Calculus II - Fall 2013

Quiz #2, October 17, 2013

In the following problems you are required to show all your work and provide the necessary explanations everywhere to get full credit.

1. Find the volume of a solid obtaining by rotating the region bounded by $y^2 = x$ and x = 2y about the y-axis.

Solution 1: We have

$$V = \int_{0}^{4} 2\pi x \left(\sqrt{x} - \frac{x}{2}\right) dx = 2\pi \int_{0}^{4} \left(x^{3/2} - \frac{x^2}{2}\right) dx = 2\pi \left[\frac{x^{5/2}}{5/2} - \frac{x^3}{6}\right]_{0}^{4} = \frac{64}{15}\pi$$

Solution 2: We have

$$V = \int_{0}^{2} \pi \left((2y)^{2} - (y^{2})^{2} \right) dy = \pi \int_{0}^{2} \left(4y^{2} - y^{4} \right) dy = \frac{64}{15}\pi$$

2. Find the volume of a solid obtaining by rotating the region bounded by $y = \sqrt{x-1}$, y = 0, x = 5 about y = 3.

Solution 1: We have

$$V = \int_{0}^{2} 2\pi (3-y) \left(5 - (y^{2}+1)\right) dy = 24\pi$$

Solution 2: We have

$$V = \int_{1}^{5} \pi (3^2 - (3 - \sqrt{x - 1})^2) dx = 24\pi$$