MATH241 Guide/Project 3

As with the second project you'll need to create an m-file, publish it, and turn in the resulting document. Check the second project directions if you've forgotten how to do this.

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Multiple Integrals

Since Matlab does integrals so well this is easy, we just nest the integrals. For example consider the following. Read it carefully from the innermost `int` outwards. Remember that when we do `int(f,x,a,b)` we integrate `f` with respect to `x` from `a` to `b`. Here `a` and `b` may also contain other variables.

For example if we wanted to do $\int_{-1}^{2} \int_{0}^{2x} \int_{x+y}^{x} x^2 y + z \, dz \, dy \, dx$:

```
syms x y z;
int(int(int(x^2*y+z,z,x+y,x),y,0,2*x),x,-1,2)
```

```
ans =

-81/2
```

Here's the integral $\int_{0}^{2\pi} \int_{0}^{\pi/4} \int_{2\sec\phi}^{5} \rho^4 \sin\phi \, d\rho \, d\phi \, d\theta$:

```
syms theta rho phi
int(int(int(rho^4*sin(phi),2*sec(phi),5),0,pi/4),0,2*pi)
```

```
ans =

-(pi*(3125*2^(1/2) - 6202))/5
```

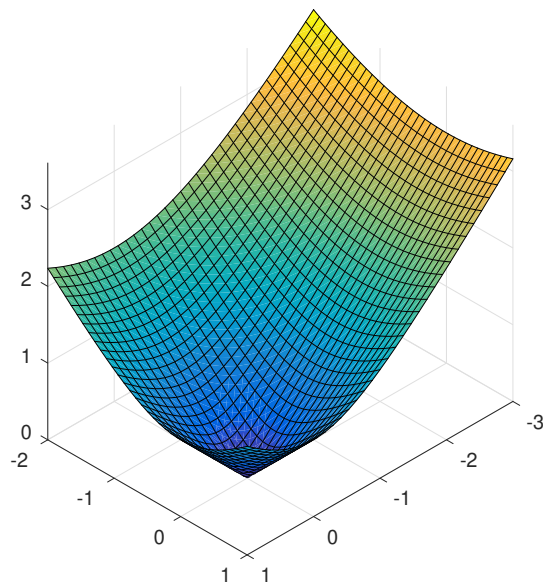**Task 1:** Calculate $\int_0^2 \int_0^x \int_0^{9-x^2-y^2} x \, dz \, dy \, dx$

Task 2: Calculate $\int_0^{\pi/2} \int_0^{\sin\theta} \int_0^{10-r\cos\theta} zr \, dz \, dr \, d\theta$

Plotting Surfaces

In section 14.9 we learn how to parametrize surfaces as vector-valued functions of two variables. This is perfect for the `fsurf` command.

Here is the part of the cone $z = \sqrt{x^2+y^2}$ with $-3 \le x \le 1$ and $-2 \le y \le 1$.

```
clear all;
syms x y;
rbar = [x,y,sqrt(x^2+y^2)];
fsurf(rbar(1),rbar(2),rbar(3),[-3,1,-2,1])
view([10 10 10])
axis equal
```

A few things to note:

- I am setting `view([10 10 10])` which puts the viewpoint in the first octant looking in.
- I have set `axis equal` which ensures that all three axis are proportionally scaled, making the pictures look better.
- We have to give the `fsurf` the three components separately. We do this by giving it `r(1)`, `r(2)` and `r(3)`.
- The restrictions are in alphabetical order so for `x`, `y` and `z` it works as expected.

