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$\frac{16}{5}$	3.200000	0.058407346	1
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$$\pi = 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{292 + \dots}}}}$$

$\mathbf{3}$	$\mathbf{0.14159}$
$\mathbf{\frac{22}{7}}$	$\mathbf{0.0619}$
$\mathbf{\frac{333}{106}}$	$\mathbf{0.93505}$
$\mathbf{\frac{355}{113}}$	$\mathbf{0.00340}$
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THEOREM 1: Let ξ be a real number and let p_i/q_i be the i-th convergent of ξ. Then

$$\left|\xi - \frac{p_i}{q_i}\right| q_i^2 < 1$$

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$\frac{104348}{33215}$	0.36586

THEOREM 2: Let ξ be a real number and let p_i/q_i be the i-th convergent of ξ. Then at least one of the following estimates is valid:

$$\left|\xi - \frac{p_i}{q_i}\right| q_i^2 < \frac{1}{2}, \quad \left|\xi - \frac{p_{i+1}}{q_{i+1}}\right| q_{i+1}^2 < \frac{1}{2}.$$

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THEOREM 3: Let ξ be a real number and let p_i/q_i be the i-th convergent of ξ. Then at least one of the following estimates is valid:

$$\left|\xi - \frac{p_i}{q_i}\right| q_i^2 < \frac{1}{\sqrt{5}}, \quad \left|\xi - \frac{p_{i+1}}{q_{i+1}}\right| q_{i+1}^2 < \frac{1}{\sqrt{5}}, \quad \left|\xi - \frac{p_{i+2}}{q_{i+2}}\right| q_{i+2}^2 < \frac{1}{\sqrt{5}}.$$

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THEOREM 4: Let ξ be an irrational number. Then there are infinitely many rational numbers p_i/q_i such that

$$\left|\xi - \frac{p_i}{q_i}\right| < \frac{1}{\sqrt{5}q_i^2}.$$

Let $\boldsymbol{\xi = \frac{-1+\sqrt{5}}{2}}$**. Then**

$$\left|\xi - \frac{1}{2}\right| \approx 0.4721359550$$

$$\left|\xi - \frac{2}{3}\right| \approx 0.4376941013$$

$$\left|\xi - \frac{3}{5}\right| \approx 0.4508497187$$

$$\left|\xi - \frac{5}{6}\right| \approx 0.4458247200$$

$$\left|\xi - \frac{8}{13}\right| \approx 0.4477440987$$

$$\left|\xi - \frac{13}{21}\right| \approx 0.4470109613$$

$$\left|\xi - \frac{21}{34}\right| \approx 0.4472909949$$

$$\left|\xi - \frac{34}{55}\right| \approx 0.4471840316$$

$$\frac{1}{\sqrt{5}} \approx 0.44721359549995793928$$

THEOREM 5: Let ξ be an irrational number. Then p/q is a best approximation of the second kind if and only if it is a convergent to ξ.

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THEOREM 6: Let ξ be an irrational number. Then p/q is a best approximation of the first kind if and only if it is either a convergent to ξ or of the form

$$\frac{p_i + cp_{i+1}}{q_i + cq_{i+1}}, \quad \text{where} \quad 1 \leq c \leq c_{i+2}.$$

THEOREM 4: Let ξ be an irrational number. Then there are infinitely many rational numbers p/q such that

$$\left|\xi - \frac{p}{q}\right| < \frac{1}{\sqrt{5}q^2} \quad \Longleftrightarrow \quad |q\xi - p| < \frac{1}{\sqrt{5}q}.$$

THEOREM 4: Let ξ be an irrational number. Then there are infinitely many rational numbers p/q such that

$$\left|\xi - \frac{p}{q}\right| \ll \frac{1}{q^2} \quad \Longleftrightarrow \quad |q\xi - p| \ll \frac{1}{q}.$$

THEOREM 4: Let ξ be an irrational number. Then there are infinitely many rational numbers p/q such that

$$\left|\xi - \frac{p}{q}\right| \ll \frac{1}{\max(|p|,\ |q|)^2} \quad \Longleftrightarrow \quad |q\xi - p| \ll \frac{1}{\max(|p|,\ |q|)}.$$

