

# GEOMETRY INSTITUTE - DAY 4

## Geometry of Shape

Time	Topic	TEKS	Approach
60	<i>The Tunnel of Samos</i>	4a,11ab,5c	
10	Reflection on <i>The Tunnel of Samos</i>		
60	Too Tall to Tell	4a,7bc,11bc	
10	Scale Factor of Mt. Rushmore	11b	
15	Astronomy and Architecture	7c,11c	
20	Squares and Cubes	8ad,9d,11abd	
10	Dollar for Your Thought	9d,11bd	
20	Dilations	5b,11a	
10	On Being the Right Size	5a,8d	
40	Trigonometric Ratios	11c	
40	Laws of Sines and Cosines	3be,4a,11bc	
15	The Shape of Space	1c	
15	Reflections & post its		
5	Give assignment		
330	<i>Total suggested time (in minutes)</i>		

## Day 4: Materials Needed:

*Sketchpad*

### *Sketches*

- “Dilation Coord.”
- “Unit Circle/Trig”
- “Sine Tracer”
- “Sin/Cos Graphs”
- “Tangent Tracer”

### *Scripts*

- Trig Investigations

- Tunnel of Samos* solution.
- Day 5 assignment.
- String
- Paperclips
- Straws
- Yard sticks or meter sticks
- Mirrors with cross hairs
- Atlas
- Cubic inch models
- Cubic foot models
- US quarter - 1 per group
- Susan B. Anthony dollar - 1 per group

# THE TUNNEL OF SAMOS

The history of mathematics reads as a series of problems requiring solutions. One class of problems involves estimating inaccessible distances and directions.

## The Tunnel of Samos

On the island of Samos, Mt. Castor lies between a source of fresh water and the capital city, today known as Pythagorean. For strategic reasons, Eupalinos decided to dig a tunnel through Mt. Castro to transport the water to the city. Eupalinos was able to find two geographical points at the same elevation. Two teams were to dig from opposite sides of the mountain, from points at the same elevation. After the level tunnel was completed, a gently sloping channel was dug inside the tunnel, along which the water flowed from its source to the city.

In 530 BC, when the tunnel was constructed, the Greeks had no magnetic compass, no surveying instruments, and no topographical maps. The problem Eupalinos faced was to determine the direction that each team must dig so their two tunnels would meet under the mountain. It is believed that he began by traveling around the mountain from the water, A, to the town, C, along a constant elevation path that was composed of perpendicular segments. Thus, he knew the net displacement in two perpendicular directions between A and C. How did this information enable him to direct the path of the construction crew?

# THE TUNNEL OF SAMOS

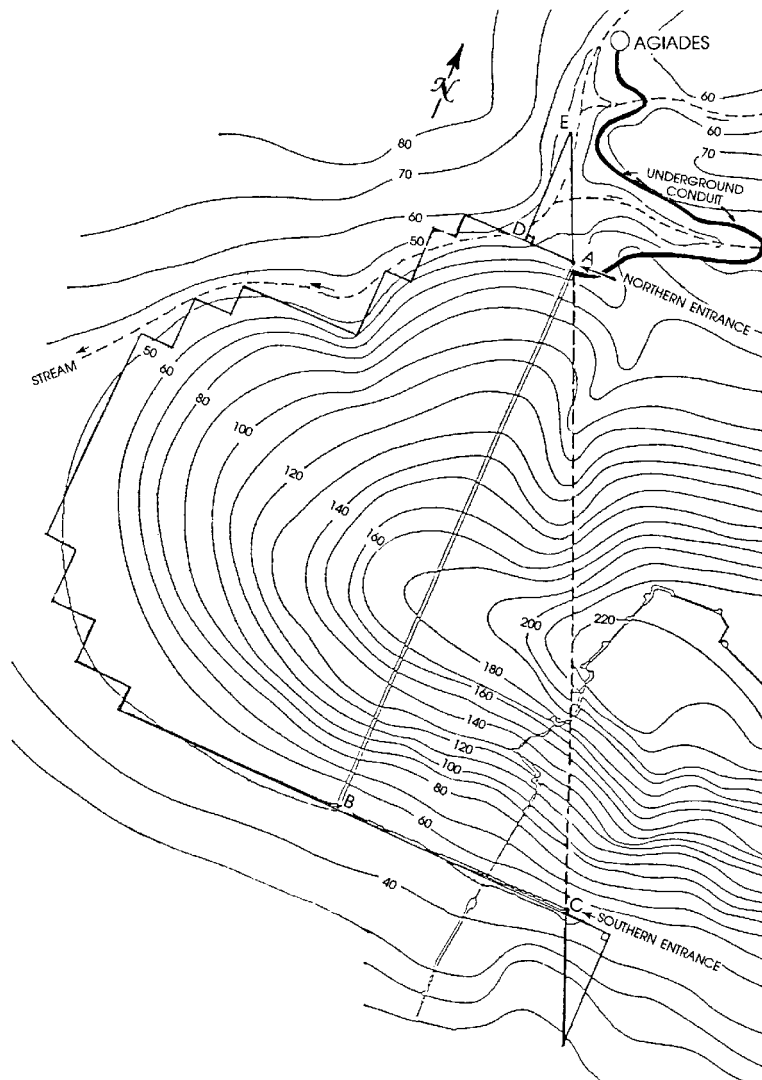
## Solution

### The Tunnel of Samos

Some 600 years after the construction, Hero reported that Eupalinos used similar right triangles and sight lines to direct construction of the tunnel.

The method involved making a right angled circuit around the mountain, always maintaining the same elevation. In this way, the legs of a right triangle beneath the mountain could be determined. The vertices of the hypotenuse are the entrances of the tunnel, A and C.

After making a right angled circuit of the mountain from A to C, the net distance traveled in one direction is AB and the net distance traveled perpendicular to that is CB. Right  $\triangle ABC$  lies under the mountain, but triangles similar to  $\triangle ABC$  can be created at each entrance of the proposed tunnel so that A and C are each one vertex of the hypotenuse of each of the similar triangles. Use the other vertex of each hypotenuse, E and G and sight along AE and CG as the tunnel is excavated.

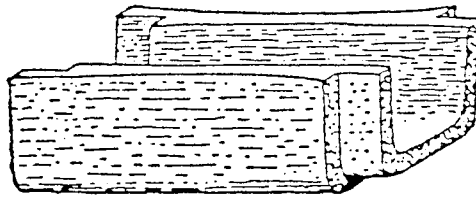


## Right angles

The Egyptians used a rope knotted into 12 equal lengths to form a 3-4-5 right triangle. The Greeks may have constructed wooden rectangles stabilized by diagonals that are equal and bisect each other.

## Constant Elevation

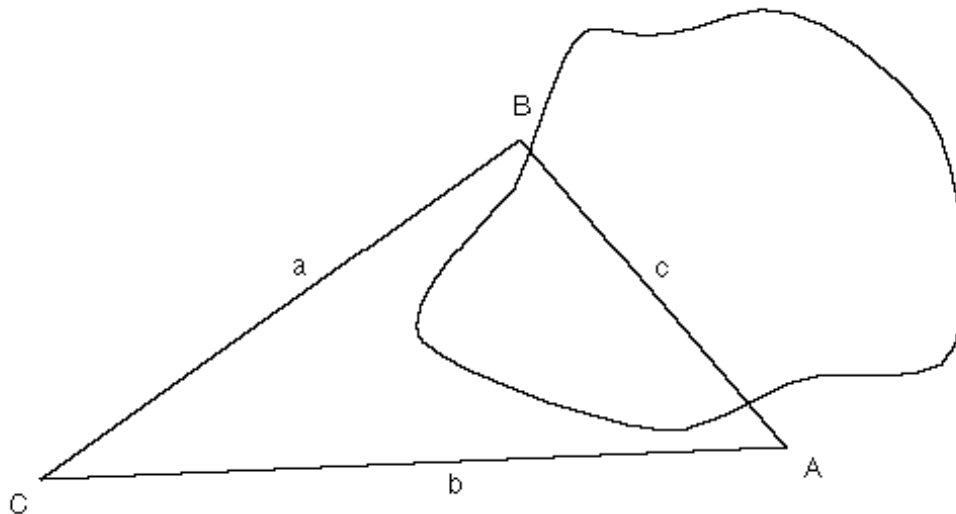
It is thought that clay gutters filled with water were used in sequence to determine the path around the mountain at a constant elevation.



## A trigonometry solution

Knowing sides  $a$ ,  $b$  and  $\angle C$ , use the Law of Cosines to find the length of side  $c$ , the length of the tunnel.

Use the Law of Sines to find  $\angle A$  and  $\angle B$ , the angles at which the tunnel should be excavated.



## Video

*The Tunnel of Samos*. Project MATHEMATICS! (1995)

# THE TUNNEL OF SAMOS

## Reflections

1. Discuss the presentation styles used in this problem and how they contributed to the problem.
  - Cooperative groups
  - Historical context
  - Video
2. Discuss the geometry learned from this problem.
3. Discuss the geometric concepts used in the solution to this problem.
  - Right triangles
  - Similarity
  - Ratios
  - Trigonometry
  - Measurement
4. How could this problem be adapted to your classroom?
5. Directions of the day. This problem leads to:
  - Measurement
  - Similarity and ratios in two and three dimensions
  - Applications
  - Trigonometry
  - Fractals (will be looked at on Day 5)



# TOO TALL TO TELL

## Teacher Guide

### GOAL:

Determine the heights of objects that cannot easily be measured directly.

### NOTES TO TEACHER:

Students should be given time to understand problem and come up with possible approaches to a solution. One may direct discussion towards a particular method or allow several methods to arise. Prior to the lesson, determine the height of flagpole (or other tall object at the school) and select other tall objects to be measured.

### PROCEDURE:

Use the flagpole overhead to present problem of measuring heights of tall objects.

Ask class to come up with indirect methods using geometry. Either the teacher should lead the discussion or the students should work in groups to discover potential methods.

Illustrate suggested methods with a diagram.

Determine what geometric principles and properties are used.

Determine what supplies are needed to measure data.

Students work in groups of 3-4 to take measurements and solve. Each group should select, or be directed to use, one method.

Compare results and discuss where errors could arise.

(Most accurate is the shadow method.)

Each of the following lessons uses one method:

- Angle of elevation and the tangent function
- Shadows and similar triangles
- Mirror and similar triangles.

# TOO TALL TO TELL

## Angle of Elevation Method

### Teacher Guide

#### GOAL:

Determine the height of objects that cannot be easily measured directly using angle of elevation and tangent function.

#### PREREQUISITES:

Trigonometry ratios and ability to use trigonometry tables or scientific calculators

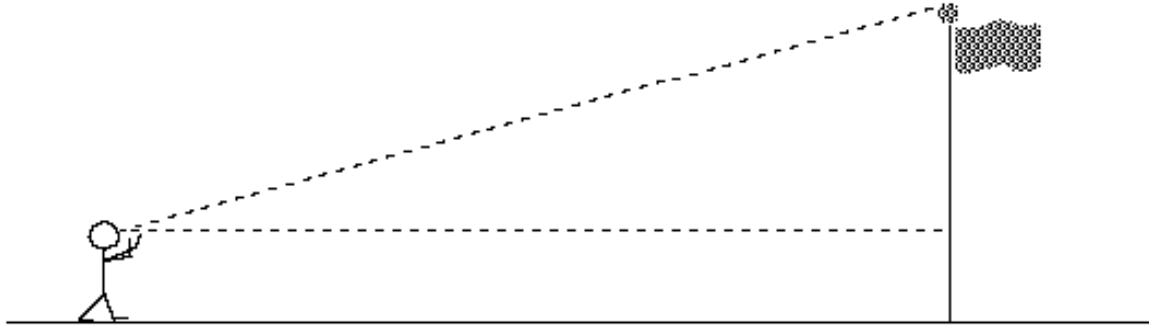
#### MATERIALS:

Measuring tape or sticks, scientific calculator or trig tables, large protractor, drinking straw, tape, string, thread, and a paper clip or small weight.

#### PROCEDURE:

1. Discuss how to use angle of elevation and tangent function to find height of tall objects.
2. Discuss what materials are needed and provide supplies. A clip board is helpful.
3. Demonstrate construction and use of angle of elevation measuring devise. Each group should construct their own devise.
4. Each group should measure one common object and some others. Provide data sheet.
5. Have each group discuss their results, including sketching one of their problems on the board or on an overhead sheet to display to the class.
6. Compare measurements of common objects and discuss where error may arise -
  - inaccurate measurements of lengths and angles
  - measuring tape may stretch
  - observer and object are at different ground elevations
  - round-off error in measurements and/or calculations





Formula: Height of object = height of observer's eye +  
(distance between object and observer)(tan of angle of elevation)

# TOO TALL TO TELL

## Angle of Elevation Method

### GOAL:

Use angle of elevation and trigonometry to calculate the height of objects that cannot easily be measured directly

### MATERIALS:

Measuring tape or sticks, scientific calculator or trig tables, large protractor, drinking straw, tape, string, and a paper clip, for shadows, mirrors.

