Teachers’ mistakes as a teaching tool

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Calculus I

Calculus I

We have:

We have:

$$\lim_{x \to -\infty} (\sqrt{x^2 + x} - x)$$

We have:

$$\lim_{x\to-\infty}(\sqrt{x^2+x}-x)=\lim_{x\to-\infty}\frac{(\sqrt{x^2+x}-x)(\sqrt{x^2+x}+x)}{\sqrt{x^2+x}+x}$$

We have:

$$\lim_{x \to -\infty} (\sqrt{x^2 + x} - x) = \lim_{x \to -\infty} \frac{(\sqrt{x^2 + x} - x)(\sqrt{x^2 + x} + x)}{\sqrt{x^2 + x} + x}$$
$$= \lim_{x \to -\infty} \frac{x^2 + x - x^2}{\sqrt{x^2 + x} + x}$$

Calculus I

We have:

$$\begin{aligned}
\lim_{x\to-\infty}(\sqrt{x^2+x}-x) &= \lim_{x\to-\infty}\frac{(\sqrt{x^2+x}-x)(\sqrt{x^2+x}+x)}{\sqrt{x^2+x}+x} \\
&= \lim_{x\to-\infty}\frac{x^2+x-x^2}{\sqrt{x^2+x}+x} \\
&= \lim_{x\to-\infty}\frac{x}{\sqrt{x^2+x}+x}
\end{aligned}$$

Calculus I

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\lim_{x\to-\infty}(\sqrt{x^2+x}-x) &= \lim_{x\to-\infty}\frac{(\sqrt{x^2+x}-x)(\sqrt{x^2+x}+x)}{\sqrt{x^2+x}+x} \\
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&= \lim_{x\to-\infty}\frac{x}{\sqrt{x^2+x}+x} \\
&= \lim_{x\to-\infty}\frac{x}{x\sqrt{1+1/x}+x}
\end{aligned}
$$

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&= \lim_{x\to-\infty}\frac{1}{\sqrt{1+1/x}+1}=\frac{1}{2}
\end{aligned}$$

We have:

$$\lim_{x\to-\infty}(\sqrt{x^2+x}-x) = \lim_{x\to-\infty}\frac{(\sqrt{x^2+x}-x)(\sqrt{x^2+x}+x)}{\sqrt{x^2+x}+x}$$

$$f(x)=\sqrt{x^2+x}-x$$

$$= \lim_{x\to-\infty}\frac{x^2+x-x^2}{\sqrt{x^2+x}+x}$$

$$= \lim_{x\to-\infty}\frac{x}{\sqrt{x^2+x}+x}$$

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$f(x) = \sqrt{x^2+x}-x$

$$= \lim_{x\to-\infty}\frac{x^2+x-x^2}{\sqrt{x^2+x}+x}$$

$f(-100)$

$$= \lim_{x\to-\infty}\frac{x}{\sqrt{x^2+x}+x}$$

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$$f(x)=\sqrt{x^2+x}-x \qquad = \lim_{x\to-\infty}\frac{x^2+x-x^2}{\sqrt{x^2+x}+x}$$

$$f(-100)\approx 199.5 \qquad = \lim_{x\to-\infty}\frac{x}{\sqrt{x^2+x}+x}$$

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$f(x)=\sqrt{x^2+x}-x$

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$f(-1000)\approx 1999.5$

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$f(x) = \sqrt{x^2+x}-x$

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$f(-10000) \approx 19999.5$

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Calculus I

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&= \lim_{x\to-\infty}\frac{1}{-\sqrt{1+1/x}+1} = +\infty
\end{aligned}$$

Calculus I

THEOREM: If $b > 0,\ b \neq 1,\ a > 0,\ c > 0$, and r is any real number, then:

- $\log_b(ac) = \log_b(a) + \log_b(c)$
- $\log_b(a/c) = \log_b(a) - \log_b(c)$
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$$\boxed{f(x \pm y) = f(x) \pm f(y)}$$

