M328K Homework Assignments

Remember that proofs should be written in complete sentences using correct punctuation and grammar.

1.1 #2, 4, 6, 8, 16
Hints:

- 1.1 #4 - Since the statement "the number α is irrational" is a negative existential statement ("irrational" means "not rational"), trying to prove the statement by contradiction is a reasonable approach.

1.3 #2, 10, 22

1.4 #8

1.5 #14 (prove your conclusion), 16 (either prove that there are no such integers, or provide an example of integers a, b, and c with the indicated property), 23, 29, 30, 36
Hints and Remarks:

- 1.5 #23 - If you can show that $-a = bc + s$ for integers c and s with $0 \leq s < b$, then the uniqueness assertion in the division algorithm allows you to conclude that c is the quotient and s the remainder when $-a$ is divided by b.

- 1.5 #29 - What does a general positive integer divisible by d look like (according to the definition of divisibility)? You will want to use that $[x/d]$ is the largest integer less than or equal to x/d.

- 1.5 #30 - You can use a calculator to compute values of the floor function (i.e. you do not need to show long division or anything). Note that the question intends to ask about integers divisible by 5, then about integers divisible by 25, etc., and not about integers divisible by all four of the given powers of 5 (indeed that would be equivalent to just asking about integers divisible by 625).

3.1 #7, 9, 12, 14, 15, 23
Hints and Remarks:

- 3.1 #14, 15 - The book is indecisive about including the $n = 0$ term of the arithmetic progression given by the formula $an + b$ (i.e. the term b). In Chapter 1, the book does include b, but in Chapter 3, it does not. This makes no difference for the statement of Dirichlet's theorem on primes in arithmetic progressions (Theorem 3.3), because removing a single term does not affect the infinitude of primes in the progression. Whether you include the term b or not also does not affect the solutions to these two problems.

3.3 #6, 8, 9, 16, 24, 30
Hints and Remarks:

- 3.3 #6 - Although the problem just asks for the greatest common divisor, you should prove your answer.

3.4 #2a-c, 4
Hints and Remarks:

- 3.4 #2a-c, 4 - You should show the steps in your computations for these two problems. Also, for 4, you only have to address the pairs in 2a-c.

3.5 #6, 8, 12, 28a-c, 38, 43, 45, 62, 66
Hints and Remarks:

- 3.5 #38 - I think the book intends to suggest using Lemma 3.5 instead of Lemma 3.4. Note that the assumption that the integers in Lemma 3.4 and Lemma 3.5 are positive is superfluous. The results are both true, with the same proof, if the integers are only assumed to be nonzero.
- 3.5 #45 - You should assume that b is not a power of p at all for this problem. Argue by contradiction and use unique factorization.
- 3.5 #62 - Theorem 3.18 can help.

4.1 #5, 10, 20, 27, 30, 33, 34

4.2 #2a-c, 8, 15, 16, 18

4.3 #15, 30

4.4 #1a-b

6.1 #18, 19, 24, 29, 42

6.3 #3, 10

7.1 #8, 14, 20, 22, 27

7.2 #40

9.1 #1a-c, 12, 16, 17, 19, 21 (here a should be greater than 1)

9.2 #9, 16

14.1 #4a-b, 11, 12, 28