

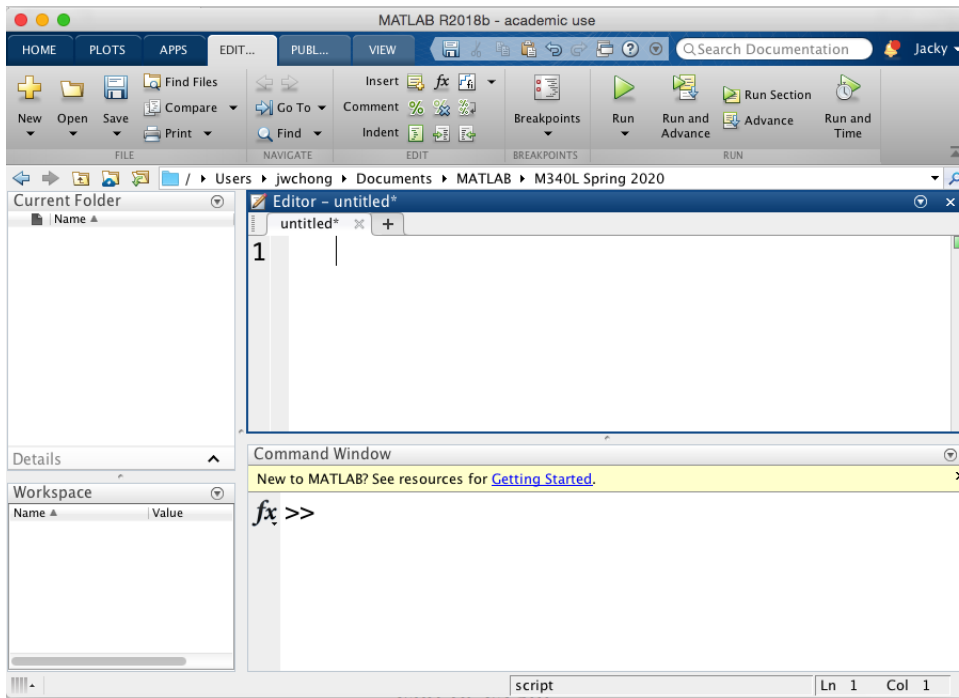
A Guide¹ to MATLAB for M340L Spring 2020 - Part 1

1. Method

The way this guide is written is that it is assumed that you will sit down at a MATLAB terminal and start. The commands that you give to MATLAB are given but the output is not. The output is, however, talked about, with the understanding that you will put in the commands and see the output. Then you can read about it and keep going.

2. Starting MATLAB

Find a system on campus or on your personal computer and run MATLAB. Once you start running MATLAB on your personal computer, you will first see that MATLAB logo while the system is loading. Immediately after (30 seconds), you will see a window with multiple sub-windows



Before you begin, make sure you are working in a designed file that has all your MATLAB files.

Let us first focus on the Command Window and the Workspace. In the Command Window, there is the symbol `>>`. This is the prompt, it's where we tell MATLAB what we want it to do.

3. Basic Stuff

By default MATLAB returns answers in decimals. We will prefer fractions for this project. To switch, type in the following and hit "return". This sets the output format to rationals; that is, fractions.

```
>> format rat
```

To return to decimal output, you could simply change the format to short by entering

```
>> format short
```

¹This is an updated version of Justin Olav Wyss-Gallifent's MATLAB guide based on an elementary linear algebra course taught at the University of Maryland, College Park.