THEOREM: If $b > 0,\ b \neq 1,\ a > 0,\ c > 0$, and r is any real number, then:

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$$\boxed{f(x \pm y) \stackrel{?}{=} f(x) \pm f(y)}$$

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- $\log_b(a^r) = r\log_b(a)$
- $\log_b(1/c) = -\log_b(c)$

$$\boxed{\sqrt{x \pm y} \neq \sqrt{x} \pm \sqrt{y}}$$

THEOREM: If $b > 0,\ b \neq 1,\ a > 0,\ c > 0$, and r is any real number, then:

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$$\boxed{(x \pm y)^2 \neq x^2 \pm y^2}$$

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- $\log_b(a^r) = r\log_b(a)$
- $\log_b(1/c) = -\log_b(c)$

$$\boxed{\mathbf{\sin(x \pm y) \neq \sin(x) \pm \sin(y)}}$$

THEOREM: If $b > 0,\ b \neq 1,\ a > 0,\ c > 0$, and r is any real number, then:

- $\cancel{\log_b(a+c) = \log_b(a) + \log_b(c)}$
  $\log_b(ac) = \log_b(a) + \log_b(c)$
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- $\log_b(a^r) = r\log_b(a)$
- $\log_b(1/c) = -\log_b(c)$

$$\boxed{\mathbf{cos}(x \pm y) \neq \mathbf{cos}(x) \pm \mathbf{cos}(y)}$$

THEOREM: If $b > 0,\ b \neq 1,\ a > 0,\ c > 0$, and r is any real number, then:

- $\cancel{\log_b(a+c) = \log_b(a) + \log_b(c)}$
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$$\boxed{\mathbf{\tan(\boldsymbol{x} \pm \boldsymbol{y}) \neq \tan(\boldsymbol{x}) \pm \tan(\boldsymbol{y})}}$$

Calculus I

Calculus I

We have:

We have:

$$\int \sin 2x dx$$

Calculus I

We have:

$$\int \sin 2x dx \;= \int 2 \sin x \cos x \, dx$$

Calculus I

We have:

$$\begin{aligned} \int \sin 2x dx &= \int 2 \sin x \cos x \, dx \\ &= \left[\sin x = u \right] \end{aligned}$$

Calculus I

We have:

$$\int \sin 2x dx = \int 2 \sin x \cos x \, dx$$

$$= \left[\begin{array}{c} \sin x = u \\ d \sin x = du \end{array} \right]$$

Calculus I

We have:

$$\int \sin 2x dx = \int 2 \sin x \cos x \, dx$$

$$= \begin{bmatrix} \sin x = u \\ d \sin x = du \\ \cos x \, dx = du \end{bmatrix}$$

Calculus I

We have:

$$\int \sin 2x dx = \int 2\sin x \cos x \, dx$$

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We have:

$$\begin{aligned}\int \sin 2x dx &= \int 2\sin x\cos x\, dx \\ &= \begin{bmatrix} \sin x = u \\ d\sin x = du \\ \cos x\, dx = du \end{bmatrix} \\ &= \int 2u du\end{aligned}$$

We have:

$$\begin{aligned}\int \sin 2x dx &= \int 2\sin x \cos x \, dx \\ &= \begin{bmatrix} \sin x = u \\ d\sin x = du \\ \cos x \, dx = du \end{bmatrix} \\ &= \int 2u du \\ &= u^2 + C\end{aligned}$$

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\int \sin 2x dx &= \int 2\sin x \cos x \, dx \\
&= \left[\begin{array}{c} \sin x = u \\ d \sin x = du \\ \cos x \, dx = du \end{array} \right] \\
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$$\int \sin 2x dx$$

