Tensor Methods: M 393C / CSE 396

University of Texas at Austin, Fall 2025

Times: TueThu 2-3:30PM CT
Location: Face-to-face in PMA 11.176
Dates: 28 classes, Tue Aug 26 – Thu Dec 4, with Tue Nov 25 and Thu Nov 27 off
Supporting Media: Canvas
Registrar Information: Unique 59255 / 70039
Instructor: Joe Kileel, jkileel@math.utexas.edu
Office Hours: Wed 1-3PM CT in POB 3.434

Description: This is a graduate topics course on tensor methods in computational mathematics and data science. The emphasis will be on low-rank decompositions (or compressions) that may be used to efficiently represent large-scale tensors, and thereby overcome the curse of dimensionality inherent in higher-order data. The course will cover algorithms, implementation details, convergence analyses and numerical conditioning issues for tensor decompositions, as well as fundamental mathematical aspects including some of the relevant geometry and algebra that underlie low-rank tensor decompositions. The material will be motivated by application areas, such as latent variable models, imaging and computer vision, high-dimensional partial differential equations, complexity theory, and quantum chemistry.

Prerequisites: Mathematical maturity is most important. That said, familiarity with numerical linear algebra and nonconvex optimization is very useful. Basic knowledge of algebraic varieties and Riemannian manifolds may help with some parts, but is not required. A bit of computer programming experience would be a plus (good software choices for tensor computations include Tensorlab and Tensor Toolbox).

Textbook: The main textbook for the course is the new book “Tensor Decompositions for Data Science" by Grey Ballard and Tamara G. Kolda. This covers computational aspects for the Tucker and CP decompositions well. We will also refer to the book “Tensors: Geometry and Applications" by J. M. Landsberg when studying the CP decomposition, which stresses relevant algebraic and geometric aspects. The book “Tensor Spaces and Numerical Tensor Calculus" by Wolfgang Hackbusch may be referred to when we study Tensor Train and Tensor Network decompositions.

Research Papers: The textbooks will be supplemented by our discussion of various research papers. The papers to be covered are not yet determined, although a list of possibilities appears at the end of the syllabus.

Homework: There will be approximately three homework assignments. Their purpose is for you to check your understanding of the basics from lectures. Since I do not have a TA for this course, you will grade each other's homework assignments through a "peer review" system in which I will serve as the associate editor analog.

Final Project: Students will work in groups of one or two on a final project. Projects will focus on a research paper not covered in class. You will write up and submit a project report (of length ~5 pages for groups of one and ~10 pages for groups of two), which summarizes the research paper and perhaps adds some worked examples or numerical experiment of your own. You will also give a slides presentation to the class during the last four class meetings, taking 20 or 30 minutes per group.

Final Grade: Since this is a graduate topics course, grading is not expected to be competitive. The homework assignments will count for 15% each, with an additional overall 5% bonus for your services as a peer reviewer. The final project will count for 50%, with 30% for the report and 20% for your presentation. Letter grade cut-offs will be finalized at the end of the semester; however they will not be stricter than the following: A (100-90), A- (89-85), B+ (84-80), B (79-75), B- (74-70), C+ (69-65), C (64-61), C- (60-57), D (56-50), F (49-0).

Tentative Course Plan:

0. Tensor Basics (~1 class)
chapters 1-3 of Ballard-Kolda

1. Tucker Decomposition (~5 classes)
chapters 4-7 of Ballard-Kolda
some research papers

2. CP Decomposition (~8 classes)
chapters 9-16 of Ballard-Kolda
a sampling of Landsberg
some research papers

3. Tensor Train Decomposition (~3 classes)
chapter 8 of Ballard-Kolda
chapter 12 of Hackbusch
some research papers

4. Tensor Network Decompositions (~3 classes)
sections 17.4.2-17.4.4 of Ballard-Kolda
a sampling of Hackbusch
some research papers

5. Tubal Tensor Decompositions (~2 classes)
section 17.4.1 of Ballard-Kolda
some research papers

6. Functional Tensor Decompositions (~2 classes)
section 17.4.5 of Ballard-Kolda
some research papers

7. Student Presentations (~4 classes)

Accommodations: The University of Texas provides, upon request, academic accommodations for qualified students with disabilities. For more information, please contact Disability and Access (D&A) at http://diversity.utexas.edu/disability/.