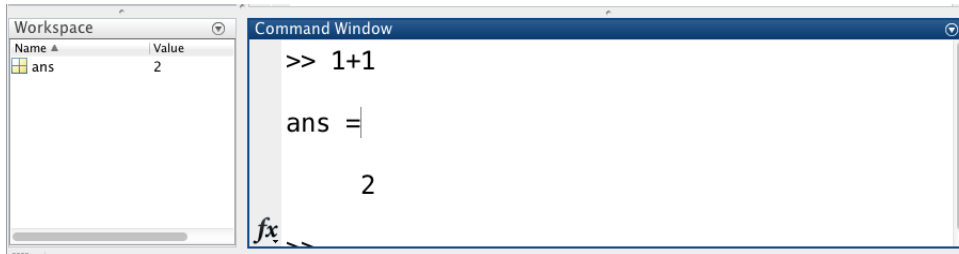
To see more digits in the decimal expansion, you could change the format to long

```
>> format long
```

4. Simple Calculation

For instance, you could use MATLAB to perform simple arithmetic such as

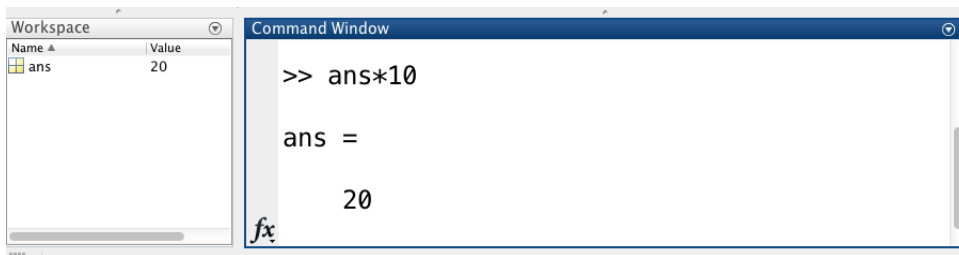
```
>> 1+1
```

 then press < Enter >. MATLAB's response will follow

Note that in the Workspace, MATLAB has stored your answer in the variable `ans`. Hence if you type

```
>> ans*10
```

MATLAB will return 20 and update `ans` and you will see



To clear the command window, type

```
>> clc
```

5. Variables Naming

Of course you could also create your own variables like `x`, `y`, `h1`, `z43`, `something_maybe`. A valid variable name starts with a letter, followed by letters, digits, or underscores. MATLAB is case sensitive, so `A` and `a` are not the same variable. The maximum length of a variable name is the value that the `namelengthmax` command returns (which seems to be 63).

For example, if we enter

```
>> x = 1+1
```

```
>> X = 2+4
```

you will see variable `x` and `X` are initiated in the Workspace



To delete a variable, you could simply select the variable and right click to choose the **Delete** option (or just select and press delete). To clear your workspace, type
 >> `clear all`

6. Solving a System

Now let's do something having to do with MATH 240.

First we'll solve a system of linear equations. We do this using MATLAB's `solve` command. Due to the quirks with which this command operates we have to make sure we interpret the result correctly. For example, suppose we wish to solve the system

$$\begin{aligned} 3x_1 + x_2 - 5x_3 &= 0 \\ x_1 - 5x_2 + x_3 &= 2 \\ x_1 + x_2 - 5x_3 &= -1 \end{aligned}$$

what we type in is the following.

```
>> [x1 x2 x3]=solve('3*x1+x2-5*x3=0','x1-5*x2+x3=2','x1+x2-5*x3=-1')
```

Note the apostrophes (single quotes), necessary because an equation must be treated as a string of characters and strings of characters in MATLAB are delimited by apostrophes.

Note here that we got three solutions. What would happen if we fed MATLAB a system with not enough equations? Let's try!

```
>> [x1 x2 x3]=solve('3*x1+x2-5*x3=0','x1-5*x2+x3=2')
```

If you try this, MATLAB chokes. The reason is that it doesn't know what to do or how to give the solution. There are ways for MATLAB to solve this but they demand that we know more than we do, too. For now we'll need to deal with the corresponding augmented matrices in order to get MATLAB to help us solve this system.