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\end{aligned}$$

We have:

$$\int \sin 2x dx = \frac{1}{2}\int 2\sin 2x dx$$

$$\begin{aligned}\int \sin 2x dx &= \int 2\sin x \cos x \, dx \\ &= \left[\begin{array}{c} \sin x = u \\ d\sin x = du \\ \cos x \, dx = du \end{array}\right] \\ &= \int 2u du \\ &= u^2 + C \\ &= \sin^2 x + C\end{aligned}$$

Calculus I

We have:

$$\int \sin 2x dx = \frac{1}{2}\int 2\sin 2x dx$$

$$= \left[\, 2x = u \,\right]$$

$$\int \sin 2x dx = \int 2\sin x\cos x\, dx$$

$$= \left[\begin{array}{c} \sin x = u \\ d\sin x = du \\ \cos x\, dx = du \end{array}\right]$$

$$= \int 2u du$$

$$= u^2 + C$$

$$= \sin^2 x + C$$

Calculus I

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$$\int \sin 2x dx = \frac{1}{2}\int 2\sin 2x dx = \begin{bmatrix} 2x = u \\ d2x = du \end{bmatrix}$$

$$\begin{aligned} \int \sin 2x dx &= \int 2\sin x \cos x \, dx \\ &= \begin{bmatrix} \sin x = u \\ d\sin x = du \\ \cos x \, dx = du \end{bmatrix} \\ &= \int 2u du \\ &= u^2 + C \\ &= \sin^2 x + C \end{aligned}$$

Calculus I

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$$\int \sin 2x dx = \frac{1}{2}\int 2\sin 2x dx = \begin{bmatrix} 2x = u \\ d2x = du \\ 2dx = du \end{bmatrix}$$

$$\int \sin 2x dx = \int 2\sin x\cos x\, dx = \begin{bmatrix} \sin x = u \\ d\sin x = du \\ \cos x\, dx = du \end{bmatrix} = \int 2u du = u^2 + C = \sin^2 x + C$$

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We have:

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$$\int \sin 2x dx = \int 2\sin x \cos x \, dx$$

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$$= \int 2u du$$

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$$\int \sin 2x dx = \int 2\sin x\cos x\, dx = \begin{bmatrix} \sin x = u \\ d\sin x = du \\ \cos x\, dx = du \end{bmatrix} = \int 2u du = u^2 + C = \sin^2 x + C$$

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We have:

$$\begin{aligned}
\int \sin 2x dx &= \frac{1}{2}\int 2\sin 2x dx \\
&= \begin{bmatrix} 2x = u \\ d2x = du \\ 2dx = du \end{bmatrix} \\
&= \frac{1}{2}\int \sin u du \\
&= -\frac{1}{2}\cos u + C
\end{aligned}$$

$$\begin{aligned}
\int \sin 2x dx &= \int 2\sin x \cos x \, dx \\
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&= u^2 + C \\
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\end{aligned}$$

We have:

$$\begin{aligned}\int \sin 2x dx &= \frac{1}{2}\int 2\sin 2x dx \\ &= \begin{bmatrix} 2x = u \\ d2x = du \\ 2dx = du \end{bmatrix} \\ &= \frac{1}{2}\int \sin u du \\ &= -\frac{1}{2}\cos u + C \\ &= -\frac{1}{2}\cos 2x + C\end{aligned}$$

$$\begin{aligned}\int \sin 2x dx &= \int 2\sin x \cos x \, dx \\ &= \begin{bmatrix} \sin x = u \\ d\sin x = du \\ \cos x \, dx = du \end{bmatrix} \\ &= \int 2u du \\ &= u^2 + C \\ &= \sin^2 x + C\end{aligned}$$

Calculus I

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2. $-\frac{1}{2}\cos 2x = \sin^2 x$.