Honor Code: "I pledge, as a member of the University of Texas community, to do my work honestly, respectfully, and through the intentional pursuit of learning and scholarship."

Possible Papers: The following is a list of research papers relevant to the course. Hint: we will not cover them all :)

Papers on Tucker

- "A Multilinear Nyström Algorithm for Low-Rank Approximation of Tensors in Tucker Format", A. Bucci, L. Robol, *SIAM Journal on Matrix Analysis and Applications, 2024*
- "Mode-Wise Tensor Decompositions: Multi-Dimensional Generalizations of CUR Decompositions", H. Cai, K. Hamm, L. Huang, D. Needell, *Journal of Machine Learning Research, 2021*
- "Weighted Low-Rank Tensor Recovery for Hyperspectral Image Restoration", Y. Chang, L. Yan, X.-L. Zhao, H. Fang, Z, Zhang, S. Zhong, *IEEE Transactions on Cybernetics, 2020*
- "A multilinear singular value decomposition", L. De Lathauwer, B. De Moor, J. Vandewalle, *SIAM Journal on Matrix Analysis and Applications, 2020*
- "Optimization Landscape of Tucker Decomposition", A. Frandsen, R. Ge, *Mathematical Programming, 2022*
- "Traffic Data Imputation via Tensor Completion Based on Soft Thresholding of Tucker Core", J. H. M. Goulart, A. Y. Kibangou, G. Favier, *Transportation Research, 2017*
- "Scalable Symmetric Tucker Tensor Decomposition" R. Jin, J. Kileel, T. Kolda, R. Ward, *SIAM Journal on Matrix Analysis and Applications, 2024*
- "Nonnegative Tucker Decomposition", Y.-D. Kim and S. Choi, *CVPR, 2007*
- "Low-Rank Tucker Decomposition of Large Tensors Using TensorSketch", O. A. Malik, S. Becker, *NeurIPS, 2018*
- "Tensor-Based Synchronization and the Low-Rankness of the Block Trifocal Tensor", D. Miao, G. Lerman, J. Kileel, *NeurIPS, 2024*
- "Randomized Algorithms for Low-Rank Tensor Decompositions in the Tucker Format", R. Minster, A. Saibaba, M. Kilmer, *SIAM Journal on Mathematics of Data Science, 2020*
- "Quasi-Newton Methods on Grassmannians and Multilinear Approximations of Tensors", B. Savas, L.-H. Lim, *SIAM Journal on Scientific Computing, 2010*
- "Low-Rank Tucker Approximation of a Tensor From Streaming Data", Y. Sun, Y. Guo, C. Luo, J. Tropp, M. Udell, *SIAM Journal on Mathematics of Data Science, 2020*
- "Streaming Low-Rank Matrix Approximation with an Application to Scientific Simulation", J. Tropp, A. Yurtsever, M. Udell, V. Cevher, *SIAM Journal on Scientific Computing, 2019*
- "A New Truncation Strategy for the Higher-Order Singular Value Decomposition", N. Vannieuwenhoven, R. Vandebril, K. Meerbergen, *SIAM Journal on Scientific Computing, 2012*

Papers on CP

- "Tensor Decompositions for Learning Latent Variable Models", A. Anandkumar, R. Ge, D. Hsu, S. M. Kakade, M. Telgarsky, *Journal of Machine Learning Research, 2014*
- "A Practical Randomized CP Tensor Decomposition", C. Battaglino, G. Ballard, and T. Kolda, *SIAM Journal on Matrix Analysis and Applications, 2018*
- "Pencil-Based Algorithms for Tensor Rank Decomposition are not Stable", C. Beltrán, P. Breiding, N. Vannieuwenhoven, *SIAM Journal on Matrix Analysis and Applications, 2019*

