Calculus II - Fall 2013

Quiz #1, September 19, 2013

In the following problems you are required to show all your work and provide the necessary explanations everywhere to get full credit.

1. Find $\int_0^1 \frac{\sqrt{x}}{1+\sqrt{x}}dx$.

Solution: We have

$$\int_0^1 \frac{\sqrt{x}}{1+\sqrt{x}}dx = \left[\begin{array}{r} 1+\sqrt{x}=u \implies \sqrt{x}=u-1 \implies x=(u-1)^2 \\ dx = d(u-1)^2 \\ dx = 2(u-1)du \end{array}\right] = \int_{1+\sqrt{0}}^{1+\sqrt{1}} \frac{u-1}{u}\cdot 2(u-1)du$$

$$= 2\int_1^2 \frac{(u-1)^2}{u}du = 2\int_1^2 \frac{u^2-2u+1}{u}du = 2\int_1^2 \left(\frac{u^2}{u}-\frac{2u}{u}+\frac{1}{u}\right)du = 2\int_1^2 \left(u-2+\frac{1}{u}\right)du$$

$$= 2\left[\frac{u^2}{2}-2u+\ln|u|\right]_1^2 = 2\left[\left(\frac{2^2}{2}-2\cdot 2+\ln 2\right)-\left(\frac{1^2}{2}-2\cdot 1+\ln 1\right)\right]$$

$$= 2\left[(2-4+\ln 2)-\left(\frac{1}{2}-2+0\right)\right] = 2\left[\ln 2-\frac{1}{2}\right] = \boxed{2\ln 2-1}$$

2. Find $\int e^x\sqrt{1-e^x}dx$.

Solution: We have

$$\int e^x\sqrt{1-e^x}dx = \left[\begin{array}{c} 1-e^x=u \\ d\left(1-e^x\right)=du \\ -e^x dx = du \\ e^x dx = -du \end{array}\right] = -\int \sqrt{u}du = -\int u^{1/2}du = -\frac{u^{1/2+1}}{1/2+1}+C$$

$$= -\frac{u^{3/2}}{3/2}+C = -\frac{2}{3}u^{3/2}+C = \boxed{-\frac{2}{3}\left(1-e^x\right)^{3/2}+C}$$

3. Find $\int\limits_0^{\pi} \tan^2 \frac{x}{3} dx$.

Solution: We have

$$\int\limits_0^{\pi} \tan^2 \frac{x}{3} dx = \int\limits_0^{\pi} \left(\sec^2 \frac{x}{3} - 1\right) dx = \begin{bmatrix} \frac{x}{3} = u \\ d\left(\frac{x}{3}\right) = du \\ \frac{1}{3} dx = du \\ dx = 3du \end{bmatrix} = 3 \int\limits_0^{\pi/3} \left(\sec^2 u - 1\right) du$$

$$= [3\tan u - 3u]_0^{\pi/3} = \left[3\tan\frac{\pi}{3} - 3\left(\frac{\pi}{3}\right)\right] - [3\tan 0 - 3(0)] = \boxed{3\sqrt{3} - \pi}$$

4. Find $\int x \cos 5x dx$.

Solution: We have

$$\int x \cos 5x dx = \left[\begin{array}{c|c} x = u & \cos 5x dx = dv \\ dx = du & \frac{1}{5}\sin 5x = v \end{array}\right] = x \cdot \frac{1}{5}\sin 5x - \int \frac{1}{5}\sin 5x dx$$

$$= \frac{1}{5} x \sin 5x - \frac{1}{5}\int \sin 5x dx = \boxed{\frac{1}{5} x \sin 5x + \frac{1}{25}\cos 5x + C}$$

5. Find $\int \frac{\sqrt{9 - x^2}}{x^2} dx$.

Solution: We have

$$\int \frac{\sqrt{9 - x^2}}{x^2} dx = \begin{bmatrix} x = 3\sin\theta \\ dx = d(3\sin\theta) \\ dx = 3\cos\theta d\theta \\ \sqrt{9 - x^2} = \sqrt{9 - 9\sin^2\theta} = \sqrt{9(1 - \sin^2\theta)} = \sqrt{9\cos^2\theta} = 3|\cos\theta| = 3\cos\theta \end{bmatrix}$$

$$= \int \frac{3\cos\theta}{9\sin^2\theta} \cdot 3\cos\theta d\theta = \int \frac{\cos^2\theta}{\sin^2\theta} d\theta = \int \cot^2\theta d\theta = \int (\csc^2\theta - 1) d\theta = -\cot\theta - \theta + C$$

Note that

$$\cot\theta = \frac{\sqrt{9 - x^2}}{x}$$

Therefore

$$\int \frac{\sqrt{9 - x^2}}{x^2} dx = \boxed{-\frac{\sqrt{9 - x^2}}{x} - \sin^{-1}\left(\frac{x}{3}\right) + C}$$