7. Putting in a Matrix

We can put in a matrix easily. To put in the matrix

$$\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$$

```
>> A=[2 3 ; 5 -1]
```

8. Matrix Operations

Suppose now you put in a matrix and you wish to manually do row operations to it. MATLAB doesn't have any functions for doing row operations, instead we have to execute low-level commands to do so. This is easier than it seems.

Here is an augmented matrix

```
>> A=[1 -1 -3 7 ; -3 1 4 -16 ; 4 -3 -5 12]
```

To get this to row-echelon form, first we'll clean out the entries below the upper-left 1. To do this we need to add 3 times row 1 to row 2. In MATLAB the code is

```
>> A(2,:)=A(2,:)+3*A(1,:)
```

Confused? I was! The expression $A(\text{rownumber}, :)$ refers to the entire row, so basically this line is saying:

$$\underbrace{A(2, :)}_{\text{Row2}} = \underbrace{A(2, :)}_{\text{Row2}} + \underbrace{3 * A(1, :)}_{3 * \text{Row1}}$$

Try it! The result will be given.

Next we'll add -4 times row 1 to row 3.

```
>> A(3,:)=A(3,:)+(-4)*A(1,:)
```

Now you'll see that we have 0s below the upper-left 1. Next we need to get a 0 in the bottom row where the 1 is. The easiest way to do this is first to interchange rows 2 and 3.

```
>> A([2 3], :)=A([3 2], :)
```

This is a confusing command but it does the job.

Now we add twice row 2 to row 3.

```
>> A(3,:)=A(3,:)+2*A(2,:)
```

The result you'll see is the matrix in row-echelon form. Remember this means any rows of 0s are at the bottom (none in this case) and the leading entries go down and right.

To get to *reduced* row-echelon form, we first have to get 1s for all our leading entries. The third row is the only problem, it needs to be multiplied by $1/9$.

```
>> A(3,:)=1/9*A(3,:)
```

This means that row 3 is $1/9$ times what it was.

Lastly we must clear out the entries above the leading 1s.

First above the rightmost leading 1.

```
>> A(1,:)=A(1,:)+3*A(3,:)
```

```
>> A(2,:)=A(2,:)+(-7)*A(3,:)
```

And then above the leading 1 to the left of that one.

```
>> A(1,:)=A(1,:)+1*A(2,:)
```

The result is now in reduced row-echelon form. Since there are no free variables we can simply read off the solution: $x_1 = 3$, $x_2 = 5$ and $x_3 = -3$.

9. Skipping to the Reduced Row-Echelon Form

Suppose now you want to solve a system of matrices by getting the augmented matrix in reduced row-echelon form but you don't want to do all that work on the previous page. The `rref` command does this in MATLAB. For example if I put in the augmented matrix (corresponding to a system of linear equations) for the previous page's problem:

```
>> A=[1 -1 -3 7 ; -3 1 4 -16 ; 4 -3 -5 12]
```

and then reduce it

```
>> rref(A)
```

we see that MATLAB saves us a lot of time compared to all the work we needed to do on the previous page.

Here is another, this one has free variables at the end. We will see that after `rref` because there are non-pivot columns corresponding to variables.

```
>> A=[1 3 4 2;1 -3 0 1]
```

and then reduce it

```
>> rref(A)
```

we get the reduced form.

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3/2 \\ 0 & 1 & 2/3 & 1/6 \end{array} \right]$$

Note that the third column is not a pivot column so x_3 is free. Hence we have the solution

$$\begin{aligned} x_1 &= \frac{3}{2} - 2x_3 \\ x_2 &= \frac{1}{6} - \frac{2}{3}x_3 \\ x_3 &= \text{free} \end{aligned}$$

10. Lastly, note that we can deal with matrices with unknown constants in them but we have to explicitly tell MATLAB that they are unknown constants. For example, suppose we wish to enter the matrix

$$\left[\begin{array}{cc|c} -2 & 3 & h \\ 5 & -1 & k \end{array} \right]$$

First we must tell MATLAB that h and k are to be dealt with symbolically. We do this with the `syms` command

```
>> syms h
```

```
>> syms k
```