Here is the portion of the sphere of radius 2 inside the cone $\phi = \pi/3$ done using r and ϕ. The restriction on `phi` must come first for alphabetical reasons.

```
clear all;
syms phi t;
rbar = [2*sin(phi)*cos(t),2*sin(phi)*sin(t),2*cos(phi)];
fsurf(rbar(1),rbar(2),rbar(3),[0,pi/3,0,2*pi])
view([10 10 10])
axis equal
```
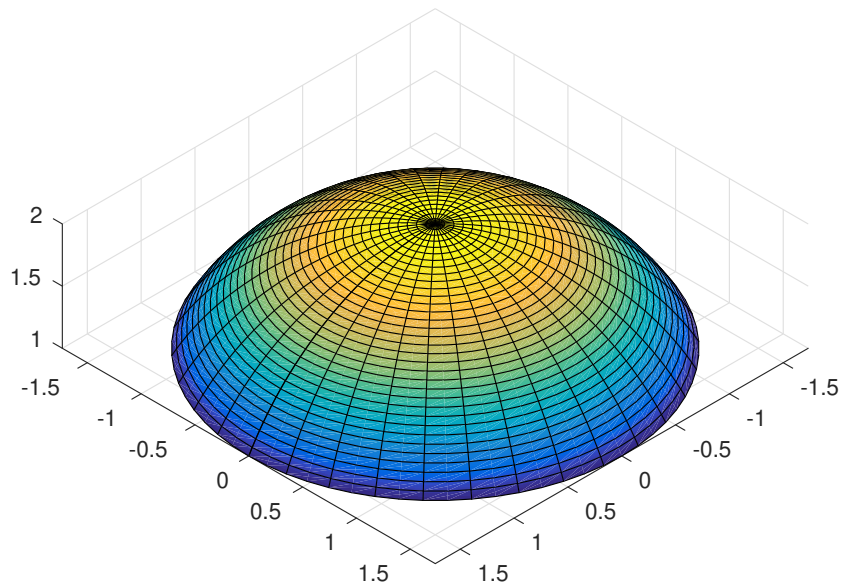
Here is something really cool, a spiral!

```
clear all;
syms t z;
rbar = [t*cos(t),t*sin(t),z];
fsurf(rbar(1),rbar(2),rbar(3),[0,6*pi,0,10])
view([10 10 10])
axis equal
```
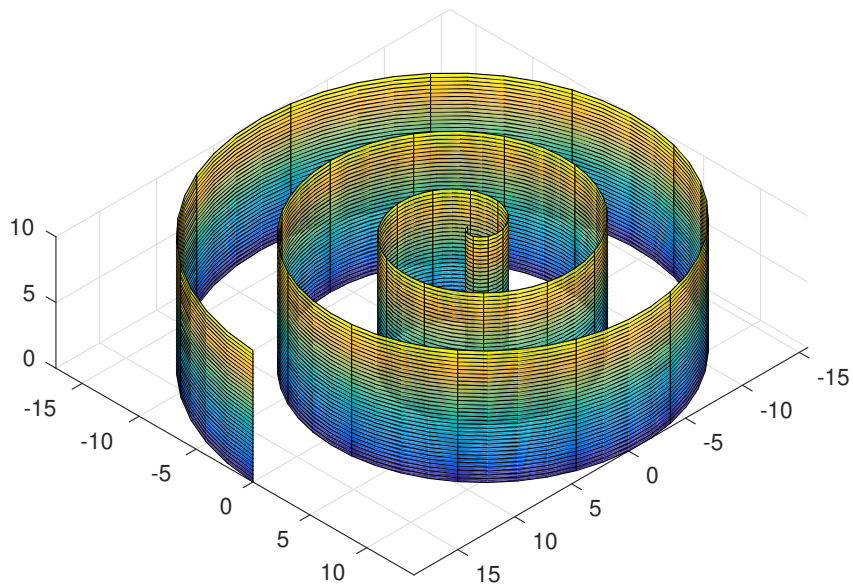

Task 3: Plot the portion of $x^2 + z^2 = 9$ above the xy-plane and between $y = -1$ and $y = 2$.

Task 4: Plot the portion of the cone $z = 9 - \sqrt{x^2 + y^2}$ inside the cylinder $r = 2$.
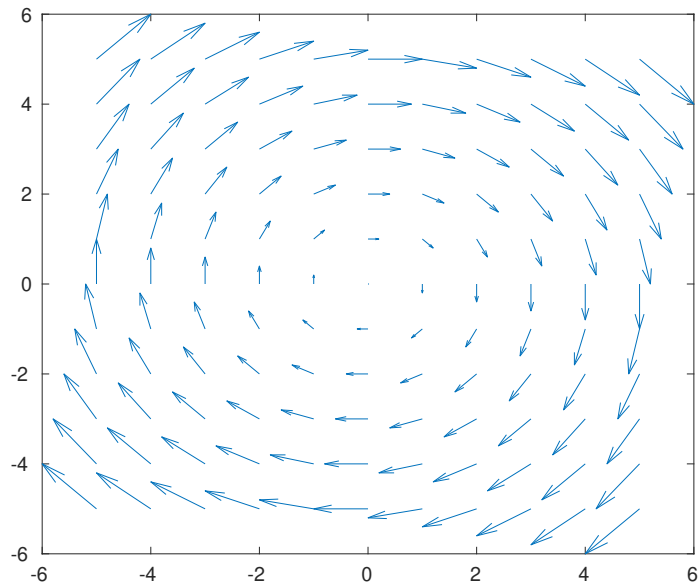
Vector Fields

Matlab can plot vector fields using the `quiver` command, which basically draws a bunch of arrows. This is not completely obvious though. First we have to set up a grid of points for which to plot arrows. In other words we have to tell it for which `x` and `y` to actually draw the vector field.

