-dimensional case.)

Remark: The existence of 3-dimensional solutions to system (1.1) is an open problem. However, under the assumption of existence of solutions $(\rho^\nu, \varphi^\nu, \Phi^\nu)$ to a 3–dimensional boundary value problem, where conditions (1.8), (1.9) and (1.10) are satisfied and $|\nabla\varphi^\nu|$ admits a ν–uniform bound denoted by $\widetilde{K}$, then $|\nabla\varphi^\nu|$ satisfies a sharper pointwise bound "close" to cavitation speed in the interior of the flow domain.

Hence, the following pointwise estimate was proven for $|\nabla\varphi^\nu(x)|$ in the interior of the 2 or 3– dimensional domain for any solution of a boundary value problem associated with system (1.8)–(1.10) and (1.13), where estimates (1.5) are satisfied ([G5]):

$$|\nabla\varphi^\nu(x_0)|^2 \le 2(K - \mathcal{R}(\varphi^\nu(x_0)) + q\Phi^\nu(x_0)) + \frac{C\nu^\beta}{(\mathrm{dist}(x_0, \partial\Omega))^2} \quad \text{for all} \quad \nu \le \nu_1 \tag{1.19}$$

where ν_1 and C both depend on $k^{-1}, K^*, K, g, q, \alpha$ the bounds of $\mathcal{R}$ and Φ. The parameter ν_1 depends on $\mathrm{dist}\{x_0, \partial\Omega\}$. The exponent β is 1 for the compressible gas model and 1/2 for the fluid Poisson system, but it is independent of the space dimension.

Furthermore, in the 2–dimensional case, we can extend the pointwise estimate to the boundary region wich includes the inflow and tangential flow regions. Therefore, for $\Omega \subset \mathbb{R}^2$

$$|\nabla\varphi^\nu(x_0)|^2 \le 2(K - \mathcal{R}(\varphi^\nu(x_0)) + q\Phi^\nu(x_0)) + \frac{C\nu^\beta}{(\mathrm{dist}(x_0, \partial_3\Omega))^2} \quad \text{for all} \quad \nu \le \nu_1 \tag{1.20}$$

where $\beta = \frac{1}{2}$ if the point x_0 lies on the tangential boundary $\partial_\tau\Omega = \partial_2\Omega \cup \partial_4\Omega$. $\beta = \frac{1}{8}$ if x_0 lies on the inflow boundary $\partial_1\Omega$ and $\beta = 1$ if $\partial_1\Omega$ is locally flat around x_0 (i.e. $\kappa_x = 0$ in a neighborhood of x_0 relative to $\partial_1\Omega$). In all cases the constants ν_1 and C depend on $k^{-1}, g, K^*, K, \mathcal{R}, q, \alpha$, data, κ_{x_0}, with κ_{x_0} the local curvature of $\partial\Omega$ at x_0.

Clearly, estimate (1.20) allows the possible formation of large boundary layers near the outflow boundary $\partial_3\Omega$ at distances less of $\mathcal{O}(\nu^{1/2})$ away from the tangential boundary, and of order $\mathcal{O}(\nu^{1/4})$ near the tangential boundary. This behavior excludes velocity overshoots above cavitation speed for the viscous solutions near shock formation for the limiting configuration, away from $\partial_3\Omega$.

We remark that an interesting aspect of this technique is that it works in 3-dimensions as well. Provided the existence of solutions with ν-independent bounds up to the boundary, these estimates seem not to depend on the conformal map but rather on the local parameterization of the boundary to a flat one. As it was described above, we do not include here boundary estimates in the 3–dimensional case due to not knowing if there exist solutions with a coarse ν–uniform bound for the speed. However estimate (1.19) holds *under the assumption* of the existence of solutions with ν–independent bounds for the speed up to the boundary of the flow domain.

Finally, in the 2–dimensional case the following conclusion holds. Assume existence of smooth solutions for the second boundary value problem presented above (case II) where φ^ν is prescribed on $\partial_3\Omega$ as an arbitrary constant above cavitation ratio (i.e. the ratio between cavitation speed and the length of the shortest curve of those that define the tangential flow walls for the domain Ω). Assume that conditions (1.8), (1.9) and (1.10) (i.e. $\partial_3\Omega$ is now an outflow boundary). Then even though we expect $\varphi^\nu \in C^{2,\alpha}(\overline{\Omega})$ and $0 < k_\nu \le |\nabla\varphi^\nu| < K_\nu$, the speed $|\nabla\varphi^\nu|$ can not be ν-uniformly bounded in $\overline{\Omega}$.