- “The Average Condition Number of Most Tensor Rank Decomposition Problems is Infinite", C. Beltrán, P. Breiding, N. Vannieuwenhoven, *Foundations of Computational Mathematics, 2023*
- “Fast Exact Leverage Score Sampling from Khatri-Rao Products with Applications to Tensor Decomposition", V. Bharadwaj, O. A. Malik, R. Murray, L. Grigori, A. Buluc, J. Demmel, *NeurIPS, 2023*
- “On Maximum, Typical and Generic Ranks", G. Blekherman, Z. Teitler, *Mathematische Annalen, 2015*
- “Symmetric Tensor Decomposition", J. Brachat, P. Comon, B. Mourrain, E. Tsigaridas, *Linear Algebra and Applications, 2009*
- “On the Alexander–Hirschowitz Theorem", M. C. Brambilla, G. Ottaviani, *Journal of Pure and Applied Algebra, 2007*
- “On the Concept of k-Secant Order of a Variety", L. Chiantini, C. Ciliberto, *Journal of London Mathematical Society, 2006*
- “Effective Criteria for Specific Identifiability of Tensors and Forms", L. Chiantini, G. Ottaviani, N. Vannieuwenhoven, *SIAM Journal on Matrix Analysis and Applications, 2017*
- “Fourth-Order Cumulant-based Blind Identification of Underdetermined Mixtures", L. De Lathauwer, J. Castaing, J.-F. Cardoso, *IEEE Transactions on Signal Processing*, 2007
- “Tensor Rank and the Ill-Posedness of the Best Low-Rank Approximation Problem", V. De Silva, L.-H. Lim, *SIAM Journal on Matrix Analysis and Applications, 2008*
- “Homotopy Techniques for Tensor Decomposition and Perfect Identifiability", J. Hauenstein, L. Oeding, G. Ottaviani, A. Sommese, *Journal für die reine und angewandte Mathematik, 2016*
- “The Expression of a Tensor or a Polyadic as a Sum of Products", F. L. Hitchcock, *Journal of Mathematics and Physics, 1927*
- “Convergence of the Alternating Least Squares Algorithm for CP Tensor Decompositions", N. Hu, M. Iwen, D. Needell, R. Wang, *arXiv, 2025*
- “Computing Linear Sections of Varieties: Quantum Entanglement, Tensor Decompositions and Beyond", N. Johnston, B. Lovitz, A. Vijayaraghavan, *FOCS, 2023*
- “Landscape Analysis of an Improved Power Method for Tensor Decomposition", J. Kileel, T. Klock, J. Pereira, *NeurIPS, 2021*
- “Subspace Power Method for Symmetric Tensor Decomposition", J. Kileel, J. Pereira, *Numerical Algorithms, 2025*
- “Overcomplete Tensor Decomposition via Koszul–Young Flattenings", P. Kothari, A. Moitra, A. Wein, *FOCS, 2025*
- “Kruskal’s Theorem", J. M. Landsberg, *arXiv, 2009*
- “Generating Polynomials and Symmetric Tensor Decompositions", J. Nie, *Foundations of Computational Mathematics, 2017*
- “Complex Best r-Term Approximations Almost Always Exist in Finite Dimensions", Y. Qi, M. Michalek, L.-H. Lim, *Applied and Computational Harmonic Analysis, 2020*
- “Secant Varieties of Segre-Veronese Varieties", C. Raicu, *Algebra Number Theory, 2012*
- “Efficient Tensor Decomposition via Moment Matrix Extension", B. Shi, J. Lindberg, J. Kileel, *arXiv, 2025*
- “A Counterexample to Comon’s Conjecture", Y. Shitov, *SIAM Journal on Applied Algebra and Geometry, 2018*
- “A Normal Form Algorithm for Tensor Rank Decomposition", S. Telen, N. Vannieuwenhoven, *ACM Transactions on Mathematical Software, 2022*
- “Tracking Online Low-Rank Approximations of Higher-Order Incomplete Streaming Tensors", L. T. Thanh, K. Abed-Meraim, N. L. Trung, A. Hafiane, *Patterns, 2023*
- “Contrastive Independent Component Analysis", K. Wang, A. Maraj, A. Seigal, *arXiv, 2024*
- “Moment Estimation for Nonparametric Mixture Models through Implicit Tensor Decomposition", Y. Zhang, J. Kileel, *SIAM Journal on Mathematics of Data Science, 2023*