### PROCEDURE:

1. Work in groups of 3 students: recorder, measurement taker, and observer.
2. Draw a sketch that models your problem.
3. Determine what information is needed to find the height of the object.
4. Build an angle measuring device:

Tape a straw along the straight edge of a large protractor. This is used to “sight” the top of the object to be measured.

Tie a paper clip to the end of a piece of string. This is your plumb line.

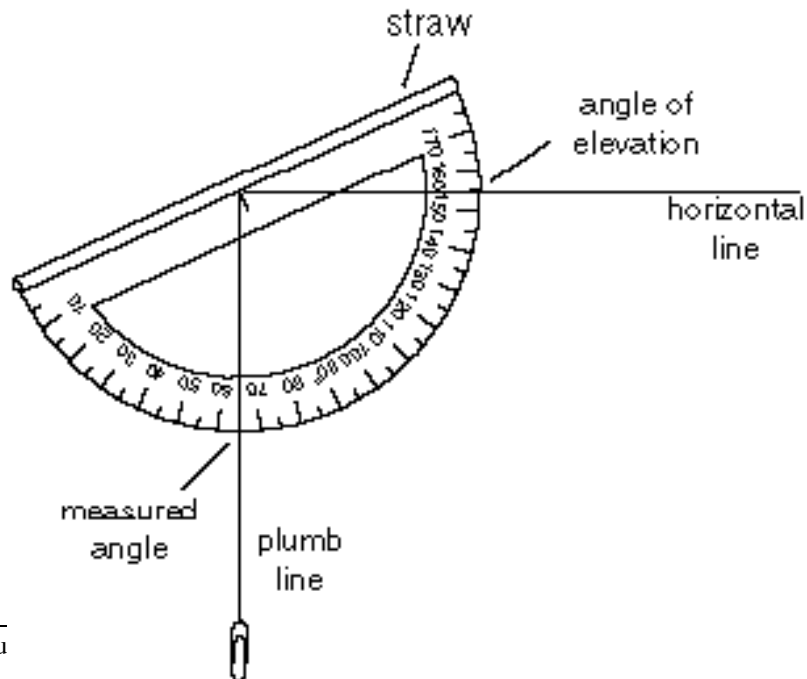
Attach plumb line to the midpoint of the straight edge of the protractor.

5. How to use the device.

Sight top of object to be measured through the straw.

Observe the acute angle at which the plumb line crosses the protractor.

This acute angle is the **complement** of the angle of elevation.



6. Gather data and fill-in the first four columns of this chart.

Object to be measured	Complement of protractor angle/ Angle of elevation	Height of observer's eye	Distance from observer to object	Height of object
1.				
2.				
3.				

7. Write a formula for computing the object's height from your measurements.

Define all variables used in the formula.

8. Calculate the heights of the objects and finish the chart. **SHOW WORK HERE.**

9. Discuss where error might have arisen.

# TOO TALL TO TELL

## Shadow Method

### Teacher Guide

#### GOAL:

Use lengths of shadows and similar triangles to determine the height of objects that cannot be measured directly.

#### PREREQUISITES:

Similar triangles, proportions.

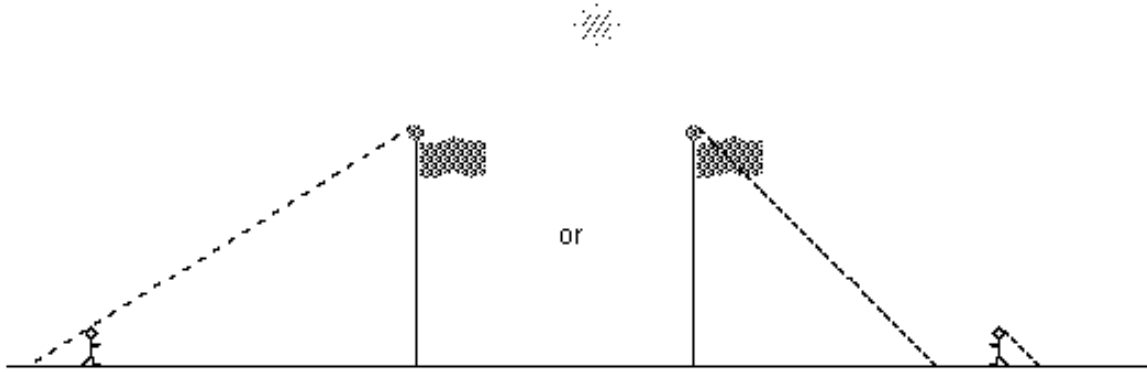
#### MATERIALS:

Measuring device: meter stick, yard stick, tape measure. Calculator.

Lab sheet for recording measurements. Sunny day.

#### PROCEDURE:

1. It's relatively easy to measure the length of a shadow of an object. How can we use that to find the actual height of an object? Discuss how to use similar triangles and shadows to find the height of a tall object.
2. Discuss what materials are needed and provide supplies. A clipboard is helpful.
3. All groups should measure one common object and each group should measure 3 other tall objects. Provide data sheet.
4. Have each group discuss their results, including sketching one of their problems on the board or on an overhead sheet to display to the class.
5. Compare measurements of common objects and discuss where error may arise -
  - inaccurate measurements of lengths
  - measuring tape may stretch
  - observer and object are at different ground elevations
  - round-off error in measurements and/or calculations
  - because of the rotation of the Earth, the angle of elevation of the sun changes continuously, hence the triangles are not similar



Formula:  $\frac{\text{Height of Object}}{\text{Shadow of Object}} = \frac{\text{Height of Person}}{\text{Shadow of Person}}$

6. Times of day that might not be good to try this experiment - noon when the sun is overhead, early morning when shadows are very long and may stretch onto other structures that prevent measurement, dusk.

# TOO TALL TO TELL

## Shadow Method

### GOAL:

Use lengths of shadows and similar triangles to determine the height of objects that cannot easily be measured directly.

### MATERIALS:

Measuring device: meter stick; yard stick; tape measure.

Lab sheet for recording measurements, calculator, sunny day.

### PROCEDURE:

1. Work in groups of 3 students: Recorder \_\_\_\_\_ ;  
Person to be measured (observer) \_\_\_\_\_ ;  
Measurement taker/equipment keeper \_\_\_\_\_ .
2. Draw a sketch that models your problem.
3. Determine what information is needed to find the height of the object.
4. Gather data and record it in the first 4 columns of this chart.

	Object to be measured	Shadow length of Object	Height of Observer	Shadow length of Observer	Height of Object
1.					
2.					
3.					

4.					
----	--	--	--	--	--

5. Write a proportion for computing the object's height from your measurements.  
Define all variables used in the proportion.

6. Calculate the heights of the objects and record in the last column.  
SHOW WORK HERE.

7. Discuss where error might have arisen.

# TOO TALL TO TELL

## Mirror Method

### Teacher Guide

#### GOAL:

Use a mirror and similar triangles to determine the height of objects that cannot easily be measured directly.

#### PREREQUISITES:

Similar triangles and proportions.

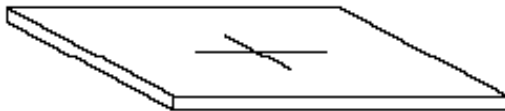
#### MATERIALS:

Measuring device: meter stick, yard stick, tape measure. Lab sheet for recording measurements. Small mirror with cross hairs (use narrow tape or overhead pens to make two lines intersecting at right angles in the middle of the mirror).

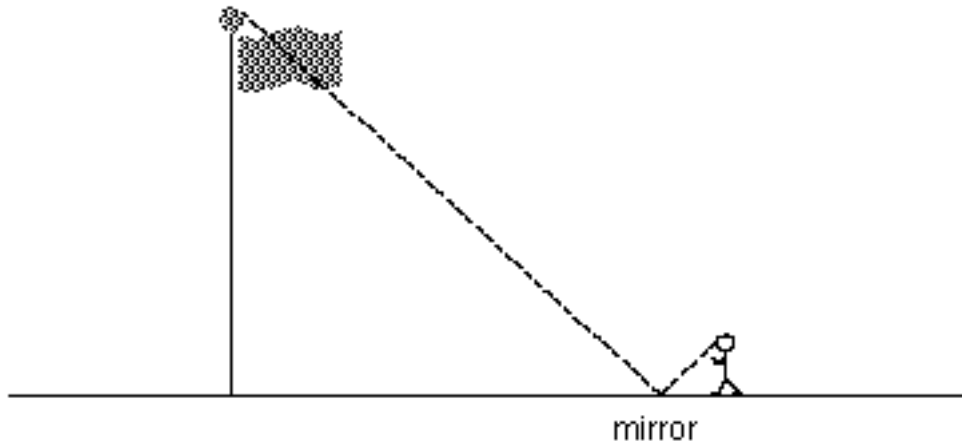
Calculator.

#### PROCEDURE:

1. Discuss “angle of incidence = angle of reflection” property of reflected light.
2. How can we use this property to find the actual height of a tall object?
3. Discuss what materials are needed and provide supplies. A clipboard is helpful.
4. Discuss how to use the mirror:
  - the position of the mirror could affect the calculations
  - the mirror must be on a flat surface - can float mirror in a pan of water
  - a cross hair should point to the object and to the observer







$$\text{Formula: } \frac{\text{Height of Object}}{\text{Distance from Object to Mirror}} = \frac{\text{Height of Observer's Eye}}{\text{Distance from Observer to Mirror}}$$

$$\text{Height of Object} = \frac{(\text{Height of Observer's Eye}) \cdot (\text{Distance from Observer to Mirror})}{\text{Distance from Object to Mirror}}$$

5. Each group should measure one common object and some others. Provide data sheet.
6. Have each group discuss their results, including sketching one of their problems on the board or on an overhead sheet to display to the class.
7. Compare measurement of common objects and discuss where error may arise:
  - inaccurate measurements of lengths
  - measuring tape may stretch
  - observer and object at different ground elevations
  - round-off error in measurements and/or calculations
  - mirror not horizontal
  - crosshairs not collinear with object and observer

**NOTE TO TEACHER:**

This lab is good for rainy/cloudy day when the shadow method cannot be used.

Objects can be measured in the classroom, cafeteria, or gym.

# TOO TALL TO TELL

## Mirror Method

### GOAL:

Use a mirror and similar triangles to determine the height of objects that cannot easily be measured directly.

### MATERIALS:

Measuring device: meter stick; yard stick; tape measure.

Lab sheet for recording measurements. Calculator.

Small mirror with cross hairs (use narrow tape or overhead pens to make two lines intersecting at right angles in the middle of the mirror).

### PROCEDURE:

1. Work in groups of 3 students: Recorder \_\_\_\_\_ ;  
Person to be measured (observer) \_\_\_\_\_ ;  
Measurement taker/equipment keeper \_\_\_\_\_ .
2. Draw a sketch that models your problem. Indicate any congruent parts.
3. Determine what information is needed to find the height of the object.
4. Position the mirror between the observer and the object so that the observer can look in the mirror and see the top of the object at the center of the crosshairs and the rest of the object appears aligned over one cross hair.

5. Gather data and record in the first 4 columns in this chart.

	Object to be Measured	Distance from Object to Mirror	Eye height of Observer	Distance from Observer to Mirror	Height of Object
1.					
2.					
3.					
4.					

6. Write a proportion for computing the object's height from your measurements.

Define all variables used in the proportion.

7. Calculate the heights of the objects and record in the last column.

SHOW WORK HERE.

8. Discuss where error might have arisen.



# THE SCALE FACTOR OF MOUNT RUSHMORE

## Teacher Guide

### GOAL:

Use the scale factor of the models of Mount Rushmore and the length of George Washington's nose to find the lengths of various features of the presidents on Mount Rushmore.

### PREREQUISITE:

Scale factor

### PROCEDURE:

Have students read the paragraphs on Borglum, then discuss how Borglum created his "pointing" machine and used it to transfer the model to the mountain. Possibly make a pointing machine using a protractor, small wire and string with a weight.

Read up on Mt. Rushmore. Students have many questions. Mt. Rushmore was started in 1927 and in 1941 Borglum died. Final touchup work was completed by his son.