Finally we look at the lower estimate l_ν for the density, from (1.17). It was shown to be of order $\exp^{-\frac{1}{\nu}}$ (see (3.24)-(3.25) in [GM]). As $\nu \to 0$ this very fast decay can be much improved for the case of gas flow past a profile.

Theorem 1. *The gas flow past a profile case Let ρ^ν be the $C^{1,\alpha}$ solution of (0.2)–(0.3) constructed in [GM] where the solvability conditions are satisfied. If, in addition to (1.8) the function $G(t)$ satisfies $0 < C_1 \le t^2\, G(t)$ for $t > 0$, then*

$$C\,\nu \le i(\rho^\nu)(x) \le L^* \quad \text{for any} \;\; x \in \overline{\Omega} \setminus \mathcal{D}_\nu, \tag{1.21}$$

where $C = C(\Omega, \gamma, C_1, K^*, L^*, d^{-2})$ for $d = dist(x, \partial_3\Omega)$, independent of ν for $\nu \le \nu_0(\mathcal{G}, \Omega)$. The set $\mathcal{D}_\nu$ satisfies $\partial\Omega \subset \mathcal{D}_\nu \subset \overline{\Omega}$ and contains a "slit" of the outer boundary $\partial_3\Omega$ such that $dist\{\overline{\Omega} \setminus \mathcal{D}_\nu, \partial_3\Omega\} = \mathcal{O}(\nu^{1/2-\delta})$, $\delta > 0$.

Theorem 2. *The charged–particle system case Let ρ^ν be the $C^{1,\alpha}$ solution of (1.13) constructed in [GM] where the solvability conditions are satisfied. If, in addition to (1.8–9–10) the function $G(t)$ satisfies $0 < C_1 \le t^2\, G(t)$ for $t > 0$, then*

$$C\,\nu^{1/2}\, d^{-2}\, \exp(-\nu^{-1/2}\frac{\gamma-1}{C_1} d^{-2}) \le i(\rho^\nu)(x) \le L^* \quad \text{for any} \;\; x \in \overline{\Omega} \setminus \mathcal{D}_\nu, \tag{1.22}$$

where $C = C(\Omega, \gamma, C_1, K^*, L^*, d^{-2})$ for $d = dist(x, \partial_3\Omega)$, independent of ν for $\nu \le \nu_0(\mathcal{G}, \Omega)$. The set $\mathcal{D}_\nu$ satisfies $\partial\Omega \subset \mathcal{D}_\nu \subset \overline{\Omega}$ and contains a "slit" of the outer boundary $\partial_3\Omega$ such that $dist\{\overline{\Omega} \setminus \mathcal{D}_\nu, \partial_3\Omega\} = \mathcal{O}(\nu^{1/4-\delta})$, $\delta > 0$.

Proofs:

The proofs of both theorems are similar, so we shall write it for the fluid–Poison case, and made the distinction when needed. Let φ^ν, ρ^ν and Φ^ν be the strong solutions constructed in [GM], omitting the subscript ν, they satisfy the equation

$$-\frac{\nu}{\gamma-1} \nabla \ln i(\rho) \cdot \frac{\nabla\varphi}{|\nabla\varphi|^2} = \frac{i(\rho) - \left(K - \frac{|\nabla\varphi|^2}{2} - \mathcal{R}(\varphi) + q\Phi\right)}{|\nabla\varphi|^2\, G(|\nabla\varphi|)} \tag{1.23}$$

because if $i(\rho) = \frac{\gamma}{\gamma-1} k \rho^{\gamma-1}$.

Now, from estimate (1.19) and (1.20)

$$\left\{ \frac{|\nabla\varphi|^2}{2} - (K - \mathcal{R}(\varphi) + q\Phi) \right\}(x) \le C\frac{\nu^\beta}{d^2}, \qquad \text{with } d = dist(x, \partial_3\Omega) \tag{1.24}$$

for any $x \in \overline{\Omega} \setminus \partial_3\Omega$, and $C = C(k^{-1}, G, K^*, K, \mathcal{R}, q, data, \kappa_x)$.

In addition, if $|\nabla\varphi|^2 G(|\nabla\varphi|) > C_1$ for all values $|\nabla\varphi|$, then the right hand side of (1.23) can be estimated yielding the following differential inequality

$$-\nu\,(\ln i(\rho))_\varphi \le \frac{\gamma-1}{C_1}\left(C\frac{\nu^\beta}{d^2} + i(\rho)\right) \tag{1.25}$$

for any $x \in \overline{\Omega} \setminus \partial_3 \Omega$.