Calculus I

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1. There is a mistake. Where???
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Calculus I

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Calculus I

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Probability

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College Algebra

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College Algebra

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$$= (-1)^{\frac{1}{6}2+\frac{1}{6}2+\frac{1}{6}2}$$

We have:

$$
\begin{aligned}
-1 = (-1)^1 &= (-1)^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} \\
&= (-1)^{\frac{1}{6}2+\frac{1}{6}2+\frac{1}{6}2} \\
&= (-1)^{\frac{1}{6}2}(-1)^{\frac{1}{6}2}(-1)^{\frac{1}{6}2}
\end{aligned}
$$

We have:

$$\begin{aligned} -1 = (-1)^1 &= (-1)^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} \\ &= (-1)^{\frac{1}{6}2+\frac{1}{6}2+\frac{1}{6}2} \\ &= (-1)^{\frac{1}{6}2}(-1)^{\frac{1}{6}2}(-1)^{\frac{1}{6}2} \\ &= (-1)^{2\frac{1}{6}}(-1)^{2\frac{1}{6}}(-1)^{2\frac{1}{6}} \end{aligned}$$

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&= (-1)^{2\frac{1}{6}}(-1)^{2\frac{1}{6}}(-1)^{2\frac{1}{6}} \\
&= \left[(-1)^2\right]^{\frac{1}{6}}\left[(-1)^2\right]^{\frac{1}{6}}\left[(-1)^2\right]^{\frac{1}{6}}
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&= \left[(-1)^2\right]^{\frac{1}{6}}\left[(-1)^2\right]^{\frac{1}{6}}\left[(-1)^2\right]^{\frac{1}{6}} \\
&= 1^{\frac{1}{6}}1^{\frac{1}{6}}1^{\frac{1}{6}}
\end{aligned}
$$

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Calculus III

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PROOF (ELEMENTARY, SHORT): Put

$$f(x_1, \ldots, x_n) = \frac{x_1 + \ldots + x_n}{n} - \sqrt[n]{x_1 \ldots x_n}$$

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THEOREM (Extreme-Value Theorem):

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THEOREM (Extreme-Value Theorem): If $f(x_1, \ldots, x_n)$ is continuous on a closed and bounded set R, then f has both an absolute maximum and an absolute minimum on R.