And now we can do

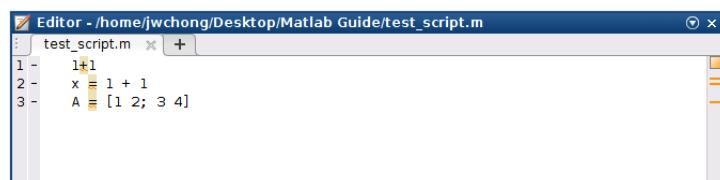
```
>> A=[-2 3 h ; 5 -1 k]
```

```
>> rref(A)
```

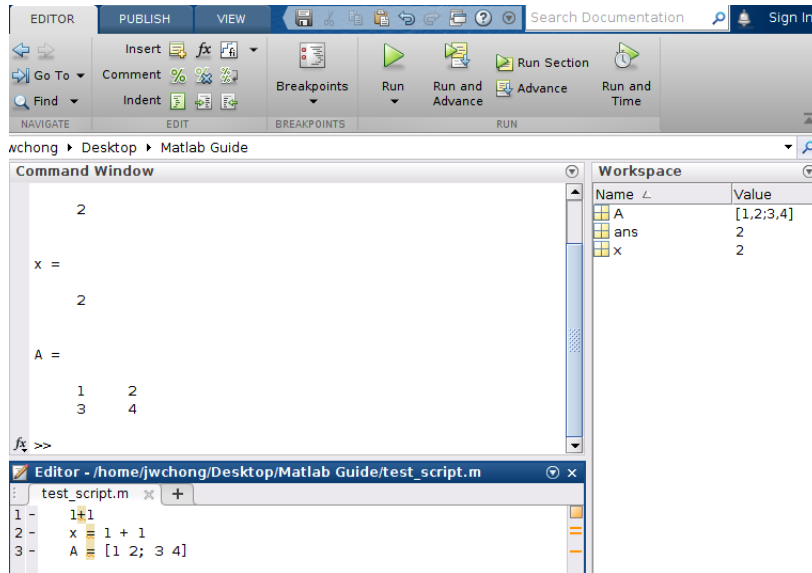
11. m-file/Publishing

Despite we having been working with MATLAB through the command window, it is actually more convenient to write an m-file, or script file, when there are a lot of calculations that need to be done. An m-file is a simple text file where you write your MATLAB commands. When the file is run, MATLAB reads the commands and executes them in the order as written as if you had typed each command line-by-line at the MATLAB prompt.

For instance, let us create a test m-file, call it `test_script.m`, in the editor window as follows

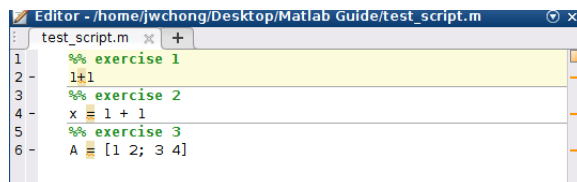


To run the above script, go to the EDITOR tab and click Run



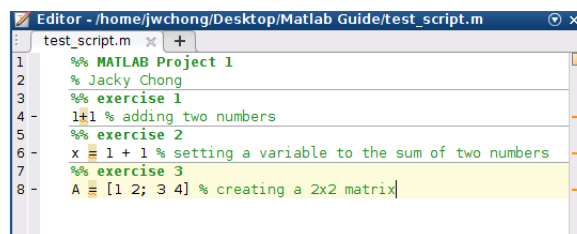
You can see your script has been executed in the command window.