Let $h = i(\rho) > 0$, $M = C\,\nu^\beta\, d^{-2}$ and $a = \frac{\gamma-1}{C_1}$, (1.25) can be written as

$$-\frac{1}{a}(\ln h)_\varphi \leq M + h,$$

or equivalently

$$-\frac{h_\varphi}{ah(M+h)} \leq 1.$$

Integrating along the streamlines in the direction of growing φ starting at $\Gamma \subset \partial_1\Omega \cup \partial_3\Omega$,

$$\ln\left(\frac{M}{h} - 1\right) \leq aM(\varphi - \varphi_0). \tag{1.26}$$

Hence, since $|\varphi| \leq \mathcal{K}(K^*)$ in $\overline{\Omega}$ uniformely in ν, it follows from (1.26) that

$$\ln\left(\frac{M}{h} - 1\right) \leq aM\mathcal{K}$$

or, equivalently,

$$\frac{1}{a\mathcal{K}} e^{-(aM\mathcal{K})} \leq h. \tag{1.27}$$

Since,

$$aM = \nu^{(\beta-1)} \frac{\gamma - 1}{C_1} d^{-2}$$

we have obtained the following estimate from below for $i(\rho)$, namely

$$\frac{\nu\, C_1}{(\gamma-1)\mathcal{K}} e^{-(\nu^{\beta-1} d^{-2} \mathcal{K} \frac{\gamma-1}{C_1})} \leq i(\rho). \tag{1.28}$$

Therefore, for the gas flow past a profile model $\beta = 1$ for all the points in the interior of the flow domain, so that taking a "slit" next to the outer boundary of lenght $\nu^{1/2}$, estimate (1.21) follows with $C = \frac{C_1}{(\gamma-1)\mathcal{K}} e^{-(d^{-2}\mathcal{K}\frac{\gamma-1}{C_1})}$. Theorem 1 is then proved.

For the charged–particle transport model, the best we can get is $\beta = 1/2$ in the interior of the flow domain, which yields estimate (1.22) when $\beta = 1/2$ is set in (1.28). Thus the proof of Theorem 2 is also completed.

Conclusions about transonic flow.

Because $|\nabla\varphi^\nu|$ and ρ^ν are uniformly bounded from above, there exists a convergent subsequence (φ^ν, ψ^ν) with a limit (φ^0, ψ^0). However this is not enough to establish that there exists a weak solution to the equations $\rho\,\varphi_x = \psi_y,\ \rho\,\varphi_y = -\psi_x$.

One method to prove the existence of weak solutions involves the method of compensated compactness, see Murat [Mu], Tartar [Ta] and Di Perna [Dp] and the applications to mixed-type systems (see Morawetz [M1], [M2]), which has been mainly used for hyperbolic problems. This would require that the speed $|\nabla\varphi^\nu|$ is uniformly bounded below from zero and above from the cavitation speed (see [M2]). Furthermore, one would also require bounds on the flow angle.

While this paper thus provides a complete proof of existence of viscous solutions with some uniform bounds which allows us to consider a convergent subsequence, still there is a major gap in showing that its limit is a solution of the inviscid problem.

Similar difficulties arise in the existence theorems of Feistauer and Nečas [FN]. They show the existence of a solution for the inviscid model under the the assumption of existence of viscous solutions to a boundary value problem provided the divergence of the viscous velocity field satisfies uniform bounds in ν. Gittel [Gi] shows existence using a variational approach to a boundary value problem for the transonic small disturbance equation and shows existence theorems assuming a priori uniform bounds and entropy conditions.

We also mention the work of Klouček and Nečas [NK] and [Kl] to find entropic solutions for the transonic flow model by the method of stabilization. There they solve a perturbed flow equation by introducing an artificial time variable. Also there, they need to assume uniform speed and entropy bounds, in order to pass to the limit.

About the semiconductor device model.

We have shown the existence of a regular solution to the boundary value problem Fluid-Poisson system in an approximating geometry to a real device and without smallness assumptions on the size of the data. The model we have considered corresponds to a fluid level approximation of a kinetic formulation for a charged particle system, with a pressure law similar to the one for an isentropic gas with $1 < \gamma < 2$, and a viscous parameter ν. In this context the "viscosity" parameter is related to the mean free path and refers to the constant coefficient related to the the nonconvective energy flux term, which has been modeled as a non linear term that involves first and second order derivatives of the velocity field.

In addition, we show sharp uniform bounds in the viscosity parameter for the speed and an improvement for the lower bound for the density, both valid for regimes where this parameter becomes small compared with the scaled coefficient of the acceleration term in the momentum equation.

This is a potential flow model obtained under the assumption that the velocity field aligns with the gradient field of τ, where $\tau > 0$ stands for the velocity relaxation time term. Potential flow can be justified if the initial state has zer vorticity.