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CONJECTURE: Let $\xi \notin \mathbb{A}_n$. Then there are infinitely many algebraic numbers $\alpha \in \mathbb{A}_n$ such that

$$|\xi - \alpha| \ll H(\alpha)^{-n-1}, \quad n \geq 1.$$

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THEOREM 7 (Davenport-Schmidt, 1967): Let $\xi \notin \mathbb{A}_2$. Then there are infinitely many algebraic numbers $\alpha \in \mathbb{A}_2$ such that

$$|\xi - \alpha| \ll H(\alpha)^{-3}.$$

THEOREM 4: Let ξ be an irrational number. Then there are infinitely many rational numbers p/q such that

$$\left|\xi - \frac{p}{q}\right| \ll \frac{1}{\max(|p|,\ |q|)^2} \quad \Longleftrightarrow \quad |q\xi - p| \ll \frac{1}{\max(|p|,\ |q|)}.$$

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$$|\xi - \alpha| \ll H(\alpha)^{-2}.$$

THEOREM 8 (Wirsing, 1961): Let $\xi \notin \mathbb{A}_n$. Then there are infinitely many algebraic numbers $\alpha \in \mathbb{A}_n$ such that

$$|\xi - \alpha| \ll H(\alpha)^{-A(n)}, \quad \lim_{n\to\infty} \left(A(n) - \frac{n}{2}\right) = 2.$$

THEOREM 4: Let ξ be an irrational number. Then there are infinitely many rational numbers p/q such that

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THEOREM 9 (Bernik-T., 1993): Let $\xi \notin \mathbb{A}_n$. Then there are infinitely many algebraic numbers $\alpha \in \mathbb{A}_n$ such that

$$|\xi - \alpha| \ll H(\alpha)^{-A(n)}, \quad \lim_{n\to\infty} \left(A(n) - \frac{n}{2}\right) = 3.$$

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THEOREM 10 (T., 2001): Let $\xi \notin \mathbb{A}_n$. Then there are infinitely many algebraic numbers $\alpha \in \mathbb{A}_n$ such that

$$|\xi - \alpha| \ll H(\alpha)^{-A(n)}, \quad \lim_{n \to \infty} \left(A(n) - \frac{n}{2}\right) = 4.$$

n	1961	1993	2001
3	3.28	3.5	3.73
4	3.82	4.12	4.45
5	4.35	4.71	5.14
6	4.87	5.28	5.76
7	5.39	5.84	6.36
8	5.9	6.39	6.93
9	6.41	6.93	7.50
10	6.92	7.47	8.06
50	26.98	27.84	28.70
100	51.99	52.92	53.84

THEOREM 11 (Davenport-Schmidt, 1969): Let $n \geq 3$. Let ξ be real, but not algebraic of degree ≤ 2 if $n = 3, 4$. Let also ξ be real, but not algebraic of degree $\leq (n-1)/2$ if $n \geq 5$. Then there are infinitely many algebraic integers α of degree $\leq n$ which satisfy

$$|\xi - \alpha| \ll H(\alpha)^{-A(n)},$$

where

$$A(n) = \begin{cases} \dfrac{3+\sqrt{5}}{2} & \text{if } n = 3 \\ 4 & \text{if } n = 4 \\ \left\lfloor \dfrac{n+1}{2} \right\rfloor & \text{if } n > 4. \end{cases}$$

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THEOREM 12 (Roy, 2001): Let ξ be a real number with continued fraction expansion $\xi = [0, m+1, m, m+2, m, m, m+2, \ldots]$ given by the Fibonacci word on $\{m, m+2\}$, $m \in \mathbb{Z}^+$. Then for any algebraic integer α of degree ≤ 3 we have

$$|\xi - \alpha| \gg H(\alpha)^{-\frac{3+\sqrt{5}}{2}}.$$

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$$|\xi - \alpha| \gg H(\alpha)^{-\frac{3+\sqrt{5}}{2}}.$$

THEOREM 13 (Roy, 2001): Let ξ be an extremal real number. Then there exists a constant $c > 0$ depending only on ξ such that for any real number $X > 1$ the inequalities

$$|x_0| \leq X, \quad |x_1| \leq X, \quad |x_0\xi^2 + x_1\xi + x_2| \leq cX^{-\frac{3+\sqrt{5}}{2}}$$

has a non-zero solution $(x_0, x_1, x_2) \in \mathbb{Z}^3$.