---

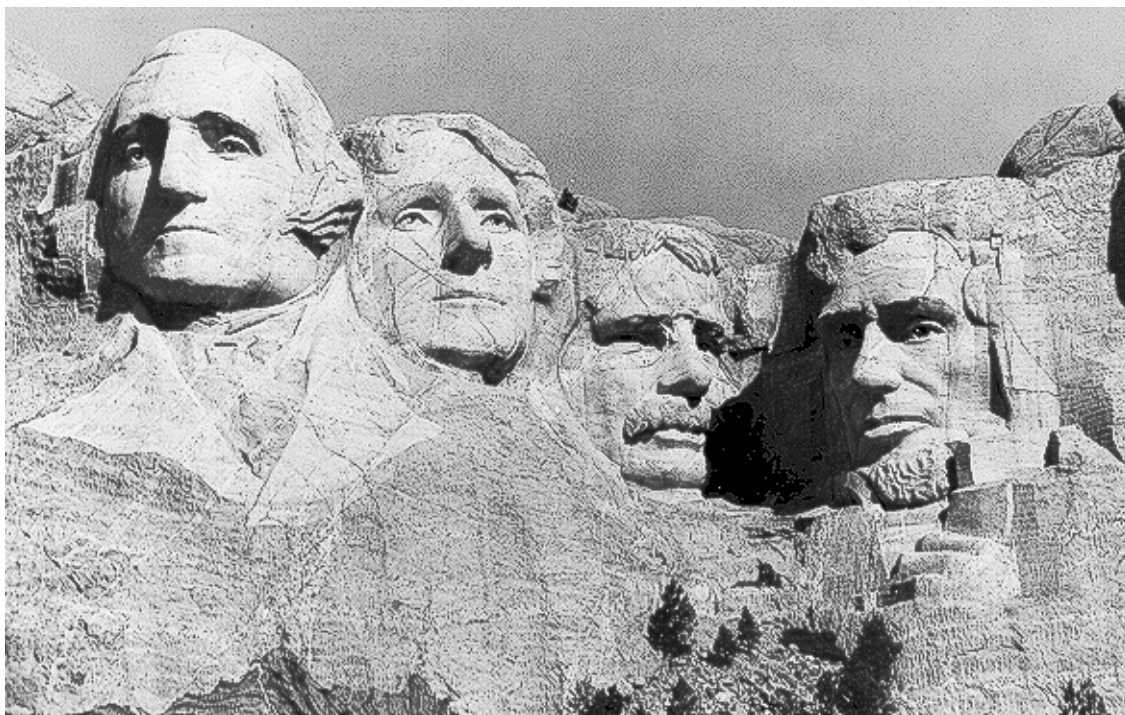
---

### SOLUTIONS:

1. 240"
2. 11'
3. 216", 18'
4. 60"
5. Between 434' and 496' (7.5 heads = 465')
6. Using  $5/16"$  get picture:mountain =  $5/16\text{in}:20\text{ft}$  or  $1/16\text{in}:4\text{ft}$  or  $1/16\text{in}:46\text{in}$
7. 4', 48"
8.  $3\frac{1}{3}'$ , 40"
9. 4', 48"
10. 48', 576"
11. Not accurate. Measurement of a three dimensional, curved object is made from a two dimensional picture. Estimate made in measurement of Washington's nose affects scale factor and hence all other lengths.



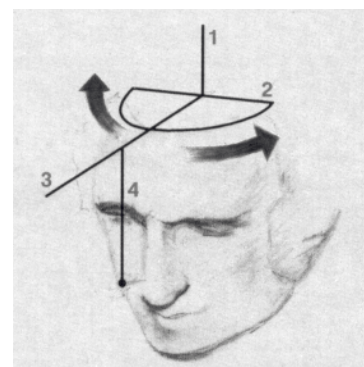
## THE SCALE FACTOR OF MT. RUSHMORE



Mount Rushmore, located in the Black Hills of South Dakota, is the masterpiece of Gutzon Borglum. To create Mt. Rushmore, Borglum made plaster models of each president based on life masks, paintings and photographs. He kept the plaster models on the mountain to guide him and the workers.

To transfer the models to the mountains, Borglum used a ratio of 1:12—one inch on the model equaled 12 inches on the mountain. Borglum created a “pointing” machine. The “pointing” machine was a metal shaft that was placed upright at the center of the top of the model’s head (1). At the base of the shaft was a plate (2) marked like a protractor. A metal bar (3) attached to the shaft pivoted on the plate so Borglum could read the angle measurement. A plumb line (4) hung from the bar and slid back and forth to measure the distance from the head to the point on the model.

Each point on the model would receive 3 separate measurements (the degree of the angle, the distance to where the plumb is connected on the bar, and the length of the plumb line). A bigger “pointing” machine was



placed on each president on the mountain. Borglum would determine the distances and lengths on the models and multiply them by 12 to transfer them to the mountain. Did he multiply the measure of the angle by 12, also?

Why or why not?

**The scale factor of the models to the mountain is 1:12**

**(one inch on the model equals twelve inches on the mountain)**

1. On the model, George Washington’s nose is 20” long.  
How many inches long is his nose on the mountain? \_\_\_\_\_
2. On the model, the length of George Washington’s eye is 11”.  
How many feet long is his eye on the mountain? \_\_\_\_\_
3. On the model, the length of Abraham Lincoln’s mouth is 1.5 feet.  
How many inches long is his mouth on the mountain? \_\_\_\_\_  
How many feet long is his mouth on the mountain? \_\_\_\_\_
4. From Theodore Roosevelt’s chin to his forehead is 60 feet on the mountain.  
How many inches is it from Roosevelt’s chin to his forehead on the model?  
\_\_\_\_\_
5. The height of an “average” human is about 7 to 8 times the length of his or her head.  
Confirm this by measuring your head \_\_\_\_\_ and height \_\_\_\_\_.  
If George Washington’s head is 62 feet long, how tall would George Washington be if a figure were made from head to toe? \_\_\_\_\_

**Use the picture from the front page and a ruler to answer the following questions.**

6. George Washington’s nose is 20 feet long on the mountain.  
Use a ruler to find the scale factor of the picture to the mountain.

**Picture : Mountain**

$$\frac{1}{16}'' : 20 \text{ ft}$$

$$\frac{1}{16}'' : \text{_____ ft}$$

$$\frac{1}{16}'' : \text{_____ in}$$

7. Using the scale factor from above, find the length of Jefferson’s nose on Mt. Rushmore.  
in feet: \_\_\_\_\_ in inches: \_\_\_\_\_
8. Find the length of Jefferson’s mouth.  
in feet: \_\_\_\_\_ in inches: \_\_\_\_\_

9. Find the length of Lincoln's nose from the bridge to the tip.  
in feet: \_\_\_\_\_ in inches: \_\_\_\_\_
10. Find the length from the outer corner of George Washington's left eye to the outer corner of his right eye.  
in feet: \_\_\_\_\_ in inches: \_\_\_\_\_
11. Could you consider these to be accurate measurements of the men on the mountain?  
Explain your answer.



# ASTRONOMY AND ARCHITECTURE

## Teacher Guide

### GOAL:

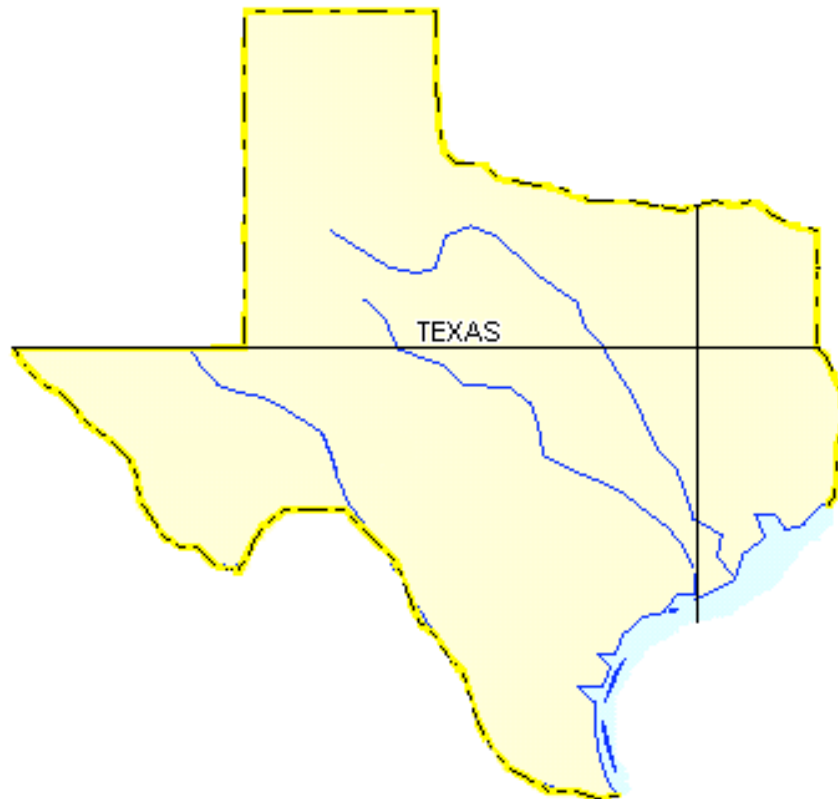
Use the angle of elevation and angle of depression to study architectural design problems.

### PREREQUISITES:

Right triangles, latitude, angle of elevation, and tangent ratios.

### MATERIALS:

Atlas.



### PROCEDURE:

Discuss astronomy related to finding the angle of elevation of the sun. (Note that  $0^{\circ}27'$  is approx.  $\frac{1}{2}^{\circ}$ .) Work with students to find their city's latitude from an atlas (either use the map or the index). Students work alone or in groups to solve the design problem and



discuss results with class. Students can also work the balcony design problem and discussion questions.

**EXTENSIONS:**

**GROUP PROJECTS FOR ANGLE OF ELEVATION:**

- Design and construct model windows for different areas of the United States.
- Research the climate to determine the best design.

**GROUP PROJECTS FOR BALCONY DESIGN:**

- Design a theater and make a scale model.
- Design a stadium.
- Design a ballpark.

**GROUP DISCUSSION:**

- What are other design problems where angle of elevation and angle of depression are important factors?

The solstices have important meanings to many religions. Research how the architects of North American Indians, pyramids, and Stonehedge were oriented those structures about astronomical events.

---

---

**SOLUTIONS:**

Length of overhang ( $x$ ) in Houston is 3 feet.

The bottom 4 feet will be in the sun in the winter.

Balcony design - 26.5 meters.

# ASTRONOMY AND ARCHITECTURE

## GOAL:

Use the angle of elevation and angle of depression to study architectural design problems.

## PASSIVE SOLAR DESIGN:

The earth revolves around the sun in a planar elliptic orbit. Meanwhile, the earth is also rotating about its North-South axis. If that axis were perpendicular to the orbital plane, the sun would always be directly above the equator and there would be no summer or winter. In fact, the axis of the earth forms an angle  $23^{\circ} 27'$  away from the perpendicular.

The Tropic of Cancer, latitude  $23^{\circ}27'$  north of the equator, delineates the northernmost point at which the sun appears directly overhead at noon. On or about June 21st, the sun's rays strike the Earth vertically at the Tropic of Cancer. This is called the summer solstice and it marks the beginning of summer. The term solstice means "sun stands still." To use our city's latitude to obtain the local angle of elevation of the sun in the summer use the following formula.

- The summer solstice

$$\text{angle of elevation} = (90 - \text{the latitude} + 23 \frac{1}{2})^{\circ}$$

The Tropic of Capricorn, latitude  $23^{\circ}27'$  south of the equator, delineates the southernmost point at which the sun appears directly overhead at noon. On or about December 22nd, the sun's rays strike the earth vertically at the Tropic of Capricorn. We call this the winter solstice and it marks the beginning of winter. To obtain the angle of elevation of the sun in the winter use the following formula.

- The winter solstice

$$\text{angle of elevation} = (90 - \text{the latitude} - 23 \frac{1}{2})^{\circ}$$

## Activity I

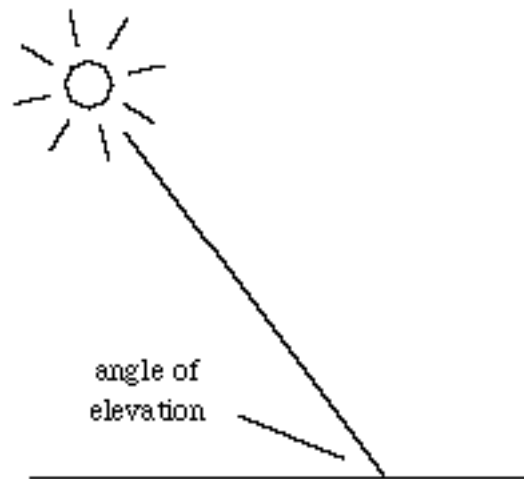
## PASSIVE SOLAR DESIGN

Passive solar homes are designed to let the sun into the windows in the winter to help heat the home and shade the windows in the summer to help cool the home.

From the latitude of the home site an architect can determine the angle of elevation of the sun in the summer and winter. For places in the northern hemisphere, between the Tropic of Cancer and the Arctic Circle, use the solstice formulas to determine the angle of elevation of the sun for the summer solstice (the longest day of the year) and the winter solstice (the shortest day of the year).