Papers on Tensor Train

- "Entrywise Tensor-Train Approximation of Large Tensors via Random Embeddings", S. Budzinskiy, *SIAM Journal on Matrix Analysis and Applications, 2025*
- "Tensor Train Based Sampling Algorithms for Approximating Regularized Wasserstein Proximal Operators", F. Han, S. Osher, W. Li, *SIAM/ASA Journal on Uncertainty Quantification, 2025*
- "A Randomized Tensor Train Singular Value Decomposition", B. Huber, R. Schneider, S. Wolf, *Compressed Sensing and its Applications, 2018*
- "Generative Modeling via Tensor Train Sketching", Y. Hur, J. Hoskins, M. Lindsey, E. M. Stoudenmire, Y. Khoo, *Applied and Computational Harmonic Analysis, 2023*
- "Multiscale Interpolative Construction of Quantized Tensor Trains", M. Lindsey, *arXiv, 2023*
- "Dynamical Approximation by Hierarchical Tucker and Tensor-Train Tensors", C. Lubich, T. Rohwedder, R. Schneider, B. Vandereycken, *SIAM Journal on Matrix Analysis and Applications, 2013*
- "Tensor-Train Decomposition", I. V. Oseledets, *SIAM Journal on Scientific Computing, 2011*
- "Matrix Product State Representations", D. Perez-Garcia, F. Verstraete, M. Wolf, J. Cirac, *Quantum Information & Computation, 2007*
- "Error Analysis of Tensor-Train Cross Approximation", Z. Qin, A. Lidiak, Z. Gong, G. Tang, M. Wakin, Z. Zhu, *NeurIPS, 2023*
- "Group-Invariant Tensor Train Networks for Supervised Learning", B. Sprangers, N. Vannieuwenhoven, *SIAM Journal on Matrix Analysis and Applications, 2023*

Papers on Tensor Networks

- "Low-Rank Tree Tensor Network Operators for Long-Range Pairwise Interactions", G. Ceruti, D. Kressner, D. Sulz, *SIAM Journal on Scientific Computing, 2025*
- "Committor Functions via Tensor Networks", Y. Chen, J. Hoskins, Y. Khoo, M. Lindsey, *Journal of Computational Physics, 2022*
- "Combining Monte Carlo and Tensor-Network Methods for Partial Differential Equations via Sketching", Y. Chen, Y. Khoo, *arXiv, 2023*
- "Efficient Construction of Tensor Ring Representations from Sampling", Y. Khoo, J. Lu, L. Ying, *SIAM Multiscale Modeling & Simulation, 2021*
- "Sampling-Based Decomposition Algorithms for Arbitrary Tensor Networks", O. A. Malik, V. Bharadwaj, R. Murray, *arXiv, 2022*
- "Efficient Tree Tensor Network States (TTNS) for Quantum Chemistry: Generalizations of the Density Matrix Renormalization Group Algorithm", N. Nakatani, G. K.-L. Chan, *Journal of Chemical Physics, 2013*
- "Stable Tensor Neural Networks for Efficient Deep Learning", E. Newman, L. Horesh, H. Avron, M. Kilmer, *Frontiers in Big Data, 2024*
- "Tensor Networks Meet Neural Networks: A Survey and Future Perspectives", M. Wang et al., *arXiv, 2023*
- "Tensor Network Ranks", K. Ye, L.-H. Lim, *arXiv, 2018*