When working with multiple problems, it is also a good practice to divide your script into different sections by using the double percentage symbol %% as follows

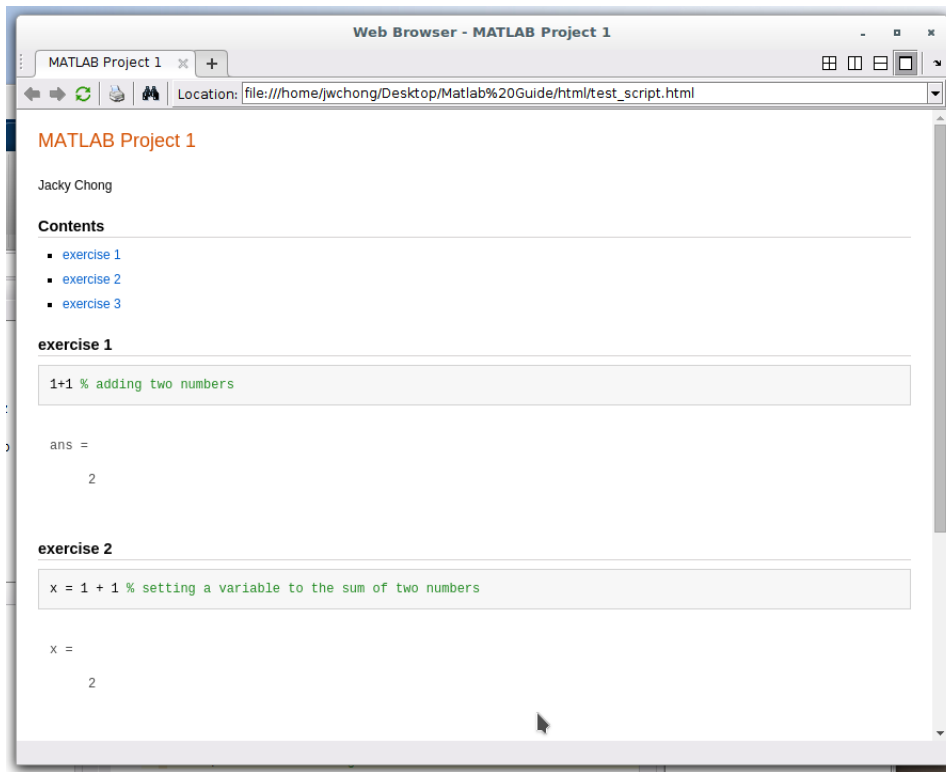


One immediate benefit of breaking your script into different sections is that you could now execute each section separately. Select the section you want execute then click the Run Section button in the EDITOR tab.

To submit your MATLAB project, you must publish your m-file to get an html file which you will print and submit to me. For instance, suppose you would like to publish the script



Go to the PUBLISH tab and click Publish. Then MATLAB will generate the following html file



Important: Please comment your code!

Directions: The following may be done in a single MATLAB session or in several. Please print out the session (the entire contents of the command window). Any question marked with a star \star indicates that that answer should be hand-written onto the printout since it is not explicitly a MATLAB result.

Important: Before each question, type `clear` to clear out the memory. At the start of the entire assignment (or each session) type `format rat`.

1. Consider the system of equations

$$\begin{aligned}x_1 + 2x_2 - x_3 &= -1 \\x_1 + 3x_2 + 2x_3 &= 17 \\2x_1 + 3x_2 - 4x_3 &= -15\end{aligned}$$

- Enter it into MATLAB as an augmented matrix named `A`.
- Use elementary row operations (as in part 7 of the guide) to reduce it to row-echelon form.
- Continue to reduced row-echelon form.
- \star Give the solution of the system.

2. Consider the system of equations

$$\begin{aligned}3x_2 - 6x_3 + 6x_4 + 4x_5 &= -5 \\3x_1 - 7x_2 + 8x_3 - 5x_4 + 8x_5 &= 9 \\3x_1 - 9x_2 + 12x_3 - 9x_4 + 6x_5 &= 15\end{aligned}$$

- Enter it into MATLAB as an augmented matrix named `B`.
- Use elementary row operations (as in part 7 of the guide) to reduce it to row-echelon form.
- Continue to reduced row-echelon form.
- \star Give the solution of the system.