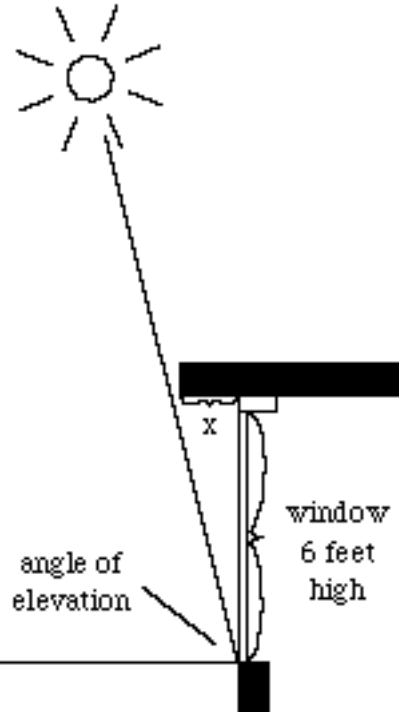
Find the angle of elevation of the sun for the summer and winter solstices for several cities.

1. Use an Atlas to find the latitude of selected northern hemisphere cities, then determine the angle of elevation of the sun in these cities for their summer and winter solstices.



City	latitude	angle elevation summer	angle elevation winter
Houston			
San Antonio			
Dallas			

2. The following window diagram is in Houston, Texas.  
What length  $x$  (the overhang above the window) will shade the entire window in the summer?



3. Utilizing  $x$  from 2, how much of the window will be in the sun at noon at the winter solstice?
4. Determine the length of the overhang for a five foot high window that will shade the window during the summer in your home town.
5. Utilizing the length of the overhang from problem 4, how much of the window will be in the sun at noon of the winter solstice?
6. Find a south facing window at your home or school and make the necessary measurements to determine if the window is designed with passive solar design in mind. Make sure you include a drawing of the window.

Group discussion: State answers in complete sentences.

7. Why is passive solar design important?
8. Are there places that passive solar design is not necessary?

9. We have only studied the summer and winter solstice designs.

What additional information would you have to take into account for architectural designs in Houston, Texas compared to those in Minneapolis, Minnesota?

## Activity II

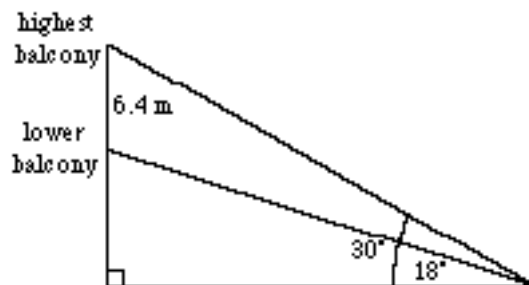
## BALCONY DESIGN

The height from which the audience views a stage production is very important. Balconies must be designed so the players on stage can look at the audience in the balcony and the audience in the balcony can see the persons on the stage. Many angry people would storm the ticket booth if they paid for balcony seats and then were much too high to view the performance on the stage.

From the stage of a theater, the angle of elevation of the lowest balcony is  $18^\circ$  and the highest balcony is 6.4 m directly above the first. The angle of elevation of the highest balcony is  $30^\circ$ .

How high above stage level is the lower balcony?

Is this design practical? Why or why not?





# SQUARES & CUBES

## Teacher Guide

### GOAL

Understand the relative sizes of various squares and cubes by directly constructing and comparing their perimeters, surface areas, and volumes.

### MATERIALS:

Bulletin board paper, yardsticks, rulers, markers, masking tape.

### PREREQUISITES:

Constructing a net for a cube, drawing a square.

### NOTE TO TEACHER:

Many students have problems comprehending the relative sizes of a square inch vs. a square foot vs. a square yard and a cubic inch vs. and cubic foot vs. a cubic yard.

The SQUARES LAB should precede lessons about area and the CUBES LAB should precede lessons about volume.

A service center may have a die cut for a one inch cube.

### PROCEDURE:

#### Squares Lab

As a class, complete part 1, then students work in groups of 4 to complete lab.

Save constructions to use in the Cubes Lab.

If your classroom has square foot tiles, point this out after the lab.

Use masking tape to mark out a square yard on the floor for further reference.

#### Cubes Lab

As a class, complete part 1. All students should know how to construct a cube from a net.

Have groups of 10 students complete the lab (4 complete part 2, 6 complete part 3).

Demonstrate that 27 cubic feet fit into one cubic yard.

Groups should fill in the table comparing various measurements of the 3 cubes.

Students should write in words or formulas the relationships that appear in the table.

*Perimeter varies directly as the edge length.*

*Surface area varies as the square of the edge length.*

*Volume varies as the cube of the edge length.*

**REFERENCES:**

Haldane, J. B. S., “On Being the Right Size” from *The World of Mathematics*.

**EXTENSIONS:**

**Housing Design:** Heat and cold are lost through the surface area of a home. Architects must take this into account when designing an energy efficient home. Study the designs of homes in different climates and investigate how energy is conserved by the design used. Some examples are igloos and apartment houses.

**Physiology:** Compare the height, surface area and volume of David and Goliath or Gulliver and a Lilliputian or the giant ants in the film *Them*. Could such very large or small creatures really exist?

**Advertising:** Find advertisements or other illustrations which demonstrate that a false impression may arise by failing to understand how volume and area vary with length of side.

---

---

**SOLUTIONS:**

**Squares Lab:**

2. C. 144
3. C. 9
4. The area varies as the square of the side.
5. A. 144, 9, 1296  
B. 288, 864, 1008, 1440  
C. 18, 36, 81, 90  
D. 72  
E.  $120 \text{ sq. ft} = 13\frac{1}{3} \text{ sq. yd. } \$160$

**Cubes Lab**

2. B. 1728
3. B. 27
4. The volume varies as the cube of the side.
5. A. 1728, 27, 46656



B. 3456, 10368, 12096

C. 54, 108, 243

D.  $(24)(27) = 648$  cu ft.  $60 \div 648 = \$.093$

6.

edge length	perimeter of one face			surface area of one face			volume of cube		
	in	ft	yd	in <sup>2</sup>	ft <sup>2</sup>	yd <sup>2</sup>	in <sup>3</sup>	ft <sup>3</sup>	yd <sup>3</sup>
1 in	4	$\frac{1}{3}$	$\frac{1}{9}$	1	$\frac{1}{144}$	$\frac{1}{1728}$	1	$\frac{1}{1728}$	$\frac{1}{46656}$
1 ft	48	4	$\frac{4}{3}$	144	1	$\frac{1}{9}$	1728	1	$\frac{1}{27}$
1 yd	144	12	4	1728	9	1	46656	27	1

8. weight is  $10^3 \times 400$  lbs = 400,000 lbs; surface area is 100 times as large

# SQUARES LAB

## GOAL:

Understand how the area of a square changes as its side length changes.

## PROCEDURE:

1. Draw a square inch:

### Complete the following tasks in groups of 4.

On the same sheet of bulletin board paper, two people complete problem 2 while two other people complete problem 3.

2. A. On bulletin board paper, construct a square foot.  
B. Divide the square foot into square inches.  
C.  $1 \text{ ft}^2 = \underline{\hspace{2cm}} \text{ in}^2$
3. A. On the same bulletin board paper, construct a square yard.  
B. Divide the square yard into square feet.  
C.  $1 \text{ yd}^2 = \underline{\hspace{2cm}} \text{ ft}^2$
4. Compare and explain results.
5. As a group answer the following questions:  
A.  $1 \text{ ft}^2 = \underline{\hspace{2cm}} \text{ in}^2$       B.  $2 \text{ ft}^2 = \underline{\hspace{2cm}} \text{ in}^2$       C.  $2 \text{ yd}^2 = \underline{\hspace{2cm}} \text{ ft}^2$   
 $1 \text{ yd}^2 = \underline{\hspace{2cm}} \text{ ft}^2$        $6 \text{ ft}^2 = \underline{\hspace{2cm}} \text{ in}^2$        $4 \text{ yd}^2 = \underline{\hspace{2cm}} \text{ ft}^2$   
 $1 \text{ yd}^2 = \underline{\hspace{2cm}} \text{ in}^2$        $7 \text{ ft}^2 = \underline{\hspace{2cm}} \text{ in}^2$        $9 \text{ yd}^2 = \underline{\hspace{2cm}} \text{ ft}^2$   
 $10 \text{ ft}^2 = \underline{\hspace{2cm}} \text{ in}^2$        $10 \text{ yd}^2 = \underline{\hspace{2cm}} \text{ ft}^2$   
D. A room that is 4 yds by 2 yds contains how many square feet?

E. Anthony wants to carpet his bedroom. The measurements of the bedroom is 10 ft by 12 ft. Carpet costs \$12 yd<sup>2</sup>.

How many square yards of carpet will Anthony need?

How much will it cost to carpet his bedroom?

# CUBES LAB

## GOAL:

Understand how the volume and surface area of a cube change as the edge of the cube increases.

## PROCEDURE:

- A. On a separate sheet of paper, draw a net that will fold into a cubic inch:  
B. Cut out the net and fold it into a cube.

**As a group, 3 - 4 people work on problem 2, 4 - 6 people work on problem 3.**

- A. Using the square feet from the SQUARES LAB, create a net for a cubic foot, then build the cubic foot. Build at least 2 cubic feet.  
B.  $1 \text{ ft}^3 = \underline{\hspace{2cm}} \text{ in}^3$
- A. Construct a net for a cubic yard, then build the cubic yard. Your group will have to find some way to keep the cube upright (be creative!). Do this with the least amount of materials. Recycle the square yards created for the SQUARES LAB.  
B.  $1 \text{ yd}^3 = \underline{\hspace{2cm}} \text{ ft}^3$
- Compare and explain results.

- As a big group complete the following:

A. $1 \text{ ft}^3 = \underline{\hspace{2cm}} \text{ in}^3$	B. $2 \text{ ft}^3 = \underline{\hspace{2cm}} \text{ in}^3$	C. $2 \text{ yd}^3 = \underline{\hspace{2cm}} \text{ ft}^3$
$1 \text{ yd}^3 = \underline{\hspace{2cm}} \text{ ft}^3$	$6 \text{ ft}^3 = \underline{\hspace{2cm}} \text{ in}^3$	$4 \text{ yd}^3 = \underline{\hspace{2cm}} \text{ ft}^3$
$1 \text{ yd}^3 = \underline{\hspace{2cm}} \text{ in}^3$	$7 \text{ ft}^3 = \underline{\hspace{2cm}} \text{ in}^3$	$9 \text{ yd}^3 = \underline{\hspace{2cm}} \text{ ft}^3$

D. A storage unit for rent is 2 yds wide, 4 yds long and 3 yds high.

How many cubic feet of storage is available?

If the storage unit rents for \$60 a month, what is the cost per cubic foot?

6. Complete the chart.

edge length	perimeter of one face			surface area of one face			volume of cube		
	in	ft	yd	in <sup>2</sup>	ft <sup>2</sup>	yd <sup>2</sup>	in <sup>3</sup>	ft <sup>3</sup>	yd <sup>3</sup>
1 in									
1 ft									
1 yd									

7. Fill in the blanks using length, square or cube.

The perimeter varies as the \_\_\_\_\_ of a side.

The surface area varies as the \_\_\_\_\_ of a side.

The volume varies as the \_\_\_\_\_ of a side.

8. A mature gorilla is 5 feet tall and weights 400 pounds. King Kong is 10 times the height of a mature gorilla.

What is King Kong's weight?

How does King Kong's surface area compare to the surface area of a mature gorilla?

# A DOLLAR FOR YOUR THOUGHTS

## Teacher Guide

### GOAL:

Experiment with size, shape and volume to understand how they are related.

### PREREQUISITES:

Volume of a cylinder.

### MATERIALS:

U.S. quarter, Susan B. Anthony dollar, compass, ruler, clay, calculator.

### PROCEDURE:

- Divide class in small groups.  
Give each group a quarter and an Anthony dollar to compare.
- Discuss why the Anthony dollar was unpopular and has essentially dropped out of circulation. *Difficult to distinguish from a quarter.*  
Discuss why a \$1 coin is useful. *Vending machines.*
- Show transparency depicting a US quarter and an Anthony dollar with measurements (width, height, weight).
- Each group is to design a circular dollar coin made of the same material as a quarter which weighs four times as much as a quarter.
- Each group then presents its coin design with a clay model of the coin, its dimensions, and an explanation of why it is a good design.

### NOTES:

This lesson allows student creativity within the restrictions of mathematical rigor. In addition, algebra skills such as formula manipulation are used and the relationship between dimensions and volume is experienced.

- Students may not realize that the weight depends directly on the volume.
- Require that students clearly define all variables and not use the same variable for different quantities.
- After presentations have been made, examine the formula for the volume of a cylinder in order to point out that the volume changes as the height and the volume changes as the square of the radius.