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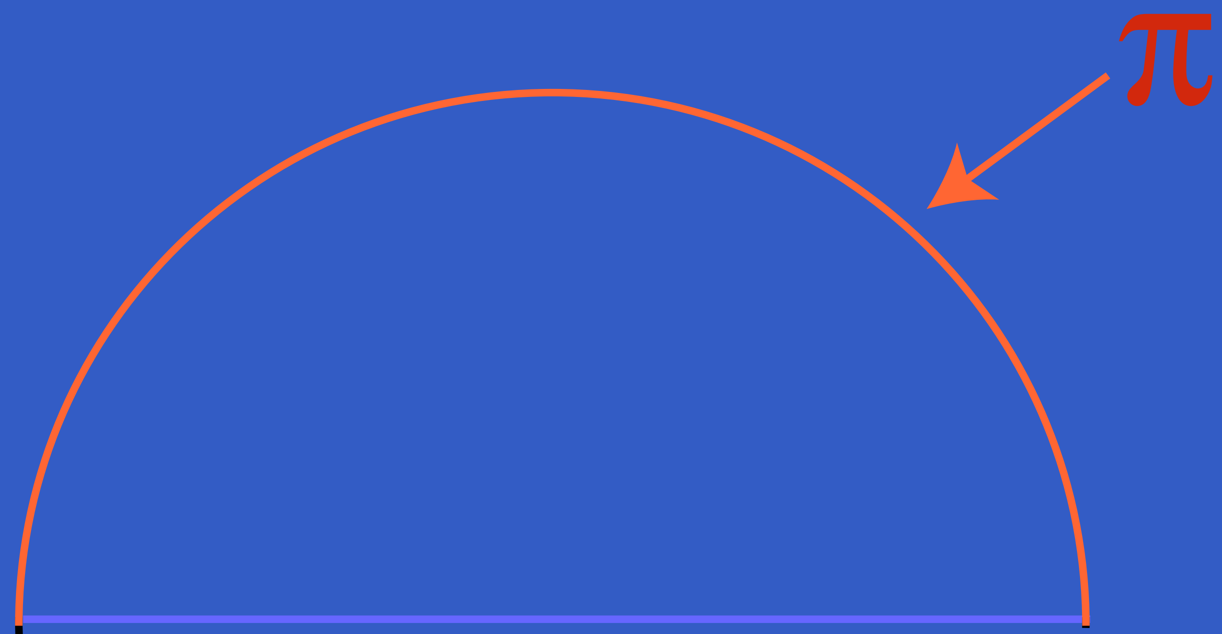