Let $p_1 = 1,\ q_1 = 0,\ q_2 = 1$ and let $p_2 \in \mathbb{R}$. Let also $a_1, a_2 \ldots \in \mathbb{R}$. Define a sequence $\{p_k/q_k\}_{k=2}^{\infty}$ as

$$p_{k+2} = a_k p_{k+1} + p_k, \quad q_{k+2} = a_k q_{k+1} + q_k \quad (k = 1, 2, \ldots).$$

THEOREM 14: Suppose $\{p_k/q_k\}_{k=2}^{\infty}$ converges. Then

$$\lim_{k\to\infty} \frac{p_k}{q_k} = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \ldots}}}.$$

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Put $P_k(x) = q_k x + p_k$ and $\xi_k = p_k/q_k$. Then

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and

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THEOREM 12′ (Roy, 2001): Let $\{a_k\} = 0, m+1, m, m+2, m, m, m+2, \ldots$ and let

$$P_{k+2} = a_k P_{k+1} + P_k \quad \text{with} \quad P_1 = 1 \quad \text{and} \quad P_2 = x.$$

Let also ξ_k be a root of P_k. Then for any algebraic integer α of degree ≤ 3 we have

$$|\xi - \alpha| \gg H(\alpha)^{-\frac{3+\sqrt{5}}{2}}, \quad \text{where} \quad \lim_{k\to\infty} \xi_k = \xi.$$

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Put $P_k(x) = q_k x + p_k$ and $\xi_k = p_k/q_k$. Then

$$P_{k+2} = a_k P_{k+1} + P_k$$

and

$$\lim_{k\to\infty} \xi_k = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \ldots}}}.$$

1	$x^2 + x - 1$	$[1, -1, -1, 1]$	0.1607132
2	$2x^2 - x$	$[1, -2, 0, 1]$	0.1900259
3	$3x^2 + 2x - 2$	$[1, -2, 0, 1]$	0.2324587
4	$4x^2 - 4x + 1$	$[1, 3, 1, 1]$	0.1221587
5	$8x^2 + 3x - 4$	$[1, -2, 0, 1]$	0.2656654
6	$5x^2 + 12x - 8$	$[1, -2, 2, 1]$	0.3249885
7	$7x^2 - 13x + 5$	$[1, 1, -1, 1]$	0.2073685
8	$20x^2 + 2x - 7$	$[1, 1, 1, 1]$	0.2668497
9	$18x^2 + 27x - 20$	$[1, -1, 1, 1]$	0.2644142
10	$9x^2 - 38x + 18$	$[1, 1, -1, 1]$	0.2847574
11	$47x^2 - 9x - 9$	$[1, 1, 1, 1]$	0.2368395
12	$56x^2 + 56x - 47$	$[1, -1, 1, 1]$	0.1828037
13	$103x^2 - 56x$	$[1, 2, 0, 1]$	0.1828035
14	$159x^2 + 103x - 103$	$[1, 2, 0, 1]$	0.2368399
15	$206x^2 - 215x + 56$	$[1, 3, -1, 1]$	0.1280084
16	$421x^2 + 150x - 206$	$[1, 2, 0, 1]$	0.2668557
17	$271x^2 + 627x - 421$	$[1, 2, -2, 1]$	0.3218088
18	$356x^2 - 692x + 271$	$[1, 1, 1, 1]$	0.2131207
19	$1048x^2 + 85x - 356$	$[1, -1, 1, 1]$	0.2657581
20	$963x^2 + 1404x - 1048$	$[1, 1, -1, 1]$	0.2593276

1	x^2+x-1	$[1,-1,-1]$	0.1607132
2	$2x-1$	$[1,-1,-1]$	0.3495121
3	$2x^2-x$	$[1,-1,-1]$	0.1900259
4	$3x^2+2x-2$	$[1,-1,-1]$	0.2324587
5	x^2+5x-3	$[1,-1,-1]$	0.3510701
6	$4x^2-4x+1$	$[1,-1,-1]$	0.1221587
7	$8x^2+3x-4$	$[1,-1,-1]$	0.2656654
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9	$7x^2-13x+5$	$[1,-1,-1]$	0.2073685
10	$20x^2+2x-7$	$[1,-1,-1]$	0.2668497
11	$18x^2+27x-20$	$[1,-1,-1]$	0.2644142
12	$9x^2-38x+18$	$[1,-1,-1]$	0.2847574
13	$47x^2-9x-9$	$[1,-1,-1]$	0.2368395
14	$56x^2+56x-47$	$[1,-1,-1]$	0.1828037
15	$-103x+56$	$[1,-1,-1]$	0.3362280
16	$103x^2-56x$	$[1,-1,-1]$	0.1828035
17	$159x^2+103x-103$	$[1,-1,-1]$	0.2368399
18	$56x^2+262x-159$	$[1,-1,-1]$	0.3496340
19	$206x^2-215x+56$	$[1,-1,-1]$	0.1280084
20	$421x^2+150x-206$	$[1,-1,-1]$	0.2668557

