Calculus II - Fall 2013

Quiz #4, December 5, 2013

In the following problem you are required to show all your work and provide the necessary explanations everywhere to get full credit.

Find the Maclaurin series for $f(x) = x^3 \cos x$.

Solution: We arrange our computation in two columns as follows:

$$\begin{array}{ll} f(x) = \cos x & f(0) = 1 \\ f'(x) = -\sin x & f'(0) = 0 \\ f''(x) = -\cos x & f''(0) = -1 \\ f'''(x) = \sin x & f'''(0) = 0 \\ f^{(4)}(x) = \cos x & f^{(4)}(0) = 1 \end{array}$$

Since the derivatives repeat in a cycle of four, we can write the Maclaurin series as follows:

$$\begin{aligned} \cos x &= f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \dots \\ &= 1 + \frac{0}{1!}x + \frac{-1}{2!}x^2 + \frac{0}{3!}x^3 + \frac{1}{4!}x^4 + \dots \\ &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \\ &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \end{aligned}$$

Therefore

$$x^3 \cos x = x^3 \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = \boxed{\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+3}}{(2n)!}}$$