

A Guide¹ to MATLAB for M340L Spring 2020 - Part 2

1. Method

The way this guide is written is different from the first since it is assumed that you know the basics. New commands are presented on this page and the project starts on the next page.

2. New Commands

- (a) Powers of matrices can be done in the obvious way. If A is a matrix then we can do:

```
>> A^2 % or
>> A^17
```

You cannot however do the inverse of a matrix this way:

```
>> A^-1
```

This will not work.

- (b) Matrix multiplication can be done with:

```
>> A*B
```

This also takes care of matrices times vectors. For example if

```
>> A=[1 2 3;4 5 6;7 8 9]
```

```
>> v=[-2 ; 1 ; 3]
```

then we can do the product Av with

```
>> A*v
```

- (c) The determinant of a matrix is done with the `det` command. This can be done either on a given matrix:

```
>> A=[1 2 3;4 5 6;7 8 9]
```

```
>> det(A)
```

or inline:

```
>> det([1 2 3;4 5 6;7 8 9])
```

- (d) The inverse of a matrix can be done with the `inv` command:

```
>> inv(A)
```

or inline as with `det`.

- (e) To extract a submatrix from a given matrix, it suffices to indicate the rows and columns you want. For instance, suppose you entered the matrix:

```
>> A=[1 2 3 4; 5 6 7 8; 9 10 11 12; 13 14 15 16]
```

If you want to create a B matrix consists of rows 2, 4 and columns 1, 3, 4, then this can be readily done by entering

```
>> B = A([2, 4], [1, 3, 4])
```

If you want to extract a block, say the first 2×2 block or the the 3×3 block, then the more efficient way is to indicate the range of the rows and columns as follows

```
>> B = A(1:2, 1:2)% or >> B = A(1:3, 1:3)
```

3. Combining Commands

MATLAB allows you to combine commands easily. For example, suppose we wish to solve the system

$$\begin{aligned}x_1 + x_2 &= 0.5 \\ -3x_1 - 5x_2 &= 10\end{aligned}$$

We can do this with a single MATLAB command line entry with an inverse matrix:

```
>> inv([2 1;-3 -5])*[0.5;10]
```

¹This is an updated version of Justin Olav Wyss-Gallifent's MATLAB guide based on an elementary linear algebra course taught at University of Maryland, College Park.

Directions: The following may be done in a single MATLAB session or in several. Please print out the session (the entire contents of the command window). Any question marked with a star ★ indicates that that answer should be hand-written onto the printout since it is not explicitly a MATLAB result.

Important: Before each question:

1. Type `clear` to clear out the memory.
2. For questions 1-4,6,10 make sure you do `format rat` first.
3. For questions 5,7,8,9 make sure you do `format long` first.

1. (Linear Dependence) It was mentioned in class that a set of vectors is linearly dependent iff one of the vectors can be written as a linear combination of *previous* vectors in the set (See Theorem 7 on page 59). Using `rref` and progressing from left to right, find the leftmost vector in the following set which can be written in this way and write it as such. Set any free variables equal to 1.

$$\left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 3 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 10 \\ -6 \\ -2 \\ 14 \\ -9 \end{bmatrix}, \begin{bmatrix} 7.5 \\ -3 \\ -4 \\ 15 \\ -6 \end{bmatrix} \right\}$$

2. Suppose a linear transformation T has the property that $T(2, 2) = (-3, 1)$ and $T(1, -1) = (0, 5)$.
 - (a) Use a single MATLAB command line entry with an inverse matrix to find $T(1, 0)$.
 - (b) Use a single MATLAB command line entry with an inverse matrix to find $T(0, 1)$.
 - (c) ★ Write down the matrix for the transformation T .

3. (Inverse Matrix and Gauss-Jordan Algorithm) Define

$$A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 2 & -2 \\ 10 & 7 & 1 \end{bmatrix}$$

- (a) Find the inverse using `inv`.
 - (b) Find the inverse by reducing the augmented matrix $[A|I]$ to reduced row-echelon form.
4. (Vandermonde Matrices)
 - (a) Symbolically define the variables `a` through `i` and then find the determinant of the following matrices

$$\text{i. } \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \quad \text{ii. } \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix} \quad \text{iii. } \begin{bmatrix} 1 & a & a^2 & a^3 \\ 1 & b & b^2 & b^3 \\ 1 & c & c^2 & c^3 \\ 1 & d & d^2 & d^3 \end{bmatrix}$$

Hint: Use `simplify`.