**EXTENSIONS:**

- Discuss other possible shapes, their dimensions and their advantages and disadvantages.
- Investigate the coins of other countries - materials, shapes, denominations, value compared to US coins.
- Report on the history of coinage (numismatics) in cooperation with a history or geography class.
- In cooperation with a chemistry or earth science class, report on the metals used for coins throughout history and why these metals were and are used.
- Investigate how a vending machine distinguishes among different coins and slugs.

**SOLUTIONS:**

Volume of a cylinder is  $V = \pi r^2 h$ .

Quarter dimensions : diameter =  $\frac{15}{16}$  in. = 0.9375 in.

height =  $\frac{1}{16}$  in. = 0.0625 in.

$$V_Q = \pi \left(\frac{15}{32}\right)^2 \cdot \left(\frac{1}{16}\right)$$

Design dollar so that  $V_D = 4V_Q$

Possible designs:

1. Keep diameter fixed and change only height.

Then  $h_D = 4 \times \left(\frac{1}{16}\right) = \frac{1}{4}$  in. A very thick coin.

2. Keep height fixed and change only diameter.

Then  $r_D = \frac{30}{32}$  in. and diameter =  $\frac{60}{32}$  in., almost 2 in. wide; a very wide coin.

3. Change both diameter and height in varying amounts.

Example, if the diameter is  $\frac{20}{16}$  in. =  $\frac{5}{4}$  in., then  $r_D = \frac{20}{32}$  in. and  $h_D = \frac{9}{64} \approx \frac{1}{8}$  in.

4. To create a dollar similar to the quarter (not necessarily the “best” solution)

$r_D = kr_Q$  and  $h_D = kh_Q$ . Then  $k^3 = 4$  and  $k \approx 1.5874$  and  $r_D = 1.4882$  in. and  $h_D = 0.09921$  in.

Many other solutions are possible.



# A DOLLAR FOR YOUR THOUGHTS

## GOAL:

Design a better one dollar coin.

## PROCEDURE:

- Each group is to mathematically design a better one-dollar coin. Requirements for the coin:
  - ◇ It must weigh four times as much as a quarter,
  - ◇ It must be round (a right circular cylinder),
  - ◇ It must be made of the same material as a quarter.
- Each group presents its coin design modeled in clay, its dimensions, and an explanation of why it is a good design.

Dimensions of a US quarter: diameter =  $\frac{15}{16}$  in. = 0.9375 in.

thickness =  $\frac{1}{16}$  in. = 0.0625 in.

## EXTENSION:

Explore other possible shapes for a one dollar coin that weighs four times as much as a quarter. Discuss their dimensions and the advantages and disadvantages of other shapes.



# DILATIONS

## Teacher Guide

### GOAL:

Explore the properties of a size transformation (dilation).

### PREREQUISITES:

Properties of similar figures.

### MATERIALS:

Computer lab worksheet and a computer. *Sketchpad* Sketch: “Dilation Coord.”  
or TI-92

### PROCEDURE:

- Put students into pairs and allow them to work through the lab worksheet.
- Have student pairs lead a discussion of the main points learned.

### REFERENCE:

[Similarity](#). Video by Project MATHEMATICS!

---

---

### SOLUTIONS:

1. E. The ratio of the areas is the square of the original scale factor.
  - I. The sides are in the ratio of  $1/3$ . The angles are congruent.
  - J. Similar triangles.
2. E. The vertices of the images and pre-images lie on the rays.
  - G. When the ratio is equal to 1, the pre-image and image are the same size and in the same location. When the ratio is less than 1, the pre-image is a reduction of the image. When the ratio is greater than 1, the pre-image is an expansion of the image.
  - I. The ratios are equal.
  - J. A dilation about O with a scale factor k maps the point P to P' so that O, P and P' are collinear and  $\frac{OP'}{OP} = k$ .

3. E. The image is shrinking or expanding proportional to the changing ratio. The rate at which the image is changing is not constant even though the ratio is changing at a constant rate. (Note: This may be an interesting related rate problem for calculus students to investigate. It is an interesting phenomenon for geometry students to notice and discuss in their own terms.)
4. C. (6, 9)  
 D. (kx, ky)  
 G. (6, 9) and (5, 7)  
 H.  $D_{(a,b),k} : (x, y) \rightarrow (x+k(a-x), y+k(b-y))$

**SOLUTIONS TO TI-92 LESSON:**

1. G and H. The ratio of the sides of the image to the pre-image is the scale factor.  
 J. The ratio of corresponding sides stays constant. The corresponding angles are congruent.  
 K. Similar triangles.
2. J. The vertices of the image and pre-image lie on the rays.  
 L. When the ratio is less than 1, the image is a reduction of the pre-image. When the ratio is equal to 1, the image and pre-image are the same size and in the same location. When the ratio is greater than 1, the image is an enlargement of the pre-image.  
 N. The ratios are equal.  
 O. A dilation about O with a scale factor k maps the point P to P' so that O, P and P' are collinear and  $\frac{OP'}{OP} = k$ .
3. F. (-4, 1)  
 G. (kx, ky)  
 H. No. The rule changes to  $D_{(a,b),k} : (x, y) \rightarrow (x+k(a-x), y+k(b-y))$   
 I. (6, 9) and (5, 7)  
 J. Descriptions will vary.

# DILATIONS

## *Geometer's Sketchpad*

### GOAL:

Explore the properties of a size transformation (dilation).

### PROCEDURE:

1. Investigate the measurements of corresponding parts of dilated polygons.

#### **A. Open a new sketch on Sketchpad.**

- Under the File menu, select New Sketch.

#### **B. Construct a random polygon.**

- Select the point tool from the toolbar.
- While holding down the shift key click the corners of the polygon. (The vertices must be clicked in a clockwise direction around the polygon.)
- Under the Construct menu, select Segment.
- While holding down the Option key, click in the white space.
- Under the Edit menu, select Select All Points.
- Under the Construct menu, select Polygon Interior.

#### **C. Dilate the polygon by a scale factor of $\frac{1}{3}$ about a point (center of dilation).**

- Select the point tool from the toolbar.
- Click in the white space.
- Under the Transform menu, select Mark Center.
- Select the arrow tool from the toolbar.
- Under the Edit menu, select Select All.
- Under the Transform menu, select Dilate...
- Type "1". (The "1" should appear in the New-box.)
- Press the Tab key.
- Type "3". (The "3" should appear in the Old-box.)
- Click on "OK".
- Under the Display menu, scroll to Color, then select blue.

#### **D. Measure the area of each polygon and calculate their ratio.**

- Click in the white space.
- While holding down the Shift key, click in the area of each polygon.
- Under the Measure menu, select Area.
- Double click either measure to bring up the calculator.
- Click on the area measure of the original polygon.
- Click on the "/" symbol of the calculator.

- Click on the area measure of the dilated polygon.
- Click on “OK”.

**E. Manipulate either polygon or the center of dilation. How does the ratio of the areas relate to the original scale factor of  $1/3$ ?**

- Click-hold-drag any point in the sketch.

**F. Measure the ratio of corresponding sides of the two polygons.**

- Click in the white space.
- While holding down the Shift key, click on a side of the gray polygon followed by the corresponding side of the blue polygon. (To assure the correct selection, check the bottom, left corner for the message “And Select Segment”.)
- Under the Measure menu, select Ratio.

**H. Measure corresponding angles of the two polygons.**

- Click in the white space.
- To measure an angle: While holding down the Shift key, select three consecutive vertices. (The angle now selected is the angle whose vertex is the middle point. Therefore, the order points are selected is very important.)
- Under the Measure menu, select Angle.
- Click in the white space.
- Measure the corresponding angle in the other polygon.

**I. Manipulate the polygons and write conclusions about the sides and angles of the two polygons.**

**J. What is the geometric term used to describe polygons with these properties?**

2. Develop a definition for a dilation.

**A. Delete the center of dilation.**

- *Click on the center of dilation.*
- *Press the Delete key.*

**B. Draw two red segments and mark them as a ratio.**

- *Under the Display menu, scroll to Color and select red.*
- *Select the segment tool from the toolbar.*
- *Click-hold-drag a short segment.*
- *Click-hold-drag a longer segment.*
- *Select the arrow tool from the toolbar.*
- *Click in the white space.*
- *While holding down the Shift key, select the short segment followed by the longer segment.*
- *Under the Transform menu, select “Mark Ratio”.*

**C. Dilate the polygon using the marked ratio.**

- *Select the point tool from the toolbar.*
- *Click in the white space.*
- *Under the Transform menu, select “Mark Center”.*
- *Select the arrow tool from the toolbar.*
- *Drag a box around the polygon to select its parts.*
- *Under the Transform menu, select Dilate...*
- *Click on “OK”.*
- *Under the Display menu, scroll to Color and select green.*

**D. Construct rays from the center of dilation through vertices of the pre-image (gray polygon).**

- *Click in the white space.*
- *Click and hold the segment tool and scroll to the ray tool.*
- *Select the arrow tool from the toolbar.*
- *While holding down the Shift key, select the center of dilation and a vertex of the pre-image.*
- *Under the Construct menu, select Ray.*
- *Repeat for at least one other vertex.*

**E. Adjust the lengths of the ratio segments. What is the relationship between the vertices of the image and the rays constructed in part D?**

**F. Measure the ratio of the dilation.**

- *Calculate the ratio of the short red segment to the longer red segment.*

**G. Drag the red segments to change the ratio of the dilation. Describe the relationship between the pre-image and image when the ratio is less than 1, equal to 1 and greater than 1.**

**H. Measure the following ratio:  $\frac{\text{distance from center to image}}{\text{distance from center to pre - image}}$ .**

- *With the arrow tool, click in the white space.*
- *While holding down the Shift key, click on the center and a vertex of the image.*
- *Under the Measure menu, select Distance.*
- *Click in the white space.*
- *While holding down the Shift key, click on the center and the corresponding vertex of the pre-image.*
- *Under the Measure menu, select Distance.*
- *Using the Calculator, compute the ratio above.*

**I. Drag the red segments to change the ratio and write a conjecture about the ratios found in parts F and H.**

**J. Use your findings to write a definition for a dilation with scale factor  $k$ , and center  $O$ . ( $D_{k, O}$ )**

3. Create an animation button to vary the ratio of the dilation.

**A. Construct a segment of variable length on the larger red segment.**

- *Click and hold the ray tool on the toolbar, and select the segment tool.*
- *Select the arrow tool on the toolbar and click in the white space.*
- *Select the longer red segment.*
- *Under the Construct menu, select Point On Object.*
- *Select the longer red segment.*
- *Under the Display menu, select Hide Segment.*
- *While holding down the Shift key, click on an endpoint of the hidden red segment and the constructed point.*
- *Under the Construct menu, select Segment.*
- *Click in the white space.*

**B. Mark the ratio of the shorter red segment to the variable red segment.**

**C. Create a new dilation using the marked ratio.**

- *Delete the previous dilation by deleting the center of dilation.*
- *Create a new center and mark it as center.*
- *Dilate the polygon interior.*

**D. Create an animation button.**

- *Under the Display menu, select Show All Hidden.*
- *While holding down the Shift key, select the point on the longer red segment (not an endpoint).*
- *Under the Edit menu, scroll to Action Button, and select Animation...*
- *Click on “Animate”.*
- *To begin the animation, double-click on the “Animate” button and to stop, click anywhere.*

**E. Describe mathematically the action on the screen.**

4. Use coordinates to define a dilation.

**A. Open the sketch titled, “Dilation Coord”.**

- *Under the File menu, select “Open...”.*
- *Find the sketch entitled “Dilation Coord” and “Open”.*

**B. Double-click the “Show origin dilation” button and investigate the relationships between the coordinates of C and C’ and the scale factor.**



C. What is the image of the point (2, 3) under a dilation of scale factor 3 about the origin, O?

$$D_{O, 3}: (2, 3) \rightarrow (\underline{\quad}, \underline{\quad})$$

D. Write a general rule for the following:

$$D_{O, k}: (x, y) \rightarrow (\underline{\quad}, \underline{\quad})$$

E. Double-click on the “Hide origin dilation” button.

F. Show the dilation about J and drag points K and J around the sketch.

G. Find the following:

$$D_{(3, 3), 3}: (4, 5) \rightarrow (\underline{\quad}, \underline{\quad})$$

$$D_{(3, 3), 2}: (4, 5) \rightarrow (\underline{\quad}, \underline{\quad})$$

H. Describe how one would find the coordinates of the image of a point under a dilation where the center is not the origin.

# DILATION

## TI-92


### GOAL:

Explore the properties of a size transformation (dilation).