3. Enter the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 & 1 \\ 2 & 3 & 1 & 3 & 2 \\ -4 & 2 & 4 & 1 & 2 \end{bmatrix}$$

- Convert it to reduced row-echelon form using `rref`.
- \star Write the solution of the system which corresponds to this matrix (as if it were augmented).

4. Suppose you wish to show that $\begin{bmatrix} a \\ b \end{bmatrix}$ is in the span of $\left\{ \begin{bmatrix} 7 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$ for any a and b .

(a) First declare a and b as symbolic (unknown constants).

(b) Enter the augmented matrix A which can help you show this.

(c) Use `rref` on the matrix A .

(d) ★ Explicitly give the weights w_1 and w_2 such that $\begin{bmatrix} a \\ b \end{bmatrix} = w_1 \begin{bmatrix} 7 \\ -2 \end{bmatrix} + w_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

5. Suppose you wish to show that the set of vectors

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ -7 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -3 \\ 1 \\ -2 \end{bmatrix} \right\}$$

is linearly dependent.

(a) Enter the augmented matrix A corresponding to the homogeneous system which can help you solve this.

(b) Use `rref` on the matrix A .

(c) ★ Give a nontrivial linear combination of the vectors which yields $\bar{\mathbf{0}}$. Hint: Make a free variable nonzero.

6. Suppose you wish to show that

$$\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \text{ is not in the span of } \left\{ \begin{bmatrix} 1 \\ 3 \\ -5 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right\}$$

(a) Use `rref` on an appropriate augmented matrix to show this fact.

(b) ★ Explain why your calculation in (a) is sufficient.

7. Suppose

$$A = \begin{bmatrix} 12 & 10 & -6 & -3 & 7 & 10 \\ -7 & -6 & 4 & 7 & -9 & 5 \\ 9 & 9 & -9 & -5 & 5 & -1 \\ -4 & -3 & 1 & 6 & -8 & 9 \\ 8 & 7 & -5 & -9 & 11 & -8 \end{bmatrix}.$$

(a) Use as many columns of A as possible to construct a matrix B with the property that the equation $B\mathbf{x} = \mathbf{0}$ has only the trivial solution. Use `rref` to solve $B\mathbf{x} = \mathbf{0}$ to verify your claim.

(b) Select a column \mathbf{v} of A that was not used in the construction of B and determine if \mathbf{v} is in the set spanned by the columns of B .

8. Suppose experimental data are represented by a set of points in the plane. An **interpolating polynomial** for the data is a polynomial whose graph passes through every point. In scientific work, such a polynomial can be used, for example, to estimate values between the known data points. Another use is to create curves for graphical images on a computer screen. One method for finding an interpolating polynomial is to solve a system of linear equations.

- (a) Find the interpolating polynomial $p(t) = a_0 + a_1t + a_2t^2$ for the data $(1, 12), (2, 15), (3, 16)$. This is, find a_0, a_1 , and a_2 such that

$$a_0 + a_1(1) + a_2(1)^2 = 12$$

$$a_0 + a_1(2) + a_2(2)^2 = 15$$

$$a_0 + a_1(3) + a_2(3)^2 = 16$$

- (b) In a wind tunnel experiment, the force on a projectile due to air resistance was measured at different velocities:

Velocity (100 ft/sec)	0	2	4	6	8	10
Force (100 lb)	0	2.90	14.8	39.6	74.3	119

Find an interpolating polynomial for these data and estimate the force on the projectile when the projectile is traveling at 750 ft/sec. Use $p(t) = a_0 + a_1t + a_2t^2 + a_3t^3 + a_4t^4 + a_5t^5$.

- (c) In part (b), what happens if you try to use a polynomial of degree less than 5? (Try a cubic polynomial, for instance.)