- (b) ★ Use part(a) to compute the determinant of

$$F = \frac{1}{\sqrt{4}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix}.$$

Remark: Here, i is the complex number such that $i^2 = -1$.

5. (Stochastic Matrix) Define the matrix

$$A = \begin{bmatrix} 1/6 & 1/2 & 1/3 \\ 1/2 & 1/4 & 1/4 \\ 1/3 & 1/4 & 5/12 \end{bmatrix}$$

- (a) Calculate A^2 , A^{10} and A^{15} .
- (b) ★ Conjecture the value of $\lim_{n \rightarrow \infty} A^n$.
- (c) Calculate A^{100} .
- (d) ★ Does your conjecture seem reasonable?

6. (Fundamental Subspaces) Define

$$A = \begin{bmatrix} 1 & 2 & -5 & 11 & -3 \\ 2 & 5 & -5 & 15 & 2 \\ 1 & 2 & 0 & 4 & 5 \\ 3 & 6 & -5 & 19 & -2 \end{bmatrix}$$

- (a) Use **rref** on A .
- (b) ★ Write down bases for $\text{Col}(A)$ and $\text{Nul}(A)$.

7. Consider the set of functions

$$\{1, \cos t, \cos^2 t, \cos^3 t\}.$$

We would like to show that this is a linearly independent set of functions. This means that the equation

$$x_1(1) + x_2 \cos t + x_3 \cos^2 t + x_4 \cos^3 t = 0$$

has only the trivial solution. The thing to keep in mind is that this equality is *functional*, meaning it must be true for *all* values of t .

- (a) ★ Write down the equation above four times using $t = 0, 0.1, 0.2$ and 0.3 .
- (b) ★ Write down the corresponding augmented matrix.
- (c) Solve the corresponding system using **rref** to see that only the trivial solution exists.

8. Use an approach similar to the above to show that the set of functions

$$\{3, 5 \cos^2 t, 8 \sin^2 t\}$$

is *not* a linearly independent set by showing that

$$x_1(3) + x_2(5 \cos^2 t) + x_3(8 \sin^2 t) = 0$$

has non-trivial solutions.

- (a) ★ Write down the equation above three times using $t = 0, 0.1$ and 0.2 .
- (b) ★ Write down the corresponding augmented matrix.
- (c) Solve the corresponding system using **rref**.
- (d) ★ Set any free variables equal to 1 and solve for x_1 , x_2 and x_3 . Write down the corresponding equation.
- (e) ★ Verify your equation using basic precalculus.

9. (Computation with Partitioned Matrices) Suppose memory or size restrictions prevent your matrix program from working with matrices having more than 32 rows and 32 columns. However, the matrices A and B in the attachment file are 50×50 . Write the commands or operations in MATLAB that will accomplish the following tasks. *Remark:* Remember to download the MATLAB file with A, B, \mathbf{b} .
- Compute $A + B$.
 - Compute AB .
 - Solve $A\mathbf{x} = \mathbf{b}$ with `rref` by considering appropriate smaller systems to solve. Notice that A can be partitioned into a 2×2 block matrix $[A_{ij}]$, with A_{11} an invertible 30×30 matrix, A_{22} an invertible 20×20 matrix, and A_{12} a zero matrix.

10. (Determinant of “Exotic” Matrices)

- (a) Use MATLAB to compute the determinants of the following matrices

$$\text{i. } \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}, \text{ ii. } \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix}, \text{ iii. } \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}$$

- (b) ★ Use the results to guess the determinant of the matrix below, and confirm your guess by using row operations to evaluate that that determinant.

$$\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 2 & \cdots & 2 \\ 1 & 2 & 3 & \cdots & 3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 2 & 3 & \cdots & n \end{bmatrix}$$

- (c) Repeat (a) to guess the determinant of the following matrix, then justify your conjecture.

$$\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 3 & 3 & \cdots & 3 \\ 1 & 3 & 6 & \cdots & 6 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 3 & 6 & \cdots & 3(n-1) \end{bmatrix}$$

Hint: Use part (b) and Exercise 14(c) in the Chapter 3 Supplementary Exercises section.