0.1 Basic facts on dynamics on the Cantor space

We collect basic facts and definitions on dynamics on the Cantor space and several lemmas on minimal homeomorphisms to which we will refer several times in the later sections.

The *support* of a homeomorphism T of the Cantor space $\mathbf{C}$ is defined by

$$supp(T) = \overline{\{x \in \mathbf{C} : T(x) \neq x\}}.$$

Generally the support of a homeomorphism does not need to be open set. However for any minimal homeomorphism T, the support of each element of the full topological group $[[T]]$ is a clopen set. This follows immediately from the definition of $[[T]]$.

A homomorphism $T \in Homeo(\mathbf{C})$ is called *periodic*, if every orbit of T is finite, and *aperiodic* if every orbit is infinite. It has *period* n if every orbit has exactly n elements.

Lemma 0.1.1. Let $T \in Homeo(\mathbf{C})$ be a periodic homeomorphism of period n. Then there exists a clopen set $A \subset \mathbf{C}$ such that

$$\mathbf{C} = \bigsqcup_{i=0}^{n-1} T^i(A)$$

Proof. For any $x \in \mathbf{C}$ let $U_x \subset \mathbf{C}$ be a clopen neighborhood such that $T^i(U_x) \cap U_x = \emptyset$ for all $1 \leq i \leq n$, where n is the period of T. Since $\mathbf{C}$ is compact there are $x_1, \ldots, x_k$ such that $\mathbf{C} = \bigcup_{1 \leq i \leq k} U_{x_i}$. Let $A_1 = U_{x_1}$ and

$$A_{j+1} = A_j \bigcup \left(U_{x_{j+1}} \setminus \bigcup_{i=1}^{n-1} T^i(A_j) \right).$$

It is trivial to check that the statement holds for A_k. □

Lemma 0.1.2. Let $T \in Homeo(\mathbf{C})$ be a minimal homeomorphism. Then for any $g \in [[T]]$ and $n \in \mathbb{N}$ the set

$$\mathcal{O}_n = \{x \in \mathbf{C} : |Orb_g(x)| = n\}$$

is clopen.

Proof. Let $\mathbf{C} = \bigcup_{i \in I} A_i$ be a finite clopen partition of $\mathbf{C}$ such that the restriction of g to each piece of the partition coincides with some power of T.

Denote by $\{B_j\}_{j\in J}$ the refinement of $\{A_i\}_{i\in I}$ and the sets $\{T^{-k}(A_i)_{i\in I}\}$, $1 \leq k \leq n$. Thus

$$g|_{B_j} = T^{m_j}|_{B_j},$$

for some $\{m_j\}_{j\in J}$.

Let $x \in \mathcal{O}_n$, we will show that there is a neighborhood of x inside $\mathcal{O}_n$. Let $j_0, \ldots, j_n$ be such that $T^k x \in B_{j_k}$ for all $0 \leq k \leq n$. Then we have $g^n(x) = x$ for all $x \in \mathcal{O}_n$ and therefore

$$T^s x = x, \text{ where } s = \sum_{0\leq k\leq n} m_{j_k}.$$

But this can happen only if $s = 0$, hence $B_{j_0} \subseteq \mathcal{O}_n$. This implies that $\mathcal{O}_n$ is open.

Moreover, we have the decomposition

$$\mathcal{O}_n = \{x \in \mathbf{C} : g^n(x) = x\} \setminus \bigcup_{m<n} \{x \in \mathbf{C} : g^m(x) = x\},$$

which implies that $\mathcal{O}_n$ is closed. □

Invariant Borel measures on the Cantor set. The set of Borel measures on a compact space X is separable, compact in the weak*-topology coming from the dual of the space of all continuous functions on X, $C(X)$.

Let T be a homeomorphism of X, denote by $\mathcal{M}(T)$ the space of all T-invariant Borel probability measures on X. The classical Krylov-Bogolyubov theorem, [16] states that $\mathcal{M}(T)$ is non-empty. For example, for a fixed point $x \in X$ it contains a cluster point μ of the following sequence

$$\mu_n = \frac{1}{n}\sum_{i=1}^{n} T^i \circ \delta_x,$$

where δ_x is a Dirac measure concentrated on a point $x \in X$. To verify that μ is T-invariant, let $f \in C(X)$, then

$$\int f d\mu_n = \frac{1}{n}\sum_{i=1}^{n} f(T^i(x))$$

and

$$\int f d(T \circ \mu_n) = \frac{1}{n} \sum_{i=1}^{n} f(T^{i+1}(x)).$$

Then

$$\left| \int f d\mu_n - \int f d(T \circ \mu_n) \right| \leq 2/n \|f\|,$$

which implies the claim.

