INTRODUCTION TO ENUMERATIVE ALGEBRAIC GEOMETRY: EXERCISE SESSION 2

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Exercise 1. Finish the verification that the Schubter cycles form the closed strata of an affine stratification of $\mathbf{G}(1,3)$.

The open stratum corresponding to Σ_1 being an affine space case should be the hardest.

Exercise 2. How many lines in $\mathbf{P}^3$ are tangent to four general quadric surfaces?

For the previous one, it might be useful to recall something we proved in lectures on Tuesday: namely, that the degree of the dual hypersurface to a degree d hypersurface in $\mathbf{P}^n$ is $d(d-1)^{n-1}$.

Exercise 3. How many elements in a general pencil of quartic surfaces contain a line?

Exercise 4. Let $C \subseteq \mathbf{P}^3$ be a general algebraic curve of degree d and genus g, and $S \subseteq \mathbf{P}^3$ a general algebraic surface of degree e. How many lines in $\mathbf{P}^3$ are tangent to both C and S?

The final one is quite a bit harder. You will need the Riemann-Hurwitz formula: if $f : X \to Y$ is a degree d map between algebraic curves, then $2g_X - 2 = r_f + d(2g_Y - 2)$ where r_f is the ramification (i.e. branching) number, and g is the genus.

Also, perhaps a helpful hint (for a certain part where it's easy to get stuck): consider the projection from a general line onto another line.

Good luck! :)

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