A Guide to MATLAB in M340L Spring 2020 - Part 3, Correction

1. **Method**

   The way this guide is written is different from the first since it is assumed that you know the basics. New commands are presented on this page and the project starts on the next page.

2. **New Commands**

   (a) Eigenvalues can be found easily. If `A` is a matrix then:

   `>> eig(A)`

   will return the eigenvalues. Note that it will return complex eigenvalues too, which we're not so concerned about. So keep an i open for those.

   (b) However the characteristic polynomial is interesting in its own right. To begin with note the useful command `eye(n)` which returns the $n \times n$ identity (eye-dentity?) matrix:

   `>> eye(5)`

   (c) So now let use `z` for λ and if we have a matrix like:

   `>> A=[8 -10 -5;2 17 2;-9 -18 4]`

   we can symbolically define `z`:

   `>> syms z`

   and then:

   `>> det(A-z*eye(3))`

   to get the characteristic polynomial for A.

   (d) We can solve it using `solve`. One useful fact is that `solve` will assume the expression equals 0 unless specified and will solve for the single variable. Therefore we can do:

   `>> solve(det(A-z*eye(3)))`

   to get the solutions to the characteristic equation.

   (e) Of course if we have an eigenvalue λ we can use `rref` on an augmented matrix to lead us to the eigenvectors.

   (f) Moreover, MATLAB can do everything in one go. If you recall from class, *diagonalizing* a matrix A means finding a diagonal matrix D and an invertible matrix P with $A = PDP^{-1}$. The diagonal matrix D contains the eigenvalues along the diagonal and the matrix P contains eigenvectors as colums, with column i of P corresponding to the eigenvalue in column i of D.

   To do this we use the `eig` command again but demand different output. The format is:

   `>> [P,D]=eig(A)`

   which assigns `P` and `D` for A, if possible. If it's not possible MATLAB returns very strange-looking output.

M340L Spring 2020 MATLAB Project 3 **Due Monday 4/20/2020**

Directions: The following may be done in a single MATLAB session or in several. Please print out the session (the entire contents of the command window). Any question marked with a star $\star$ indicates that that answer should be hand-written onto the printout since it is not explicitly a MATLAB result.

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1. Consider the matrix

$$A = \begin{bmatrix} -4 & -4 & 20 & -8 & -1 \\ 14 & 12 & 46 & 18 & 2 \\ 6 & 4 & -18 & 8 & 1 \\ 11 & 7 & -37 & 17 & 2 \\ 18 & 12 & -60 & 24 & 5 \end{bmatrix}.$$

   (a) Compute the characteristic polynomial $p_A(z)$ of A.

   (b) Find the roots of the characteristic polynomial found in part (a). These are the eigenvalues of A.

   (c) Alternatively, use the `eig` command to find the eigenvalues of A.

2. Given the matrix

$$A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}.$$

   (a) Find the eigenvalues using `eig`.

   (b) For each eigenvalue λ, reduce the appropriate augmented matrix $[A - \lambda I \,|\, \bar{0}]$ using `rref`.

   (c) $\star$ For each reduced matrix, calculate a basis for the eigenspace for the eigenvalue.

3. Consider the following symmetric matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 2 & 3 \\ 3 & 2 & 5 & 0 \\ 4 & 3 & 0 & 3 \end{bmatrix}.$$

   (a) Find a diagonalization for A, i.e. compute P and D such that $A = PDP^{-1}$.

   (b) Check that A is indeed equal to PDP^{-1} and that D is equal to $P^{-1}AP$.

   (c) How is P^{-1} related to P^T?

4. Consider the matrix $A = \begin{bmatrix} -3 & -2 & 0 \\ 14 & 7 & -1 \\ -6 & -3 & 1 \end{bmatrix}$.

   (a) Use `eig` to compute for some P and D matrices.

   (b) From part (a), compute $AP - PD$ and $A - PDP^{-1}$. Discuss your results.

   (c) Compute the characteristic polynomial of A and use `solve` to find its roots.

   (d) Use `rref` to find a basis for each of the eigenspaces.

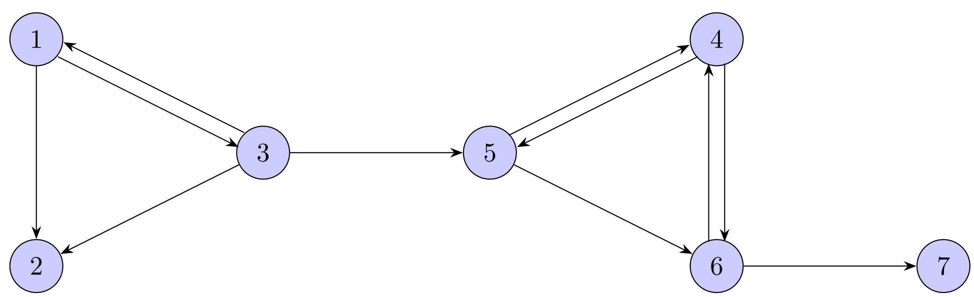
   (e) $\star$ Using part (d), find a diagonalization of A if possible. Otherwise, explain what you see.

5. Consider the following matrix

$$A = \begin{bmatrix} -1.4 & -2.0 & -2.0 & -2.0 \\ -1.3 & -0.8 & -0.1 & -0.6 \\ 0.3 & -1.9 & -1.6 & -1.4 \\ 2.0 & 3.3 & 2.3 & 2.6 \end{bmatrix}.$$

Find a factorization of A in the form $A = PCP^{-1}$, where $C = \begin{bmatrix} C_1 & \mathbf{0} \\ \mathbf{0} & C_2 \end{bmatrix}$ is a 4×4 block-diagonal matrix with 2×2 blocks C_1, C_2 of the form shown in Example 6 in §5.5 of the course textbook. (For each conjugate pair of eigenvalues, use the real and imaginary parts of one eigenvector in $\mathbb{C}^4$ to create two columns of P.)

6. Consider the following webpages hyperlinked by the given directed graph.



(a) Construct a transition matrix S for the random walk on the above directed graph.

(b) If a random surfer starts at page 1, find the probability that the surfer will be at page 4 after 4 clicks.

(c) If the random surfer starts at page 1 and you left her alone for a long period of time to surf the graph, what are the probabilities that the surfer will be on each of the 7 pages when you return?

(d) What happens if the surfer started at a different page? Find the long-term probability vector for starting at each of the other 6 pages. Does the long-term probability vector depend on where the random surfer starts?

(e) $\star$ Is S a regular matrix? Explain your answer.

(f) Compute the Google matrix for the above directed graph.

(g) Use part (f) to find the Pagerank for each of the webpages.

(h) What happens to the Pagerank if instead of .85/.15 split you use .6/.4 split? Discuss your findings.

(i) Compute the standard Pagerank for the above directed graph with the hyperlink between page 3 and page 5 removed (i.e. you have a disconnected graph).

(j) $\star$ Can you modify the above undirected graph (add hyperlinks or remove hyperlinks between webpages) so that at least one of the webpage will have a zero Pagerank?