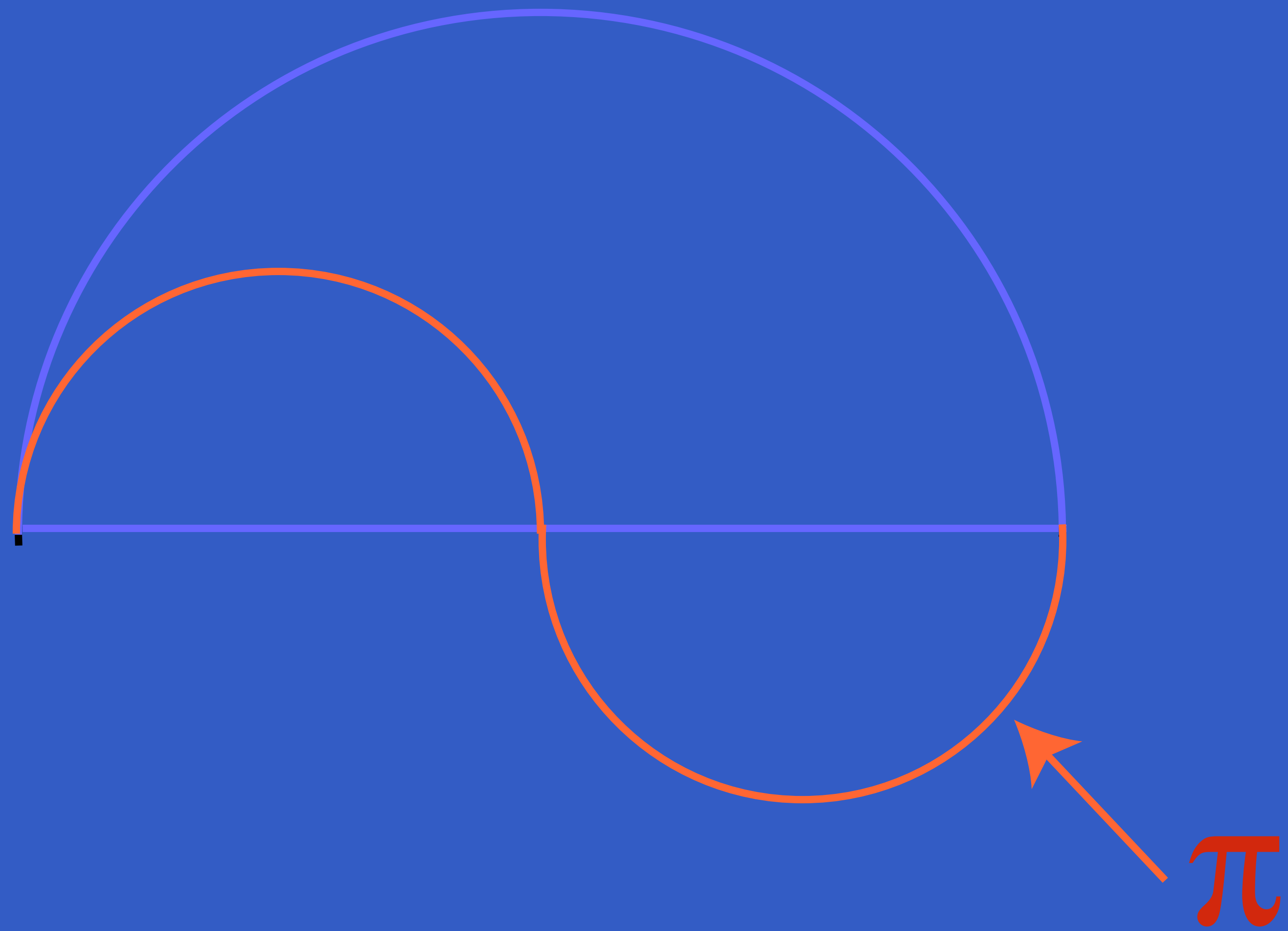
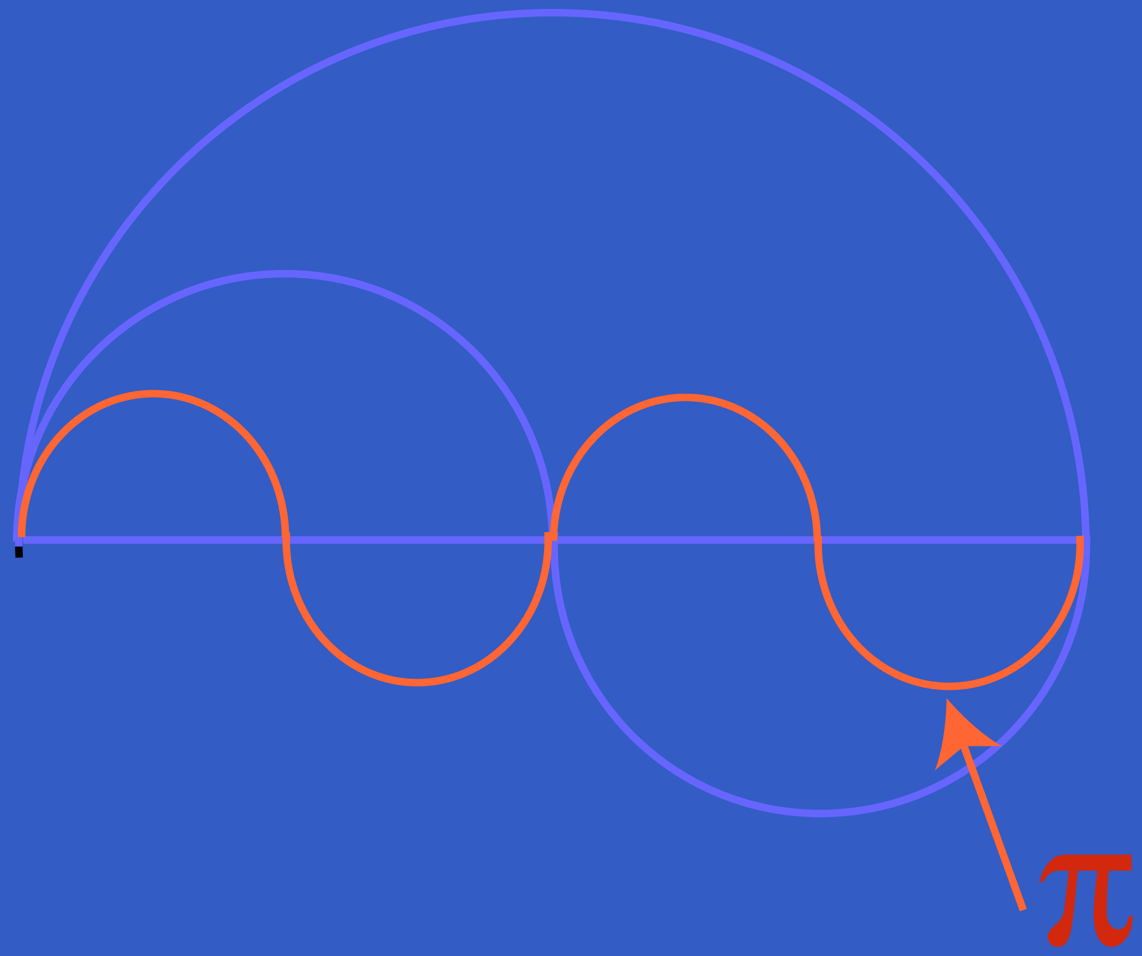
Calculus III