Suppose we want our grid to have `x` going from `-5` to `5` in steps of 1 and the same for `y`. We first do the following:

```
clear all;
[x,y] = meshgrid(-5:1:5,-5:1:5);
```

Now suppose we wish to plot the vector field `F(x,y)=(y/5)i-(x/5)j`. To do this we type

```
quiver(x,y,y/5,-x/5,0);
```

A window will pop up with the vector field on it. The `0` at the end just ensures that Matlab does not do any tricky rescaling of the vectors, something it usually does to make things fit nicely.

Just so you know the first two entries `x,y` indicate that vectors should be placed at `x,y`. The second two entries form the vector itself.

A note of caution. Suppose we want the vector field `F(x,y)=(y/5)i-(x/y)j`. Since `x` and `y` are collections of numbers, to divide them we cannot use `/`, we must use `./` instead. This is a Matlab quirk since we are not working with individual numbers. Thus we would need
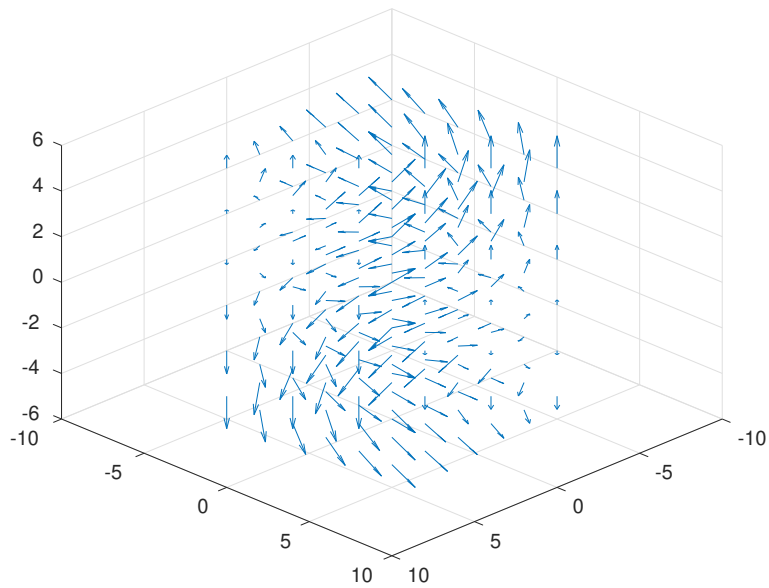
```
quiver(x,y,y./5,-x./y,0)
```
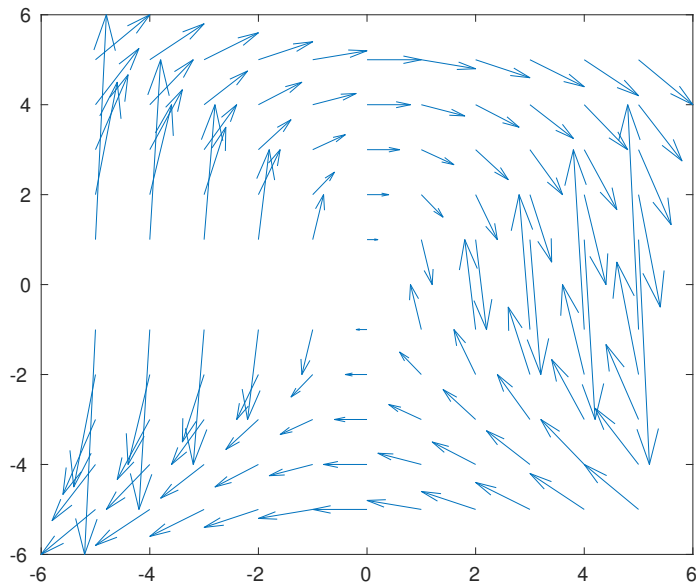

Note that this is a pretty ugly vector field. Try it!

Task 5: Plot the vector field $\bar{F}(x,y) = 0.2(x+y)\imath + 0.2(x-y)\jmath$.

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Vector Fields in 3D

For 3D vector fields we use `quiver3`. Here is an example. It's the only one we'll do because they can be pretty overwhelming.

```
[x,y,z] = meshgrid(-5:2:5,-5:2:5,-5:2:5);
quiver3(x,y,z,-y./sqrt(x.^2+y.^2),x./sqrt(x.^2+y.^2),z./5,0)
view([10 10 10])
```

Line Integrals of Functions.

Line integrals of functions are really easy - we just tell Matlab exactly what to do, step by step. For example here is example 1 from page 1006 of the text.

```
clear
syms t x y z;
rbar = [t,-3*t,2*t];
f = x+y^2-2*z;
mylength = @(u) sqrt(u*transpose(u));
mag = simplify(mylength(diff(rbar,t)));
sub = subs(f,[x,y,z],rbar);
int(sub*mag,t,0,1)


ans =

(3*14^(1/2))/2
```

Read this carefully to see how it works! Especially note the line `mylength=...` which creates a new length function for vectors. The Matlab `norm` command only works on numerical vectors and not on vectors with variables in them. Our new `mylength` function will do the job. Even if you don't understand it for now just use it - all it does is finds the norm of a vector even if that vector contains variables.