Papers on Tubal Algebra

- "Demystifying Tubal Tensor Algebra", H. Avron, *arXiv, 2025*
- "Randomized Kaczmarz Methods for t-Product Tensor Linear Systems with Factorized Operators", A. Castillo et al., *BIT Numerical Mathematics, 2025*
- "Third-Order Tensors as Operators on Matrices: A Theoretical and Computational Framework with Applications in Imaging", M. Kilmer, K. Braman, N. Hao, R. Hoover, *SIAM Journal on Matrix Analysis and Applications, 2013*
- "Tensor-Tensor Algebra for Optimal Representation and Compression of Multiway Data", M. Kilmer, L. Horesh, H. Avron, E. Newman, *Proceedings of the National Academy of Sciences, 2021*

- "Optimal Matrix-Mimetic Tensor Algebras via Variable Projection", E. Newman and K. Keegan, *SIAM Journal on Matrix Analysis and Applications, 2025*

Papers on Functional Decompositions

- "Guaranteed Functional Tensor Singular Value Decomposition", R. Han, P. Shi, A. Zhang, *Journal of the American Statistical Association, 2024*
- "Functional Tensor Regression", T. Li, F. Yao, A. Zhang, *arXiv, 2025*
- "Functional Tensor Decompositions for Physics-Informed Neural Networks", S. Vemuri, T. Büchner, J. Niebling, J. Denzler, *ICPR, 2024*
- "Functional Tensor Singular Value Decomposition", C. Wang, X.-L. Zhao, Y.-B. Zheng, B.-Z. Li, and M. Ng, *SIAM Journal on Scientific Computing, 2025*

Miscellaneous Papers

- "Moment Varieties for Mixtures of Products", Y. Alexandr, J. Kileel, B. Sturmfels, *ISSAC, 2023*
- "The Spacey Random Walk: A Stochastic Process for Higher-Order Data", A. Benson, D. Gleich, L.-H. Lim, *SIAM Review*, 2017
- "The Fascinating World of $2\times2\times2$ Tensors: Its Geometry and Optimization Challenges", G. Brown, J. Kileel, T. Kolda, *arXiv, 2025*
- "Second-Order Methods for Quartically-Regularised Cubic Polynomials, with Applications to High-Order Tensor Methods", C. Cartis, W. Zhu, *Mathematical Programming*, 2025
- "Decompositions of a Higher-Order Tensor in Block Terms—Part II: Definitions and Uniqueness", L. De Lathauwer, *SIAM Journal on Matrix Analysis and Applications, 2008*
- "Discovering Faster Matrix Multiplication Algorithms with Reinforcement Learning", A. Fawzi et al., *Nature, 2022*
- "Nuclear Norm of Higher-Order Tensors", S. Friedland, L.-H. Lim, *Mathematics of Computation, 2016*
- "A Random Matrix Perspective on Random Tensors", J. H. M. Goulart, R. Couillet, P. Comon, *Journal of Machine Learning Research, 2022*
- "Most Tensor Problems are NP-Hard", C. Hillar, L.-H. Lim, *Journal of the ACM*, 2013
- "Faster Johnson-Lindenstrauss Transforms via Kronecker Products", R. Jin, T. Kolda, R. Ward, *Information and Inference: A Journal of the IMA, 2020*
- "Tensor Decompositions and Applications", T. Kolda, B. Bader, *SIAM Review, 2009*
- "An Adaptive Shifted Power Method for Computing Generalized Tensor Eigenpair", T. Kolda, J. Mayo, *SIAM Journal on Matrix Analysis and Applications, 2014*
- "Tensor Moments of Gaussian Mixture Models: Theory and Applications", J. Pereira, J. Kileel, T. Kolda, *arXiv, 2022*
- "Uniqueness of Nonnegative Tensor Approximations", Y. Qi, P. Comon, L.-H. Lim, *IEEE Transactions on Information Theory, 2016*
- "Tensor Decomposition for Signal Processing and Machine Learning", N. Sidiropoulos et al., *IEEE Transactions on Signal Processing, 2017*