

**INTRODUCTION TO ENUMERATIVE ALGEBRAIC GEOMETRY:  
EXERCISE SESSION 1**

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- Exercise 1.** (a) Find an affine stratification on  $\mathbf{P}^r \times \mathbf{P}^s$  and use it to compute its Chow ring.
- (b) Use that to define the notion of *bidegree*  $\in \mathbf{Z} \times \mathbf{Z}$  of a hypersurface  $X \subseteq \mathbf{P}^r \times \mathbf{P}^s$  in analogy with the degree of a projective hypersurface.
- (c) For  $r = s$ , give an interpretation of the bidegree in terms of the two rulings on the smooth quadric surface  $Q$  and the isomorphism  $\mathbf{P}^1 \times \mathbf{P}^1 \cong Q$ .

**Exercise 2.** Study linear systems of plane conics.

- (a) Describe the stratification by type of conic.
- (b) How many elements of a general pencil are singular?
- (c) General linear systems of what dimension do you need to take to start finding double lines in them? How many double lines will be in such a system?

**Exercise 3.** Finish the analysis of plane cubics from today's lecture. That is to say, determine the correct dimension of general linear systems to have a finite number of elements of the following loci, and compute these numbers, for:

- (a) The locus  $\mathfrak{h} \subseteq \mathbf{P}^9$  of *thorns*, i.e. non-transverse intersections of a line and a conic.
- (b) The locus  $\mathfrak{X} \subseteq \mathbf{P}^9$  of the unions of a double line and a line.
- (c) The locus  $\text{III} \subseteq \mathbf{P}^9$  of triple lines.

**Exercise 4.** Study linear systems of quadric surfaces in 3-space. What does the space of quadric surfaces look like, in terms of the type stratification? What dimension of linear systems do you need to take to start finding:

- (a) reducible quadric surfaces?
- (b) double planes?
- (c) cones?

How many of those will there be in such a general linear system?