Judiciuos Guessing

1. Solving ODE's of the form
$$ay'' + by' + cy = p(t)$$
where $a, b, c \in \mathbb{R}$ and $p(t)$ is a polynomial of degree n.
   (a) If $c \neq 0$, initial guess is
   $$\psi(t) = A_n t^n + A_{n-1}t^{n-1} + \cdots + A_1 t + A_0$$
   (b) If $c = 0$ and $b \neq 0$, initial guess is
   $$\psi(t) = \left(A_n t^n + A_{n-1}t^{n-1} + \cdots + A_1 t + A_0\right) t$$
   (c) If $c = b = 0$, initial guess is
   $$\psi(t) = \left(A_n t^n + A_{n-1}t^{n-1} + \cdots + A_1 t + A_0\right) t^2$$
2. Solving ODE's of the form
$$ay'' + by' + cy = p(t)e^{\alpha t}$$
where $a, b, c, \alpha \in \mathbb{R}$ and $p(t)$ is a polynomial of degree n.
Recall that $ar^2 + br + c$ is the characteristic polynomial of the equation.
   (a) If α is *not* a root of the characteristic polynomial, initial guess is
   $$\psi(t) = \left(A_n t^n + A_{n-1}t^{n-1} + \cdots + A_1 t + A_0\right) e^{\alpha t}$$
   (b) If α *is* a root of the characteristic polynomial with multiplicity $m_\alpha = 1$, initial guess is
   $$\psi(t) = \left(A_n t^n + A_{n-1}t^{n-1} + \cdots + A_1 t + A_0\right) te^{\alpha t}$$
   (c) If α is a root of the characteristic polynomial with multiplicity $m_\alpha = 2$, initial guess is
   $$\psi(t) = \left(A_n t^n + A_{n-1}t^{n-1} + \cdots + A_1 t + A_0\right) t^2 e^{\alpha t}$$
3. Solving ODE's of the form
$$ay'' + by' + cy = p(t)e^{\alpha t}\cos\beta t$$
or
$$ay'' + by' + cy = p(t)e^{\alpha t}\sin\beta t$$
where $a, b, c, \alpha, \beta \in \mathbb{R}$ and $p(t)$ is a polynomial of degree n.
Recall that $ar^2 + br + c$ is the characteristic polynomial of the equation.
Note that
$$p(t)e^{(\alpha+i\beta)t} = \left(p(t)e^{\alpha t}\cos\beta t\right) + i\left(p(t)e^{\alpha t}\sin\beta t\right)$$
so we will solve
$$ay'' + by' + cy = p(t)e^{(\alpha+i\beta)t}$$
as in case 2.
   (a) If $\alpha + i\beta$ is *not* a root of the characteristic polynomial, initial guess is
   $$\Psi(t) = \left(A_n t^n + A_{n-1}t^{n-1} + \cdots + A_1 t + A_0\right) e^{(\alpha+i\beta)t}$$
   (b) If $\alpha + i\beta$ *is* a root of the characteristic polynomial, initial guess is
   $$\Psi(t) = \left(A_n t^n + A_{n-1}t^{n-1} + \cdots + A_1 t + A_0\right) te^{(\alpha+i\beta)t}$$

After solving, $g(t) = p(t)e^{\alpha t}\cos\beta t \Rightarrow \psi(t) = \mathrm{Re}(\Psi(t))$ and $g(t) = p(t)e^{\alpha t}\sin\beta t \Rightarrow \psi(t) = \mathrm{Im}(\Psi(t))$.