In order to extend our results to the case $\gamma = 1$ (the isothermal model) modifications would be required, especially to obtain the uniform bounds. This case is however often used in modeling, see [MN] and [PR].

In particular for two dimensional models in MOSFET geometry it can be shown that the behavior of any solution at the boundary points will depend on the behavior and regularity at the boundary of the domain under the conformal transformation that takes the domain of the device into a rectangular domain where the source and drain contacts are transformed into opposite walls the transformed domain. Therefore, a singularity is expected to be found in the electric field at the boundary points corresponding to rough boundaries and to junctions between contact and oxide regions. In these case, the hydrodynamic model worked out here for charged–particle transport can not handle these boundary singularities for the electric field unless very restricted assumptions are given on the data. Then becomes necessary to look at models derived for transport regimes under strong force effects, see Barenger and Wilkins [B1], [B2], Poupaud [P1], [P2], Stichel and Strothmann [StS], and Cercignani, Gamba and Levermore [C1], [C2], for some mathemathical work related to these models.

Appendix

The semi-conductor

There are many models for semiconductors. They include the quantum, kinetic and fluid level formulation.

The most accurate is the quantum one where the particles are represented by wave functions solving the Schrödinger equations with a Hamiltonian that incorporates potentials due to the semiconductor lattice, Coulomb interactions, the applied bias and particle-phonons interactions. Although such models have begun to be used for numerical simulations of small parts of the semiconductor devices where quantum effects are important, up to now it is impossible to formulate a model for a complete device at this level.

Then there is the kinetic level of modeling and the application of Monte-Carlo methods at the particle level where the dynamics of the particles is described by the evolution of distribution functions for electrons and holes respectively, both depending on time, position and wave-vector, where the latter belongs to a periodic lattice in $\mathbb{R}^3$. This evolution is dictated by a semiclassical Boltzmann equation which incorporates the electric field and collisions effects. These models are presented in Markovich *et all* [MR]; lately there have been some analytical results regarding existence of solutions for some boundary value problems (see Poupaud [P3]), but these models remain costly as numerical solutions.

Finally there is the fluid level of modeling. First is the one based on the drift diffusion equations of parabolic type for the concentration of both carriers (electrons and holes). These models can be rigorously derived from the kinetic formulation under the assumption of low electric fields (see Golse and Poupaud [GP]) and have been extensively studied from the mathematical and numerical point of view (see Jerome [J] and Markowich, Ringhofer and Schmeiser[MRS] and references therein.) They give very good results for components whose typical length scale is of the order of a micron, but they do not seem to be valid for submicron devices or for high electric fields. Therefore more sophisticated models have been introduced. They are based on electro-hydrodynamic equations (also called energy balanced equations or extended drift diffusion models) which are intented to take into account high field effects. They have been obtained by closing the moments equations derived from the Boltzmann equation with a phenomenological assumption on the distribution function. The distribution is assumed to be isotropic around its mean velocity (see, for instance, Blotekjaer [Bo], Azoff [Az], Bringer and Schön [BS] and Odeh, Gnudi and Rudan[OGR]). Then, fluid equations are obtained with source terms modeling relaxation processes and electric fields effects coupled with the Poisson equation. Some recent numerical work include papers Chen, Z. *et al* [CC] in two space dimensions, and Gardner [Gr1], [Gr2], Gardner, Jerome and Shu [GJ] and Fatemi *et al* [FG]] in one space dimension.

Analysis addressing issues of existence, uniform bounds and boundary layer formation for steady-state one dimensional transonic models have been studied by Gamba [G1], [G3] and [G4]. Asher, *et al* [AM] presented a phase plane analysis for some special models. The isothermal one space dimension time dependent inviscid problem has been recently analyzed by Marcati and Natalini [MN], Poupaud, Rascle and Vila [PR] and Zhang [Z]. In addition, Cordier, *et al* [CD] analyzed traveling waves and jump relations for an Euler-Poisson model in the quasineutral limit. Markowich [Mk] treated a 2-dimensional steady Euler-Poisson system in subsonic regimes, under a smallness assumption of the prescribed outflow velocity (small boundary current) and under a smallness assumption of the variation of the velocity relaxation time. Degond and Markowich [DM] considered a three dimensional potential inviscid flow model, where they prove existence and uniqueness results in a bounded domain for small Dirichlet data. Both papers deal with systems of equations that remain essentially elliptic under assumptions either on the size of the data, or on the size of the parameters under consideration.

Acknowledgement: The research of the first author is supported by the National Science Foundation under grant DMS 9623037.

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