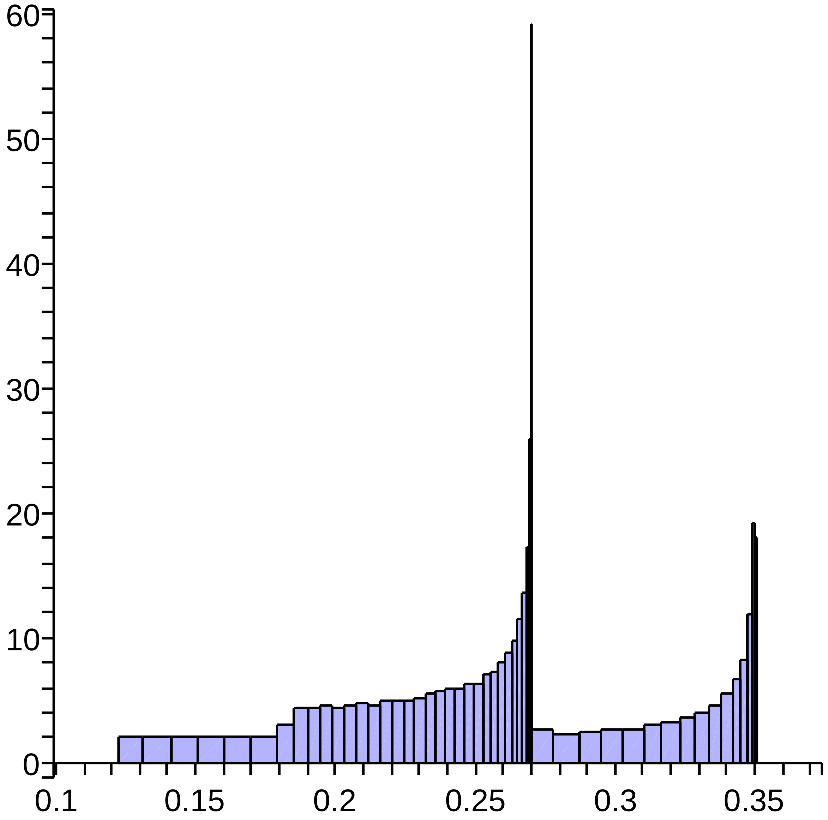
$x^2 + x - 1$	*YES*	*YES*	0
$2x - 1$	*NO*	*YES*	$HIGHT - WRONG$
$2x^2 - x$	*YES*	*YES*	0
$3x^2 + 2x - 2$	*YES*	*YES*	0
$x^2 + 5x - 3$	*NO*	*YES*	$HIGHT - WRONG$
$4x^2 - 4x + 1$	*YES*	*YES*	0
$8x^2 + 3x - 4$	*YES*	*YES*	0
$5x^2 + 12x - 8$	*YES*	*YES*	0
$7x^2 - 13x + 5$	*YES*	*YES*	0
$20x^2 + 2x - 7$	*YES*	*YES*	0
$18x^2 + 27x - 20$	*YES*	*YES*	0
$9x^2 - 38x + 18$	*YES*	*YES*	0
$47x^2 - 9x - 9$	*YES*	*YES*	0
$56x^2 + 56x - 47$	*YES*	*YES*	0
$-103x + 56$	*NO*	*YES*	$HIGHT - WRONG$
$103x^2 - 56x$	*YES*	*YES*	0
$159x^2 + 103x - 103$	*YES*	*YES*	0
$56x^2 + 262x - 159$	*NO*	*YES*	$HIGHT - WRONG$
$206x^2 - 215x + 56$	*YES*	*YES*	0
$421x^2 + 150x - 206$	*YES*	*YES*	0