Task 6: A piece of wire is in the shape of the circle $x^2 + y^2 = 1$. The density at any point is given by $\delta(x, y) = x^2 + y^4$. Find the mass.

Task 7: Evaluate the line integral $\int_C (x + y) \, ds$ where C is the straight line segment from $(0, 1, 1)$ to $(3, 2, 2)$.

Line Integrals of Vector Fields.

Just like line integrals of functions these are easy, we just tell Matlab exactly what we want it to do. Here's example 6 from page 1011 of the text. Again read carefully and understand!

```
clear all
syms t x y z;
rbar = [t,t^2,t^3];
F = [x*y,3*z*x,-5*x^2*y*z];
sub = subs(F,[x,y,z],rbar);
int(dot(sub,diff(rbar,t)),0,1)


ans =

-1/4
```

Task 8: Evaluate the line integral $\int_C yz\ dx + yz\ dy + y\ dz$ where C is the top half of $y^2 + z^2 = 4$ in the yz-plane traveling from left to right.

Surface Integrals of Functions

The nice thing about having a surface in Matlab as a vector is that it's easy to work with.

For example suppose we want to find $\int\int_\Sigma xy\, dS$ where Σ is the part of $z = x^2 + y^2$ having $0 \le x \le 2$ and $0 \le y \le 3$.

As part of this procedure we need the magnitude of the cross products of the derivatives of **r**. Here is how we do just that bit:

```
clear all;
syms x y;
rbar = [x,y,x^2+y^2];
mylength = @(u) sqrt(u*transpose(u));
simplify(mylength(cross(diff(rbar,x),diff(rbar,y))))


ans =

(4*x^2 + 4*y^2 + 1)^(1/2)
```

So now here's the really nice thing. To do our entire integral involves plugging the **i**, **j**, **k** components in for **x**, **y**, **z**, multiplying by that magnitude and integrate over the appropriate limits. Here it is all together:

```
clear all;
syms x y z;
rbar = [x,y,x^2+y^2];
f = x*y;
mylength = @(u) sqrt(u*transpose(u));
mag = simplify(mylength(cross(diff(rbar,x),diff(rbar,y))));
subresult = subs(f,[x,y,z],rbar);
int(int(subresult*mag,x,0,2),y,0,3)


ans =

(2809*53^(1/2))/240 - (1369*37^(1/2))/240 - (289*17^(1/2))/240 + 1/240
```

Make sure you read this carefully (after the `clear all`) to see what each line does. The second line sets the symbolic variables. The third line is the parametrization of the surface. The fourth line is the function to integrate. The fifth line creates a length function we'll need later. The sixth line finds the magnitude of the cross product of the derivatives. The seventh line substitutes

the components from the parametrization into the real-valued function we want to integrate. The eighth and final line does the double integral required.

Task 9: Suppose Σ is the portion of the plane $z = 10 - x - y$ inside the cylinder $x^2 + y^2 = 1$. The surface Σ is submerged in an electric field such that at any point the electric charge density is $\delta(x, y, z) = x^2 + y^2$. Find the total amount of electric charge on the surface.

Surface Integrals of Vector Fields.

Similarly we can take the surface integral of a vector field. We only need to be careful in that Matlab can't take care of orientation so we'll need to do that and instead of needing the magnitude of the cross product we just need the cross product. Here is problem 6 from the 15.6 exercises.

```
clear
syms t x y z;
rbar = [cos(t),y,sin(t)];
F = [x,y,z];
kross = simplify(cross(diff(rbar,t),diff(rbar,y)));
sub = subs(F,[x,y,z],rbar);
int(int(dot(sub,kross),y,-2,1),t,0,2*pi)


ans =

-6*pi
```

Again read this carefully. After the `clear` the first line sets the symbolic variables. The second sets the parametrization and the third sets the vector field. The fourth finds the cross product of the derivatives. The fifth substitutes the parametrization into the vector field. The sixth does the double integral of the dot product as required for the surface integral of a vector field.

Task 10: A fluid is flowing through space following the vector field $\bar{F}(x, y, z) = y\mathbf{i} - x\mathbf{j} + z\mathbf{k}$. A filter is in the shape of the portion of the paraboloid $z = x^2 + y^2$ having $0 \le x \le 3$ and $0 \le y \le 3$, oriented inwards (and upwards). Find the rate at which the fluid is moving through the filter.