### PROCEDURE:

1. Investigate the measurements of corresponding parts of dilated polygons.


#### A. Open a new geometry session.

- **APPS** 8:Geometry 3:New
-  "session name"
- **ENTER** **ENTER**


#### B. Construct a random polygon.

- **EQ** 4:Polygon
-  **ENTER** (Repeat n times)
- **ENTER**



#### C. Construct a point to be the center of dilation.

- **EQ** 1:Point
-  **ENTER**

#### D. Using the numerical edit command, create a scale factor of .25 for the dilation.

- **EQ** 6:Numerical Edit
-  **ENTER**
- Type or edit the desired number

#### E. Dilate the polygon about the center using the scale factor. Draw a sketch of the image on the screen.



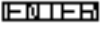
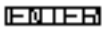
- **EQ** 3:Dilation
-  (Move until *DILATE THIS POLYGON* appears) **ENTER**
-  (Move until *WITH RESPECT TO THIS POINT* appears) **ENTER**
-  (Move until *USING THIS FACTOR* appears) **ENTER**

#### F. Measure a pair of corresponding sides of the polygons, using the distance between two points command. Label the measurements "s1" and "s2".

- **EQ** 1:Distance&Length
-  (Move until *DISTANCE FROM THIS POINT* appears) **ENTER**




-  (Move until *TO THAT POINT* appears) 
- Type the desired label

**G. Calculate the ratio of the dilated polygon side length to the original polygon side length. Compare this ratio with the scale factor.**



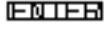

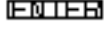

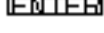
-  6:Calculate
-    
 Press the cursor pad to scroll through measurements on the screen.  
 Press  to display the variable name of the selected measurement in the calculation entry line.  
 The calculation entry is the interface for entering mathematical expressions involving geometric measurements.

- 



**H. Using the numerical edit command, change the scale factor and compare the ratio of the sides to the scale factor. Summarize your comparison in a concise statement.**

-  6:Numerical Edit
-  
- Type or edit the desired number

**I. Measure corresponding angles of the two polygons.**

-  3:Angle  
 Select the three points of the angle so that the vertex, of the desired angle, is the middle point selected
-  (Move until *THIS POINT* appears) 
-  (Move until *THIS POINT* appears) 
-  (Move until *THIS POINT* appears) 

**J. Manipulate the polygons by dragging any point on the original polygon or the center and write conclusions about the sides and angles of the two polygons.**

-  1:Pointer
-  (Move until *THIS VERTEX POINT* appears)
- Hold  



**K. What is the geometric term that is used to describe polygons with these properties?**

2. Develop a definition for a dilation.
  - A. Clear the screen.
    - 8:Clear All
  - B. Construct a random polygon.
    - See step 1B
  - C. Construct to be the center of dilation.
    - See step 1C
  - D. In top left corner of the screen, draw two segments.
    - 5:Segment
    - Repeat for second segment.
  - E. Measure the two segments.
    - 1:Distance&Length
    - (Move until LENGTH OF THIS SEGMENT appears) Repeat for second segment.
  - F. Calculate the ratio of the two lengths.
    - 6:Calculate
    - /
    -
  - G. Dilate the polygon about the center using the ratio of the segments as the scale factor.
    - See step 1E.
  - H. Construct rays from the center of dilation through the vertices of the original polygon.
    - 6:Ray
    - to the center of the dilation (until THIS POINT appears) to a vertex of the pre-image (until THIS POINT appears) Repeat for the remaining vertices
  - I. Animate one of the endpoints of either ratio-segments to vary the scale factor.
    - 3:Animation
    - to an endpoint of the ray (THIS POINT will appear), while holding the LOCK key, drag ( ) the animation spring in the opposite direction of the intended animation, release the LOCK key. (NOTE: The animation spring will appear when you begin dragging.)
  - J. What is the relationship between the vertices of the dilated image and the rays?

**K. Repeat the animation using different endpoints and pulling the spring in different directions.**

**L. Describe the relationship between the image and the pre-image when the ratio is less than one, equal to one, and greater than one.**

**M. Calculate the following ratio: distance from center to image / distance from center to pre-image.**

- :Distance&Length
-  to the center until *DISTANCE FROM THIS POINT* appears   
 to a vertex on the image until *TO THAT POINT* appears   
*Repeat to find the distance from the center to the corresponding vertex on the pre-image.*
- :Calculate
- /
- 

**N. Repeat the animation from step 2I. Compare the two ratios calculated.**

**O. Use your findings to write a definition for a dilation with scale factor  $k$  and center  $O$ .  $(D_{k, O})$**

3. Use coordinates to define a dilation.

**A. Clear the screen.**

- See step 2A

**B. Turn on Coordinate Axes and Grid**

- $\boxed{=}$  9:Format...*RIGHT CURSOR* 2:Rectangular  $\boxed{\rightarrow}$  *RIGHT CURSOR*  
2:On  $\boxed{=}$

**C. Construct a point dilated around the origin with a scale factor 3**

- See step 1B through 1E

**D. Measure the coordinates of the pre-image and image points.**

- $\boxed{=}$  5:Equation & Coordinate
- $\boxed{\rightarrow}$  (Move until *COORDINATE OF THIS POINT* appears)  $\boxed{=}$   
Repeat for other point.

**E. Drag the pre-image point around and investigate relationship between the coordinates.**

**F. What is the image of the point (-2, 0.5) under a dilation of scale factor 2 about the origin, O?**

$$D_{O, 2}: (-2, 0.5) \rightarrow (\underline{\quad}, \underline{\quad})$$

**G. Write a general rule for the following:**

$$D_{O, k}: (x, y) \rightarrow (\underline{\quad}, \underline{\quad})$$

**H. Does the rule above apply if the center of dilation is not the origin? Use the calculator to investigate this.**

**I. Find the following:**

$$D_{(3, 3), 3}: (4, 5) \rightarrow (\underline{\quad}, \underline{\quad})$$

$$D_{(3, 3), 2}: (4, 5) \rightarrow (\underline{\quad}, \underline{\quad})$$

**J. Describe how one would find the coordinates of the image of a point under a dilation where the center is not the origin without the calculator.**

# ON BEING THE RIGHT SIZE

## Teacher Guide

### GOAL:

Understand a simple but profound aspect of nature from a mathematical point of view. Read an example of well written, lively mathematics.

### PREREQUISITES:

Similarity, ratios of areas and volumes of similar figures.

### MATERIALS:

Photographs of animals mentioned in the article.

### NOTE TO THE TEACHER:

This article first appeared in *Possible Worlds*, published in 1928. It is reprinted in *The World of Mathematics*, edited by James R. Newman. Newman calls “On Being the Right Size” a “sparkling little essay” whose “mathematics is profound but simple.” It is an example of very clear and lively writing about a long studied problem that arises in many fields. Haldane explains that the size and shape of creatures is based on geometric as well as physiological considerations. He even includes some curious ideas about the ideal size of nation states. Haldane was a geneticist who also made contributions to biology, physiology, preventive medicine, botany, hematology, statistical theory and other fields.

### PROCEDURE:

Display pictures of animals of various sizes and shapes.

Ask students to imagine those creatures either much larger or much smaller.

Could they exist at those sizes?

What modifications might their bodies have to make?

Could they continue to exist in their usual environment?

Assign reading of “On Being the Right Size.”

Ask students to explain what is meant by “Being the Right Size.”

### EXTENSIONS:

- View movies such as *Them* (about giant ants) or *Honey I Blew Up the Kids* or *Honey I Shrank the Kids* and discuss whether such large and small creatures could exist as represented in those films.

- Study the bone structure and size of very large vs. very small creatures
- Galileo also wrote about this subject. Research his writings.



# ON BEING THE RIGHT SIZE

By J. B. S. Haldane

The most obvious difference between different animals are differences of size, but for some reason the zoologists have paid singularly little attention to them. In a large textbook of zoology before me I find no indication that the eagle is larger than the sparrow, or the hippopotamus bigger than the hare, though some grudging admissions are made in the case of the mouse and the whale. But yet it is easy to show that a hare could not be as large as a hippopotamus, or a whale as small as a herring. For every type of animal there is a most convenient size, and a large change in size inevitably carries with it a change of form.

Let us take the most obvious of possible cases, and consider a giant man sixty feet high—about the height of Giant Pope and Giant Pagan in the illustrated *Pilgrim's Progress* of my childhood. These monsters were not only ten times as high as Christian, but ten times as wide and ten times as thick, so that their total weight was a thousand times his, or about eighty to ninety tons. Unfortunately the cross sections of their bones were only a hundred times those of Christian, so that every square inch of giant bone had to support ten times the weight borne by a square inch of human bone. As the human thigh-bone breaks under about ten times the human weight, Pope and Pagan would have broken their thighs every time they took a step. This was doubtless why they were sitting down in the picture I remember. But it lessens one's respect for Christian and Jack the Giant Killer.

To turn to zoology, suppose that a gazelle, a graceful little creature with long thin legs, is to become large, it will break its bones unless it does one of two things. It may make its legs short and thick, like the rhinoceros, so that every pound of weight has still about the same area of bone to support it. Or it can compress its body and stretch out its legs obliquely to gain stability, like the giraffe. I mention these two beasts because they happen to belong to the same order as the gazelle, and both are quite successful mechanically, being remarkably fast runners.

Gravity, a mere nuisance to Christian, was a terror to Pope, Pagan, and Despair. To the mouse and any smaller animal it presents practically no dangers. You can drop a mouse down a thousand-yard mine shaft; and, on arriving at the bottom, it gets a slight shock and walks away. A rat would probably be killed, though it can fall safely from the eleventh story of a building; a man is killed, a horse splashes. For the resistance presented to movement by the air is proportional to the surface of the moving object. Divide an animal's length, breadth, and height each by ten; its weight is reduced to a thousandth, but its surface only to a hundredth. So the resistance to falling in the case of the small animal is relatively ten times greater than the driving force.

An insect, therefore, is not afraid of gravity; it can fall with out danger, and can cling to the ceiling with remarkably little trouble. It can go in for elegant and fantastic forms of support like that of the daddy-long-legs. But there is a force which is as formidable to an insect as gravitation to a mammal. This is surface tension. A man coming out of a bath carries with him a film of water of about one-fiftieth of an inch in thickness. This weighs roughly a pound. A wet mouse has to carry about its own weight of water. A wet fly has to lift many times its own weight and, as every one knows, a fly once wetted by water or any other liquid is in a very serious position indeed. An insect going for a drink is in as great danger as a man leaning out over a precipice in search of food. If it once falls into the grip of the surface tension of the water—that is to say, gets wet—it is likely to remain so until it drowns. A few insects, such as water-beetles, contrive to be unwettable, the majority keep well away from their drink by means of a long proboscis.

Of course tall land animals have other difficulties. They have to pump their blood to greater heights than a man and, therefore, require a larger blood pressure and tougher blood-vessels. A great many men die from burst arteries, especially in the brain, and this danger is presumably still greater for an elephant or a giraffe. But animals of all kinds find difficulties in size for the following reason. A typical small animal, say a microscopic worm or rotifer, has smooth skin through which all the oxygen it requires can soak in, a straight gut with sufficient surface to absorb its food, and a simple kidney.

Increase its length ten times and its weight is increased a thousand times, so that if it is to use its muscles as efficiently as its miniature counterpart, it will need a thousand times as much of waste products.

Now if its shape is unaltered its surface will be increased only a hundredfold, and ten times as much oxygen must enter per minute through each square millimeter of skin, ten times as much food through each square millimeter of intestine. When a limit is reached to their absorptive powers their surface has to be increased by some special device. For example, a part of the skin may be drawn out into tufts to make gills or pushed in to make lungs, thus increasing the oxygen-absorbing surface in proportion to the animal's bulk. A man, for example, has a hundred square yards of lung. Similarly, the gut, instead of being smooth and straight, becomes coiled and develops a velvety surface, and other organs increase in complication. The higher animals are not larger than the lower because they are more complicated. They are more complicated because they are larger. Just the same is true of plants, such as the green algae growing in stagnant water or on the bark of trees, are mere round cells. The higher plants increase their surface by putting out leaves and roots. Comparative anatomy is largely the story of the struggle to increase surface in proportion to volume.

Some of the methods of increasing the surface are useful up to a point, but not capable of a very wide adaptation. For example, while vertebrates carry the oxygen from the gills or lungs all over the body in the blood, insects take air directly to every part of their body by tiny blind tubes called tracheae which open to the surface at many different points. Now, although by their breathing movements they can renew the air in the outer part of the tracheal system, the oxygen has to penetrate the finer branches by means of diffusion. Gases can diffuse easily through very small distances, not many times larger than the average length traveled by a gas molecule between collisions with other molecules. But when such vast journeys—from the point of view of a molecule—as a quarter of an inch have to be made, the process becomes slow. So the portions of an insect's body more than a quarter of an inch from the air would always be short of oxygen. In consequence hardly any insects are much more than half an inch thick. Land

crabs are built on the same general plan as insects, but are much clumsier. Yet like ourselves they carry oxygen around in their blood, and are therefore able to grow far larger than any insects. If the insects had hit on a plan for driving air through their tissues instead of letting it soak in, they might well have become as large as lobsters, though other considerations would have prevented them from becoming as large as man.