The following lemma will be useful in the process of constructing new elements in the full topological group.

Lemma 0.1.3. Let $A \subset \mathbf{C}$ be a clopen set and $T \in Homeo(\mathbf{C})$ be a minimal homeomorphism. Then for every $\varepsilon > 0$ there exists a partition of A into clopen sets

$$A = \bigsqcup_{i \in I} A_i$$

such that for every $\mu \in \mathcal{M}(T)$ and $i \in I$ we have $\mu(A_i) < \varepsilon$.

Proof. It is sufficient to show the statement for $A = \mathbf{C}$. Let $\varepsilon = 1/n$ and choose a clopen set $D \subset \mathbf{C}$ such that the sets $T^k(D)$, $1 \leq k \leq n$, are pairwise disjoint. Since μ is T-invariant, we have $\mu(D) \leq 1/n$. Now the Kakutani-Rokhlin partition of $\mathbf{C}$ over D satisfies the statement. □

The commutator subgroup of the group Γ is the group generated by all elements of the form $[g, h] = ghg^{-1}h^{-1}$ and denoted by Γ'. In this section we will show simplicity of $[[T]]'$ for a Cantor minimal system $(T, \mathbf{C})$. This is a result of Matui, [69]. We will follow a simplified proof of Bezuglyi and Medynets, [13].

We start with the following theorem of Glasner and Wiess, [38], which will be crucial in the proof.

Theorem 0.1.4 (Glasner-Weiss)**.** Let T be a minimal homeomorphism of a Cantor set $\mathbf{C}$ and let $A, B \subseteq \mathbf{C}$ be clopen subsets such that for every $\mu \in \mathcal{M}(T)$, we have $\mu(B) < \mu(A)$. Then there exists $g \in [[T]]$ with $g(B) \subset A$ and $g^2 = id$.

Proof. Let $f = \chi_A - \chi_B$, then f is continuous and by the assumptions $\int f d\mu > 0$ for all $\mu \in \mathcal{M}(T)$. There exist a constant $c > 0$ such that

$$\inf(\int f d\mu : \mu \in \mathcal{M}(T)) > c.$$

Indeed, assume that this is not the case and the infimum reaches 0 on some sequence of measures in $\mathcal{M}(T)$. If μ is a cluster point of this sequence in the weak*-topology, we obtain $\mu(A) = \mu(B)$, which is a contradiction.

Let us show now that there exists n_0, such that for all $x \in \mathbf{C}$ and all $n \geq n_0$ we have

$$\frac{1}{n}\sum_{i=0}^{n-1} f(T^i x) \geq c. \tag{1}$$

To reach a contradiction assume that there exists a increasing sequence $\{n_k\}$ of natural numbers and a sequence of points $\{x_k\}$ for which

$$-1 \leq \frac{1}{n_k}\sum_{i=0}^{n_k-1} f(T^i x_k) \leq c.$$

As in the proof of Krylov-Bogoliubov theorem, we set

$$\mu_k = \frac{1}{n_k}\sum_{i=0}^{n_k-1} T^i \circ \delta_{x_k}.$$

Let μ be a cluster point of μ_k in the weak*-topology. Then μ is in $\mathcal{M}(T)$, on the other hand we have

$$\int f d\mu \leq c,$$

which is a contradiction.

Let n_0 be such that (1) holds for all $n \geq n_0$ and all $x \in \mathbf{C}$. Choose $D \subset \mathbf{C}$ be such that $T^i(D) \cap D = \emptyset$ for all $i \leq n_0$. This implies that the hight of each tower over D is greater then n_0. Let $D_1, \dots, D_N$ be a refinement of Kakutani-Rokhlin partition of D:

$$\begin{aligned}\mathbf{C} =&(D_1 \sqcup T(D_1) \sqcup \dots \sqcup T^{k_1}(D_1))\sqcup \\ &\sqcup (D_2 \sqcup T(D_2) \sqcup \dots \sqcup T^{k_2}(D_2))\sqcup \\ &\dots \sqcup (D_N \sqcup T(D_N) \sqcup \dots \sqcup T^{k_N}(D_N)),\end{aligned}$$

with property that each piece of the partition is contained in one of the sets $A\backslash B, B\backslash A, A \cap B$ or $(A \cup B)^c$. By assumptions, $k_i \geq n_0$ for all $1 \leq i \leq N$. Now taking any x in D the inequality (1) implies that for a fixed j the number of the components $T^i(D_j)$, $1 \leq i \leq k_j$ that belong to A is greater then the number of components that belong to B. We will define g by mapping between the pieces of the partition. Define g on D_j as a symmetry that maps $T^i(D_j)$ that belong to B onto a component that belong to A by applying a power of T. Since Kakutani-Rokhlin partition is clopen, we can define g as a trivial map on the rest of the sets. Obviously, g is in $[[T]]$ and $g^2 = id$. □

