Calculus II - Spring 2014

Quiz #1, February 12, 2014

In the following problems you are required to show all your work and provide the necessary explanations everywhere to get full credit.

1. Find $\displaystyle\int \left(x^3 + 3^x + \sin x + \sec^2 x + \frac{1}{\sqrt{x}} + \frac{1}{1+x^2} + \frac{1}{\sqrt{1-x^2}}\right) dx.$

Solution: We have

$$\int \left(x^3 + 3^x + \sin x + \sec^2 x + x^{-1/2} + \frac{1}{1+x^2} + \frac{1}{\sqrt{1-x^2}}\right) dx$$

$$= \frac{x^4}{4} + \frac{3^x}{\ln 3} - \cos x + \tan x + 2\sqrt{x} + \arctan x + \arcsin x + C$$

2. Find $\displaystyle\int \frac{2x^4}{\sqrt{1-3x^5}} dx.$

Solution: We have

$$\int \frac{2x^4}{\sqrt{1-3x^5}} dx = \begin{bmatrix} 1-3x^5 = u \\ d(1-3x^5) = du \\ -15x^4 dx = du \\ 2x^4 dx = -\dfrac{2}{15} du \end{bmatrix} = \int \frac{1}{\sqrt{u}} \left(-\frac{2}{15}\right) du = -\frac{2}{15} \int u^{-1/2} du = -\frac{2}{15} \cdot \frac{u^{-1/2+1}}{-1/2+1} + C$$

$$= -\frac{2}{15} \cdot \frac{u^{1/2}}{1/2} + C = -\frac{4}{15} u^{1/2} + C = -\frac{4}{15}\sqrt{1-3x^5} + C$$

3. Find $\displaystyle\int_0^{\pi/8} \sin^2 x dx.$

Solution: We have

$$\int_0^{\pi/8} \sin^2 x dx = \int_0^{\pi/8} \frac{1}{2}(1-\cos 2x) dx = \frac{1}{2}\int_0^{\pi/8} (1-\cos 2x) dx = \begin{bmatrix} 2x = u \\ d(2x) = du \\ 2dx = du \\ dx = \dfrac{1}{2} du \end{bmatrix} = \frac{1}{2} \int_{2(0)}^{2(\pi/8)} (1-\cos u) \left(\frac{1}{2}\right) du$$

$$= \frac{1}{4}\int_0^{\pi/4} (1-\cos u) du = \frac{1}{4}\left[u - \sin u\right]_0^{\pi/4} = \frac{1}{4}\left(\frac{\pi}{4} - \sin\frac{\pi}{4}\right) - \frac{1}{4}(0 - \sin 0) = \frac{1}{4}\left(\frac{\pi}{4} - \frac{\sqrt{2}}{2}\right)$$

4. Find $\displaystyle\int \ln^2 x dx$.

Solution: We have

$$\int \ln^2 x dx = \left[\begin{array}{c|c} \ln^2 x = u & dx = dv \\ d(\ln^2 x) = du & x = v \\ 2\ln x \cdot \frac{1}{x} dx = du & \end{array}\right] = x\ln^2 x - \int x \cdot 2\ln x \cdot \frac{1}{x} dx = x\ln^2 x - 2\int \ln x dx$$

$$= \left[\begin{array}{c|c} \ln x = u & dx = dv \\ d(\ln x) = du & x = v \\ \frac{1}{x} dx = du & \end{array}\right] = x\ln^2 x - 2\left(x\ln x - \int x \cdot \frac{1}{x} dx\right)$$

$$= x\ln^2 x - 2\left(x\ln x - \int dx\right)$$

$$= x\ln^2 x - 2\left(x\ln x - x\right) + C$$

$$= x\ln^2 x - 2x\ln x + 2x + C$$

5. Find $\displaystyle\int \frac{dx}{\sqrt{x^2+1}}$.

Solution: We have

$$\int \frac{dx}{\sqrt{x^2+1}} = \left[\begin{array}{c} x = \tan\theta \\ dx = d(\tan\theta) \\ dx = \sec^2\theta d\theta \\ \sqrt{x^2+1} = \sqrt{\tan^2\theta + 1} = \sqrt{\sec^2\theta} = |\sec\theta| = \sec\theta \end{array}\right] = \int \frac{\sec^2\theta d\theta}{\sec\theta}$$

$$= \int \sec\theta d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$

Since $\sec\theta = \sqrt{x^2+1}$ and $\tan\theta = x$ by the above, it follows that

$$\int \frac{dx}{\sqrt{x^2+1}} = \ln|\sec\theta + \tan\theta| + C = \ln(\sqrt{x^2+1} + x) + C$$