$271\,x^2 + 627\,x - 421$	*YES*	*YES*	0
$356\,x^2 - 692\,x + 271$	*YES*	*YES*	0
$1048\,x^2 + 85\,x - 356$	*YES*	*YES*	0
$963\,x^2 + 1404\,x - 1048$	*YES*	*YES*	0
$441\,x^2 - 2011\,x + 963$	*YES*	*YES*	0
$2452\,x^2 - 522\,x - 441$	*YES*	*YES*	0
$2893\,x^2 - 2533\,x + 522$	*YES*	*NO*	$SUM - YES$
$2974\,x^2 + 2893\,x - 2452$	*YES*	*YES*	0
$81\,x^2 + 5426\,x - 2974$	*NO*	*YES*	$HIGHT - WRONG$
$5345\,x^2 - 3055\,x + 81$	*YES*	*YES*	0
$8400\,x^2 + 5264\,x - 5345$	*YES*	*YES*	0
$3136\,x^2 + 13745\,x - 8400$	*NO*	*YES*	$HIGHT - WRONG$
$10609\,x^2 - 11536\,x + 3136$	*YES*	*YES*	0
$22145\,x^2 + 7473\,x - 10609$	*YES*	*YES*	0
$14672\,x^2 + 32754\,x - 22145$	*YES*	*YES*	0
$18082\,x^2 - 36817\,x + 14672$	*YES*	*YES*	0
$-18082 + 54899\,x^2 + 3410\,x$	*YES*	*YES*	0
$51489\,x^2 + 72981\,x - 54899$	*YES*	*YES*	0
$21492\,x^2 - 106388\,x + 51489$	*YES*	*YES*	0

1	x^2+x-1	$[1,-1,-1,1]$	0.1478990
2	$2\,x^2-1$	$[1,-1,1,1]$	0.2754301
3	$2\,x^2+3\,x-3$	$[1,3,0,1]$	0.1968671
4	$5\,x^2+x-3$	$[1,2,0,1]$	0.2545991
5	$8\,x^2-4\,x-1$	$[1,-2,2,1]$	0.3034471
6	$4\,x^2+9\,x-8$	$[1,2,-1,1]$	0.2620481
7	$12\,x^2+5\,x-9$	$[0,1,1,1]$	0.2168926
8	$21\,x^2-7\,x-5$	$[1,2,0,1]$	0.3092480
9	$7\,x^2+26\,x-21$	$[1,2,-1,1]$	0.3234507
10	$28\,x^2+19\,x-26$	$[0,1,1,1]$	0.1746480
11	$54\,x^2-9\,x-19$	$[1,2,0,1]$	0.3024274
12	$63\,x^2+64\,x-73$	$[1,2,0,1]$	0.1755733
13	$136\,x^2+x-64$	$[1,2,0,1]$	0.2837114
14	$200\,x^2-135\,x-1$	$[1,-2,2,1]$	0.2856572
15	$135\,x^2+201\,x-200$	$[1,2,-1,1]$	0.1968658
16	$335\,x^2+66\,x-201$	$[0,1,1,1]$	0.2545974
17	$536\,x^2-269\,x-66$	$[1,2,0,1]$	0.3034450
18	$269\,x^2+602\,x-536$	$[1,2,-1,1]$	0.2611780
19	$805\,x^2+333\,x-602$	$[0,1,1,1]$	0.2174310
20	$1407\,x^2-472\,x-333$	$[1,2,0,1]$	0.3092459

1	$-x+1$	$[1,-1,0]$	0.3176722
2	x^2-x	$[1,-1,0]$	0.2167566
3	x^2+x-1	$[1,-1,0]$	0.1478990
4	$x^2-2\,x+1$	$[1,-1,0]$	0.4036625
5	$2\,x^2-1$	$[1,-1,0]$	0.2754301
6	$3\,x-2$	$[1,-1,0]$	0.4228507
7	$3\,x^2-2\,x$	$[1,-1,0]$	0.2885228
8	$2\,x^2+3\,x-3$	$[1,-1,0]$	0.1968671
9	$3\,x^2-5\,x+2$	$[1,-1,0]$	0.3731331
10	$5\,x^2+x-3$	$[1,-1,0]$	0.2545991
11	$x^2-8\,x+5$	$[1,-1,0]$	0.4447233
12	$8\,x^2-4\,x-1$	$[1,-1,0]$	0.3034471
13	$4\,x^2+9\,x-8$	$[1,-1,0]$	0.2620481
14	$9\,x^2-12\,x+4$	$[1,-1,0]$	0.3178715
15	$12\,x^2+5\,x-9$	$[1,-1,0]$	0.2168926
16	$5\,x^2-21\,x+12$	$[1,-1,0]$	0.4532250
17	$21\,x^2-7\,x-5$	$[1,-1,0]$	0.3092480
18	$7\,x^2+26\,x-21$	$[1,-1,0]$	0.3234507
19	$26\,x^2-28\,x+7$	$[1,-1,0]$	0.2559591
20	$28\,x^2+19\,x-26$	$[1,-1,0]$	0.1746480

