Exercise Set #1

Exercise 1: Let D denote a diagram for a knot K in S^3. Show that the framing coefficient of the blackboard framing in D equals the writhe $w(D)$, the signed number of self-crossings of D.

Exercise 2: Let $T_{p,q}$ denote the (p,q)-torus knot. Recall that the exterior $T_{p,q}$ in S^3 is a Seifert fibered space of type $\mathbb{D}(|p|,|q|)$. Let F be the isotopy class of a Seifert fiber in $\partial N(T_{p,q})$. Show that the framing coefficients of F is pq.

Exercise 3: Let (M,ξ) be a contact 3-manifold *Legendrian link* L in (M,ξ) is a link in M whose tangent vectors all lie in ξ. The *canonical framing* on L is the framing induced by any vector field on L transverse to ξ.

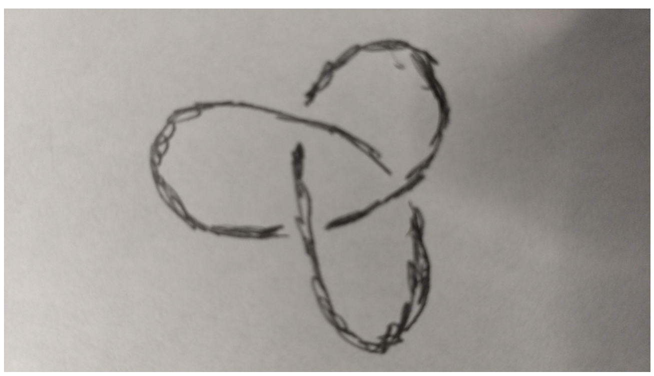
i Discuss why any link (S^3,ξ_c), where ξ_c is the standard contact structure $\ker(dz + xdy)$, is isotopic to a Legendrian link. *(Hint: Introduce cusps.)*

ii Let D be a Legendrian link diagram of K in the standard structure on (S^3,ξ_c). The Thurston-Bennequin invariant $tb(K)$ of a Legendrian knot K is defined to be the canonical framing of K in the standard structure (S^3,ξ_c). Show that

$$tb(K) := w(D) - \lambda(D)$$

where $\lambda(D)$ is the number of left cusps.

Exercise 4: Let Y be $+1$ surgery on the right-handed trefoil in S^3. Compute $\pi_1(Y)$.



Exercise 5: Prove that surgery on each the framed links in Figure 3.27 yields $\Sigma(2,3,5)$.

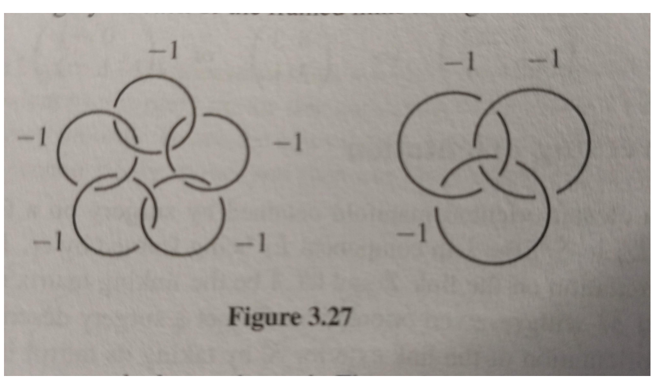


Figure 3.27

Exercise 6: Which lens space is $+5$ surgery on RHT?