Below we give a sequence of lemmas due to Bezuglyi and Medynets, from which we deduce the main result of this section: the simplicity of the full topological group of a Cantor minimal system.

Lemma 0.1.5. Let T be a minimal homeomorphism of the Cantor set. For any $g \in [[T]]$ and $\delta > 0$ there exists a decomposition $g = g_1 g_2 \dots g_n$ such that $\mu(\operatorname{supp}(g_i)) \leq \delta$ for all $\mu \in \mathcal{M}(T)$.

Proof. Assume firstly that $g \in [[T]]$ is periodic. Since $g \in [[T]]$ then by Lemma 0.1.1 and Lemma 0.1.2 we can find clopen sets A_k, $k \in I$, such that $g|_{A_k}$ has order k and

$$\mathbf{C} = \bigsqcup_{k \in I} \bigsqcup_{i=0}^{k-1} g^i(A_k).$$

By Lemma 0.1.3 we can partite A_k into clopen sets

$$A_k = \bigsqcup_{j=1}^{n_k} B_j^{(k)}$$

such that $\mu(B_j^{(k)}) < \delta/k$ for all $B_j^{(k)}$ and $\mu \in \mathcal{M}(T)$.

Now set

$$C_{k,j} = \bigsqcup_{i=0}^{k-1} g^i(B_j^{(k)}).$$

Define $g_{k,j}$ to be g on $C_{k,j}$ and identity on its complement. Since all sets are clopen $g_{k,j}$ is continuous and $g_{k,j} \in [[T]]$. Obviously, $g = \prod_{k,j} g_{k,j}$ and $\mu(supp(g_{k,j})) < \delta$.

Assume now that g is non-periodic. Let $k \in \mathbb{N}$ be such that $1/k < \delta$ and define

$$\mathcal{O}_{\geq k} = \{x \in \mathbf{C} : Orb_g(x) \text{ has at least } k \text{ elements}\}$$

By Lemma 0.1.3 we have that the complement of $\mathcal{O}_{\geq k}$ is clopen and thus $\mathcal{O}_{\geq k}$ is clopen. Therefore, for any $x \in \mathcal{O}_{\geq k}$ there exists a clopen neighborhood U_x such that $g^i(U_x) \cap U_x = \emptyset$ for all $1 \leq i < k$. By compactness there are $x_1, \ldots, x_n \in \mathcal{O}_{\geq k}$ such that $\mathcal{O}_{\geq k} = \bigcup_{1 \leq i \leq n} U_{x_i}$. Define $B_1 = U_{x_1}$ and

$$B_{i+1} = B_i \bigsqcup \left(U_{x_{i+1}} \setminus \bigcup_{l=-k+1}^{k+1} g^l(B_i) \right).$$

Then $B = B_n$ meets every orbit of g in $\mathcal{O}_{\geq k}$. Moreover, $g^i(B) \cap B = \emptyset$ for all $1 \leq i < k$, which implies $\mu(B) \leq 1/k < \delta$ for all $\mu \in \mathcal{M}(T)$. Since the transformation T is minimal we have that the function

$$F : x \mapsto \min\{l \geq 1 : g^l(x) \in B\}$$

is continuous. Define

$$g_B(x) = \begin{cases} g^k(x), & \text{if } x \in B \text{ and } k = F(x), \\ x, & x \notin B. \end{cases}$$

It is easy to see that $g_B \in [[T]]$, $\mu(supp(g_B)) < \delta$ and $g_B^{-1} \circ g$ is periodic. Thus the statement of the lemma follows from the previous case. $\square$

Lemma 0.1.6. Let $T \in Homeo(\mathbf{C})$ be a minimal homeomorphism. Then for any $f \in [[T]]'$ and $\delta > 0$ there exists $g_1, \ldots, g_n, h_1, \ldots, h_n \in [[T]]$ such that $f = [g_1, h_1] \ldots [g_n, h_n]$ and $\mu(supp(g_i) \cup supp(h_i)) < \delta$ for all $\mu \in \mathcal{M}(T)$.