Exactly the same difficulties attach to flying. It is an elementary principle of aeronautics that the minimum speed needed to keep an airplane of a given shape in the air varies as the square root of its length. If its linear dimensions are increased four times, it must fly twice as fast. Now the power needed for the minimum speed increases more rapidly than the weight of the machine. So the larger airplane, which weighs sixty-four times as much as the smaller, needs one hundred and twenty-eight times its horsepower to keep up. Applying the same principles to the birds, we find that the limit to their size is soon reached. An angel whose muscles developed no more power weight for weight than those of an eagle or a pigeon would require a breast projecting for about four feet to house the muscles engaged in working its wings, while to economize in weight, its legs would have to be reduced to mere stilts. Actually a large bird such as an eagle or kite does not keep in the air by moving its wings. It is generally to be seen soaring, that is to say balanced on a rising column of air. And even soaring becomes more difficult with increasing size. Were this not the case eagles might be as large as tigers and as formidable to man as hostile airplanes.

But it is time that we passed to some of the advantages of size. One of the most obvious is that it enables one to keep warm. All warm-blooded animals at rest lose the same amount of heat from a unit area of skin, for which purpose they need a food-supply proportional to their surface and not to their weight. Five thousand mice weight as much as a man. Their combined surface and food or oxygen consumption are about seventeen times a man's. In fact a mouse eats about one quarter its own weight of food every day, which is mainly used in keeping it warm. For the same reason small animals cannot live in cold countries. In the arctic regions there are no reptiles or amphibians, and no small mammals. The smallest mammal in Spitzbergen is the fox. The small birds fly away in

the winter, while the insects die, though their eggs can survive six months or more of frost. The most successful mammals are bears, seals, and walruses.

Similarly, the eye is a rather inefficient organ until it reaches a large size. The back of the human eye on which an image of the outside world is thrown, and which corresponds to the film of a camera, is composed of a mosaic of 'rods and cones' whose diameter is little more than a length of an average light wave. Each eye has about half a million, and for two objects to be distinguishable their images must fall on separate rods or cones. It is obvious that with fewer but larger rods and cones we should see less distinctly. If they were twice as broad two points would have to be twice as far apart before we could distinguish them at a given distance. But if their size were diminished and their number increased we should see no better. For it is impossible to form a definite image smaller than a wave-length of light. Hence a mouse's eye is not a small-scale model of a human eye. Its rods and cones are not much smaller than ours, and therefore there are far fewer of them. A mouse could not distinguish one human face from another six feet away. In order that they should be of any use at all the eyes of small animals have to be much larger in proportion to their bodies than our own. Large animals on the other hand only require relatively small eyes, and those of the whale and elephant are little larger than our own.

For rather more recondite reasons the same general principle holds true of the brain. If we compare the brain-weights of a set of very similar animals such as the cat, cheetah, leopard, and tiger, we find that as we quadruple the body-weight the brain-weight is only doubled. The larger animal with proportionately larger bones can economize on brain, eyes, and certain other organs.

Such are a very few of the considerations which show that for every type of animal there is an optimum size. Yet although Galileo demonstrated the contrary more than three hundred years ago, people still believe that if a flea were as large as a man it could jump a thousand feet into the air. As a matter of fact the height to which an animal can jump is more nearly independent of its size than proportional to it. A flea can jump about two

feet, a man about five. To jump a given height, if we neglect the resistance of the air, requires an expenditure of energy proportional to the jumper's weight. But if the jumping muscles form a constant fraction of the body, the energy developed per ounce of muscle is independent of the size, provided it can be developed quickly enough in the small animal. As a matter of fact an insect's muscles, although they can contract more quickly than our own, appear to be less efficient; as otherwise a flea or grasshopper could rise six feet into the air.

And just as there is a best size for every animal, so the same is true for every human institution. In the Greek type of democracy all the citizens could listen to a series of orators and vote directly on questions of legislation. Hence their philosophers held that a small city was the largest possible democratic state. The English invention of representative government made a democratic nation possible, and the possibility was first realized in the United States, and later elsewhere. With the development of broadcasting it has once more become possible for every citizen to listen to the political views of representative orators, and the future may perhaps see the return of the national state to the Greek form of democracy. Even the referendum has been made possible only by the institution of daily newspapers.

To the biologist the problem of socialism appears largely as a problem of size. The extreme socialists desire to run every nation as a single business concern. I do not suppose that Henry Ford would find much difficulty in running Andorra or Luxembourg on a socialistic basis. He has already more men on his pay-roll than their population. It is conceivable that a syndicate of Fords, if we could find them, would make Belgium Ltd. or Denmark Inc. pay their way. But while nationalization of certain industries is an obvious possibility in the largest of states, I find it no easier to picture a completely socialized British Empire or United States than an elephant turning somersaults or a hippopotamus jumping a hedge.

# TRIGONOMETRY

## Institute Notes

Lesson	Pages	Time	TEKS
Trigonometric Ratios		40 min.	7b
Law of Sines and Cosines		40 min.	7b,c

### MATERIALS AND SETUP

*Geometer's Sketchpad* sketches: "Unit Circle/Trig", "Sine Tracer", "Sin/Cos Graphs", and "Tangent Tracer", Trig Investigations Script

### LEARNING STYLES

Cooperative learning, technology

### MATH LINKS

Similar triangles, coordinate graphs, algebra, vectors

### OTHER LINKS

Navigation, surveying, architecture, physics, astronomy

### LESSON OVERVIEW

Trigonometry is a tool for solving problems which occur often in the physical world where limited information about distances and angles is given. These problems include applications of harmonic motion, navigation and aviation problems, and finding unknown distances and angles. In the lessons which follow, definitions for trigonometric ratios are developed, their graphs are investigated and theorems/ properties are developed using trigonometric definitions.

#### Trigonometric Ratios and Sketches

The Trigonometric Ratios activity explores the six trigonometric ratios using similar triangles and *Geometer's Sketchpad*. Participants move through parts A - E and the extension at their own pace to gain familiarity with *Sketchpad* features. This should take no more than 15 minutes.

Use the remaining time to allow participants to investigate the four "Trigonometric *Sketchpad* Sketches" listed in the materials section above. Refer teachers to the "Sine and Cosine Graphs" worksheet to guide them through the "Sin/Cos Graphs" sketch.



## **The Law of Sines and Cosines**

These activities enable the students to work with angles and lengths of nonright triangles. There are many practical applications.

Trainer presents Pond problem and Triangle Lot problems and overheads.

Participants will work in groups to derive the Law of Sines. Trainer will derive the Law of Cosines with participant input. Participants will apply the Laws of Sines and Cosines in order to work problems 2 and 7.

## NOTES



# TRIGONOMETRIC RATIOS

## Teacher Guide

### GOAL:

Explore the trigonometric ratios using similar right triangles.

### PREREQUISITES:

Parts of a right triangle: leg, hypotenuse; adjacent and opposite sides to an angle of a triangle; similar triangles

### MATERIALS:

Computer lab worksheet, data sheet, and a computer.

Scripts: Trig Investigation

### PROCEDURE:

- Put students in pairs to work on the computer lab, giving each student a copy of the worksheet and data sheet to record their work.
- Allow students to work through the lab.
- Ask student pairs to lead the discussion of each question after everyone has finished the lab. Encourage students to offer alternate conjectures and let the class discuss the validity of each conjecture.

---

---

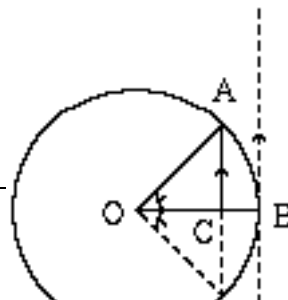
### SOLUTIONS:

H. As the length of the sides change, each ratio remains constant.

J. For each measure of angle A, each of the six ratios remains constant.

#### Possible Justification:

Any two right triangles with a common acute angle are similar by the Angle-Angle Similarity Postulate. Therefore, the ratios of corresponding sides of the triangles are equal by the definition of similar triangles. So, for a given angle measure the ratio of the corresponding sides will be a constant.



## **HISTORICAL NOTE:**

Sine was originally the length of a line segment drawn from one end of a circular arc (parallel to the tangent at the midpoint of the arc) terminated by the radius to the point of tangency,  $\overline{AC}$ . In modern use, sine is the ratio of this line segment to the radius of the circle. The word sine was first used in 1591 and, in 1593, abbreviated sin.

Much effort went into computing trigonometry tables for finer and finer subdivisions of angles. This was done by using a circle with a radius of some assigned length, the more units in the radius, the more accurate the ratio of the lengths without the need to use decimal or sexagesimal fractions (more common since the arc of a circle is divided into  $360^\circ$ ,

1 deg. = 60 min., 1 min. = 60 sec.).

Euler (1748) was the first to use radius = 1, and the idea of sines and other trigonometric functions as ratios (numbers) as opposed to lengths of seconds.

The study of trigonometry began with the Alexandrian Greeks—Hipparchus (d. 127 BC), Menelaus (c. 98 AD), and Ptolemy. It was developed to predict paths and positions of heavenly bodies, to aid the telling of time at night, and for calendar reckoning, navigation, and geography. It was studied as part of spherical geometry. Only much later was trigonometry used for surveying.

## **RESOURCES:**

Kline, Morris. *Mathematical Thought from Ancient to Modern Times*.

Struik, Dirk J. *A Concise History of Mathematics*.

# TRIGONOMETRIC RATIOS

## GOAL:

Explore the trigonometric ratios using similar right triangles.

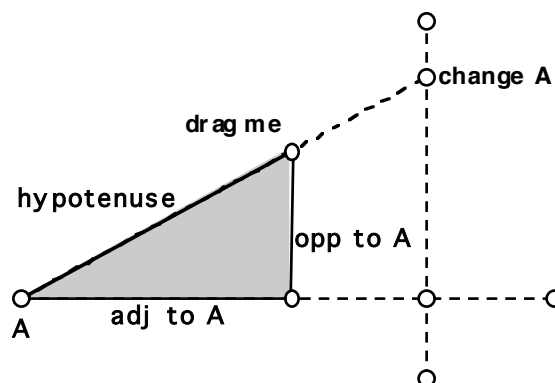
## PROCEDURE:

### A. Open a new sketch on Sketchpad.

- Under the File menu, select New Sketch.

### B. Use the Trig Investigation script tool to construct a right triangle.

- Use the script tool of the toolbar, select-hold-drag down to Institute Scripts, Triangles, then Trig Investigation, and release.
- In the sketch, click-release-drag-click to create the picture.



### C. Measure the length of the leg opposite angle A, the leg adjacent to angle A, and the hypotenuse.

- Select the arrow tool from the toolbar.
- Click on the segment opposite angle A.
- While holding down the Shift key, click on the segment adjacent to angle A and the hypotenuse.
- Under the Measure menu, select Length.

### D. Measure angle A.

- With the arrow tool, select the vertex "drag me".
- While holding down the Shift key, select vertex A and the last vertex.
- Under the Measure menu, select Angle.

**E. Find the ratio of “opp” to “hyp”.**

- *With the arrow tool from the toolbar, select segment “opp to A”.*
- *While holding down the Shift key, and select segment “hypotenuse”.*
- *Under the Measure menu, select Ratio.*

**F. Calculate the other five ratios possible. Record your data in the first line of the chart in the Data Sheet.**

**G. Drag the point “drag me” to change the lengths of the sides of the triangle, keeping angle A constant. Record several sets of data in the chart.**

**H. What do you notice about the each ratio as the length of the sides change?**

**I. Drag the point “change A” to change the measure of angle A. Repeat step G for several measures of angle A.**

**J. How can you modify your conclusion about each ratio in part H to include changes in the measure of angle A? Justify your conclusion using similar triangles.**

## Data Sheet - Trigonometric Ratios

$m \angle A$	opp	adj	hyp	$\frac{opp}{hyp}$	$\frac{adj}{hyp}$	$\frac{opp}{adj}$	$\frac{hyp}{opp}$	$\frac{hyp}{adj}$	$\frac{adj}{opp}$

These ratios are known as *trigonometric ratios*. Each ratio has a specific name.

### Trigonometric Ratios for Right Triangles

$$\text{Sine } A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}}$$

$$\text{Cosecant } A = \frac{\text{length of hypotenuse}}{\text{length of leg opposite } \angle A}$$

$$\text{Cosine } A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}}$$

$$\text{Secant } A = \frac{\text{length of hypotenuse}}{\text{length of leg adjacent } \angle A}$$

$$\text{Tangent } A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A}$$

$$\text{Cotangent } A = \frac{\text{length of leg adjacent } \angle A}{\text{length of leg opposite to } \angle A}$$

#### EXTENSION:

*Sketchpad* may be used to directly evaluate trigonometric functions.

- Double-click on the angle measure to bring up the calculator.
- Under the Functions option, select a trig. function from the menu (“sin [“, “cos [“ or “tan [“).
- Move arrow back to the angle measure and click on it.  
(The angle name should appear in the calculator display.)
- Press “ ) “ to exit the trig. option, THEN click on “OK”.