$-x+1$	*NO*	*YES*	*HIGHT − WRONG*
x^2-x	*NO*	*YES*	*HIGHT − WRONG*
x^2+x-1	*YES*	*YES*	0
x^2-2x+1	*NO*	*YES*	*HIGHT − WRONG*
$2x^2-1$	*YES*	*YES*	0
$3x-2$	*NO*	*YES*	*HIGHT − WRONG*
$3x^2-2x$	*NO*	*YES*	*HIGHT − WRONG*
$2x^2+3x-3$	*YES*	*YES*	0
$3x^2-5x+2$	*NO*	*YES*	*HIGHT − WRONG*
$5x^2+x-3$	*YES*	*YES*	0
x^2-8x+5	*NO*	*YES*	*HIGHT − WRONG*
$8x^2-4x-1$	*YES*	*YES*	0
$4x^2+9x-8$	*YES*	*YES*	0
$9x^2-12x+4$	*NO*	*YES*	*HIGHT − WRONG*
$12x^2+5x-9$	*YES*	*YES*	0
$5x^2-21x+12$	*NO*	*YES*	*HIGHT − WRONG*
$21x^2-7x-5$	*YES*	*YES*	0
$7x^2+26x-21$	*YES*	*YES*	0
$26x^2-28x+7$	*NO*	*YES*	*HIGHT − WRONG*



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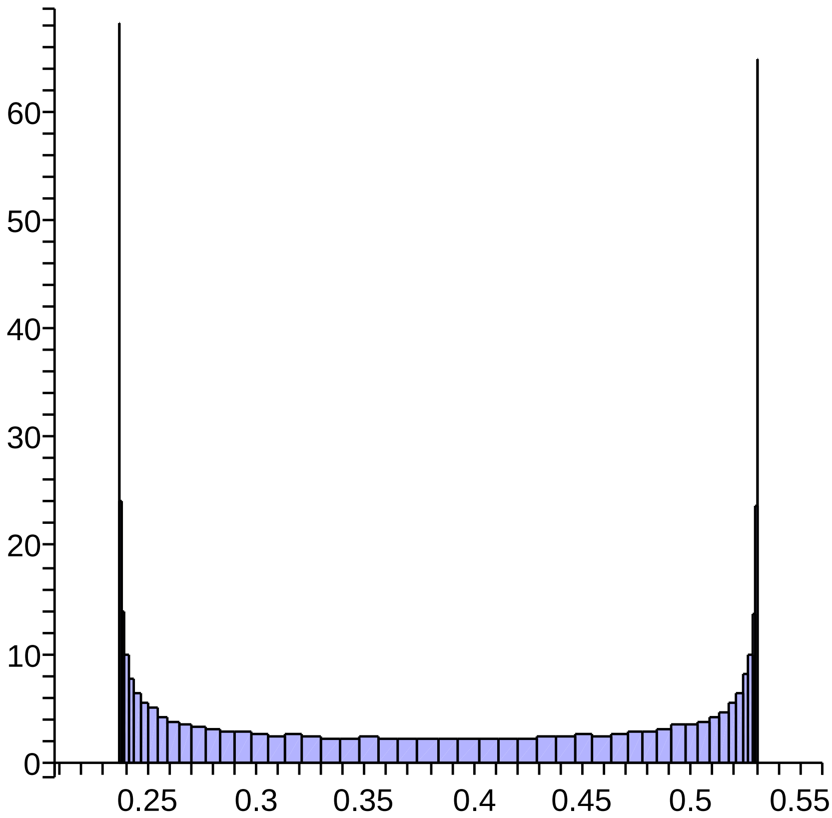

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