Proof. Let $f = [g, h]$ for some $g, h \in [[T]]$. By Lemma 0.1.5 we can find $g_1, \ldots, g_n$ and $h_1, \ldots, h_n$ in $[[T]]$ such that $g = g_1 \ldots g_n$, $h = h_1 \ldots h_n$ with $\mu(supp(g_i)) < \delta/2$ and $\mu(supp(h_i)) < \delta/2$ for all $\mu \in \mathcal{M}(T)$. Since f is in the group generated by $[g_i, h_j]$, $1 \leq i, j \leq n$, we obtain the statement. $\square$

The following is a generalization of Glasner-Weiss to the commutator subgroup.

Lemma 0.1.7. Let $T \in Homeo(\mathbf{C})$ be a minimal homeomorphism. If A and B are clopen subsets of $\mathbf{C}$ such that $3\mu(B) < \mu(A)$ for all $\mu \in \mathcal{M}(T)$, then there exists $f \in [[T]]'$ such that $f(B) \subset A$.

Proof. By replacing A by $A\backslash B$ we have $2\mu(B) < \mu(A)$ for all $\mu \in \mathcal{M}(T)$ and $A \cap B = \emptyset$. Now by Theorem 0.1.4 we can find a symmetry $g \in [[T]]$ such that $g(B) \subset A$. Then

$$\mu(g(B)) = \mu(B) < \mu(A) - \mu(B) = \mu(A\backslash g(B))$$

thus again by Theorem 0.1.4 we can find a symmetry $h \in [[T]]$ such that $h(g(B)) \subset A\backslash g(B)$. It is trivial to check, using the properties of g and h, that $g = (hg)h^{-1}(hg)^{-1}$, we obtain $hg = [h, hg]$ and $hg(B) \subseteq A$, which implies the statement. □

Theorem 0.1.8. Let T be a minimal homeomorphism and let Γ be either $[[T]]$ or $[[T]]'$. Then for every normal subgroup H of Γ, we have $\Gamma' \leq H$.

Proof. We will show that for all elements g, h in Γ their commutator $[g, h]$ is in H. Let $f \in H$. Let E be a clopen non-empty set such that such that $f(E) \cap E = \emptyset$. By the compactness of $\mathcal{M}(T)$ we have

$$3\delta = \inf(\mu(E) : \mu \in \mathcal{M}(T)) > 0$$

By Lemma 0.1.5 and Lemma 0.1.6 we can find $g_i, h_j \in \Gamma$ such that $g = g_1 \dots g_n$ and $h = h_1 \dots h_n$ and

$$\mu(supp(g_i)) < \delta/2, \quad \mu(supp(h_i)) < \delta/2$$

for all $\mu \in \mathcal{M}(T)$. We claim that for all g and h in Γ with $\mu(supp(g) \cup supp(h)) < \delta$ we have $[g, h]$ are in H. Since the commutator $[g_1 \dots g_n, h_1 \dots h_n]$ belongs to the group generated by $[g_i, h_j]$, the claim implies the statement.

To prove the claim put $F = supp(g) \cup supp(h)$, then $3\mu(F) < \mu(E)$. Thus we can apply Lemma 0.1.7 to find an element α in $[[T]]'$ such that $\alpha(F) \subseteq E$. Since H is normal, we have

$$q = \alpha^{-1} f \alpha \in H.$$

Thus

$$\overline{h} = [h, q] = (h\alpha^{-1} f \alpha h^{-1})\alpha^{-1} f^{-1} \alpha$$

and $[g, \overline{h}]$ are in H.

Since $q(F) \cap F = \emptyset$, the elements g^{-1} and $qh^{-1}q^{-1}$ commute. Hence, we have

$$[g, \overline{h}] = g(hqh^{-1}q^{-1})g^{-1}(qhq^{-1}h^{-1}) = [g, h] \in H,$$

which proves the claim. □

Corollary 0.1.9 (Matui, '06)**.** Let $T \in Homeo(\mathbf{C})$ be a minimal, then $[[T]]'$ is simple.

Proof. Since $[[T]]''$ is a normal subgroup of $[[T]]$, we can apply the theorem to obtain that $[[T]]' \leq [[T]]''$. Thus, $[[T]]'' = [[T]]'$. Let now H be a normal subgroup of $[[T]]'$. Then $[[T]]'' \leq H$, therefore $[[T]]' = H$. $\square$

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