# TRIGONOMETRIC *SKETCHPAD* SKETCHES

## Teacher Guide

### GOAL:

Make connections between the trigonometric functions and the unit circle.

The unit circle will be used to trace the trigonometric functions on coordinate axes.

### PREREQUISITES:

Definition of the trigonometric ratios on the Cartesian coordinate system, tangent line to a circle.

### MATERIALS:

*Sketchpad* Sketches: “Unit Circle/Trig”, “Sine Tracer”, “Sin/Cos Graphs”, and “Tangent Tracer”; Instruction Sheet for Sin/Cos Graphs.

### NOTE TO TEACHER:

General Note: These sketches may be used as teacher demonstrations or student explorations. The sketches should be presented in the sequence below.

- The Unit Circle and the Trigonometric Functions
- Sine as a Function of Time
- Sine and Cosine Graphs (Requires a student handout: Sine and Cosine Graphs)
- Tangent Tracer.

## The Unit Circle and the Trigonometric Functions:

- Students should open the sketch "Unit Circle/Trig".
- In this lesson, students will investigate the unit circle, A and discover that the length of segment  $DF = \sin A$ ,  $AF = \cos A$  and  $CE = \tan A$ . The connection should be made that CE is also part of the tangent line to circle A, hence the name.
- Students should record their answers to the questions on the screen on their own paper.

File Edit Display Construct Transform Measure Graph Work

Unit Circle/Trig

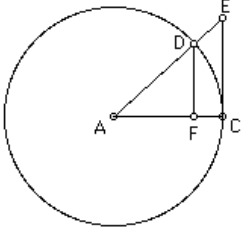
### The Unit Circle and the Trigonometric Functions

Circle A below is called a "unit circle" because it has a radius of length 1. Unit circles are often used when describing the trigonometric functions since letting  $r=1$  simplifies the trigonometric ratios. The sine, cosine and tangent of angle CAE are measured below. Certain segments in the unit circle have the same lengths as the values of certain trigonometric functions.

1. Measure all the line segments in the diagram.
2. Arrange the table to align any possible matching measures.
3. State the relationship between the trigonometric functions and the line segments.
4. Why do these relationships occur?
5. Drag point D around circle A to investigate this phenomena for various right triangles.

$m\angle CAE = 42.03^\circ$

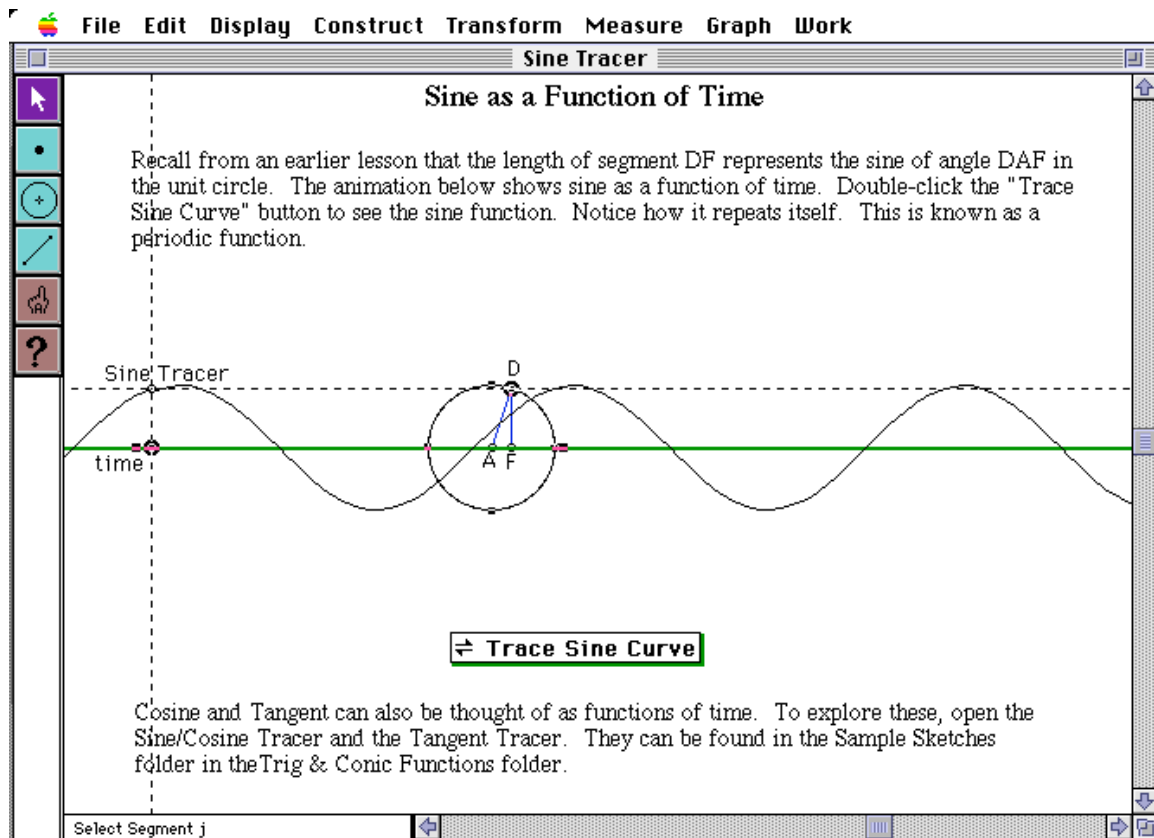
Trig. Functions	Line Segments
$\sin(m\angle CAE) = 0.67$ $\cos(m\angle CAE) = 0.74$ $\tan(m\angle CAE) = 0.90$	$m\overline{CA} = 1.00$ inches



Select & Translate

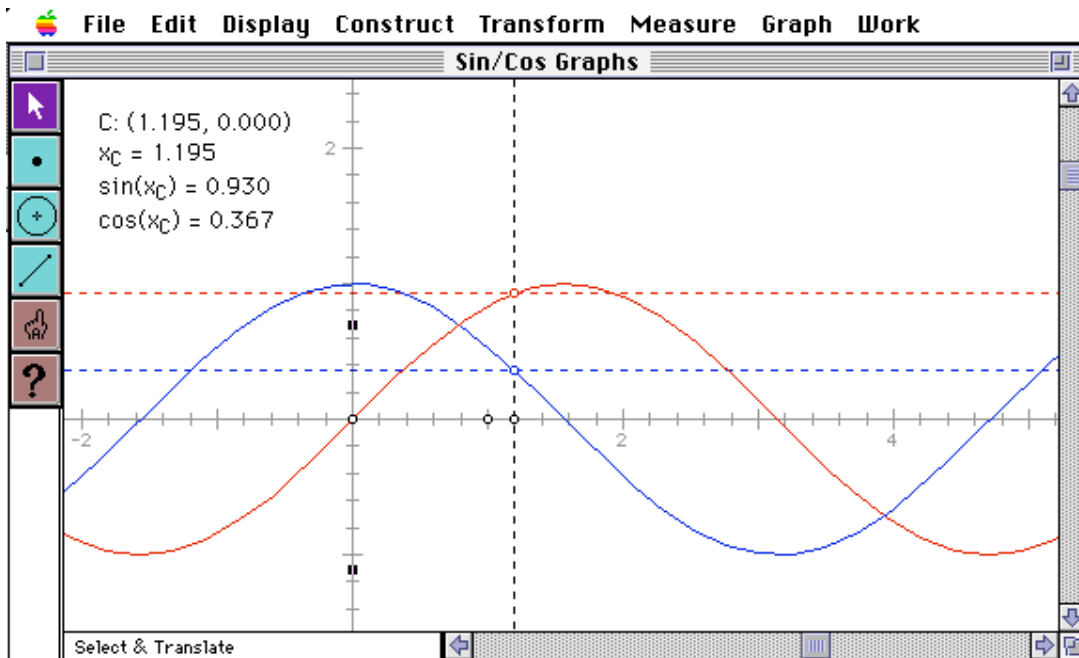
### Sine as a Function of Time:

- Students should open the sketch "Sine Tracer".
- Students will use the knowledge from the previous sketch.
- The sketch animates a point around a circle while the point "time" is animated along the horizontal axis. As the students animate the sketch, the sine function is traced.
- The idea of a periodic function can now be discussed along with concepts such as period and amplitude.



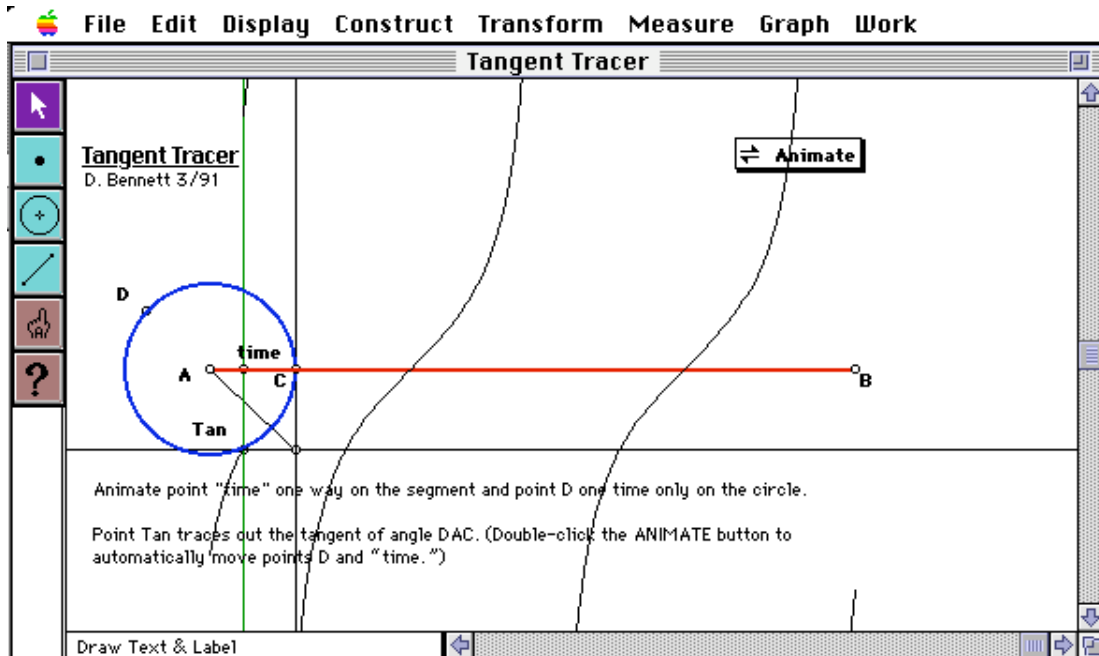
### Sine and Cosine Graphs:

- Students should open the sketch “Sin/Cos Graphs.”
- In this sketch, students will create the graphs of Sine and Cosine.
- Give each pair of students an instruction sheet to create the sketch and allow them to explore the graphs. (Or use the instructions as a teacher demonstration.)
- Students should realize that the cosine graph is a translation of the sine graph.  
(Note: these graphs are created using radian measures.)



### Tangent Tracer:

- Students should open the sketch “Tangent Tracer.” It is located in The Geometer’s Sketchpad: Sample Sketches: Trig & Conic Functions: Tangent Tracer.
- This sketch traces the tangent function.



# SINE AND COSINE GRAPHS

## GOAL:

To investigate the relationship between the graphs of the sine and cosine functions.

## PREREQUISITES:

“Sine as a Function of Time” sketch

## PROCEDURE:

### A. Open the sketch “Sin/Cos Graphs”.

- Under the File menu, choose “Open”.
- Find the sketch entitled “Sin/Cos Graphs” and “open”.

### B. Calculate the x-coordinate of C.

- Double-click on the coordinate measure of C to bring up the calculator.
- Under the “Values” menu, choose “Point C”, then choose “ $X_c$ ”.

### C. Plot the x-coordinate measure on the x-axis.

- Select the x-coordinate measure.
- Under the Graph menu, choose “Plot Measurement”, then click in the button corresponding to “On horizontal (x) axis”.

### D. Calculate the sine of the x-coordinate of C. Plot the measurement on the y-axis.

- Double-click the x-coordinate measure of point C to bring up the calculator.
- Under the “Functions” menu, choose “Sin[”.
- Under the “Values” menu, choose “ $X_c$ ”.
- Click on the “)” key, then “OK”.
- Select the “Sin[ $X_c$ ]” measure.
- Under the “Graph” menu, choose “Plot Measurement”, then click in the button corresponding to “On vertical (y) axis”.

### E. Create a point at the intersection of the two dotted lines by clicking on the intersections.

### F. Construct the locus of points which define $\sin(x)$ .

- Select the intersection of the two dotted lines and C.
- Under the Construct menu, choose “locus”.

### G. Repeat steps D through F substituting “Cos” for “Sin” to graph $\cos(x)$ .