π

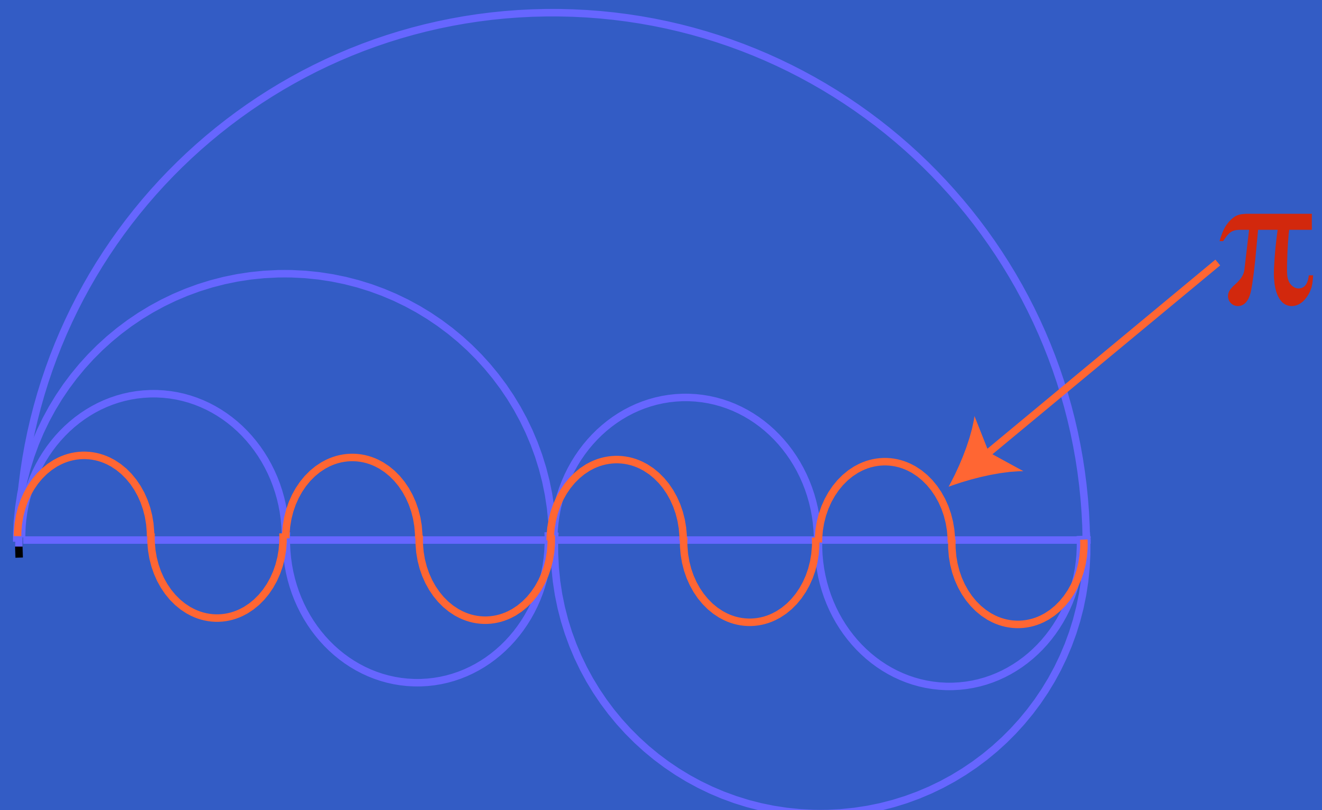
Calculus III



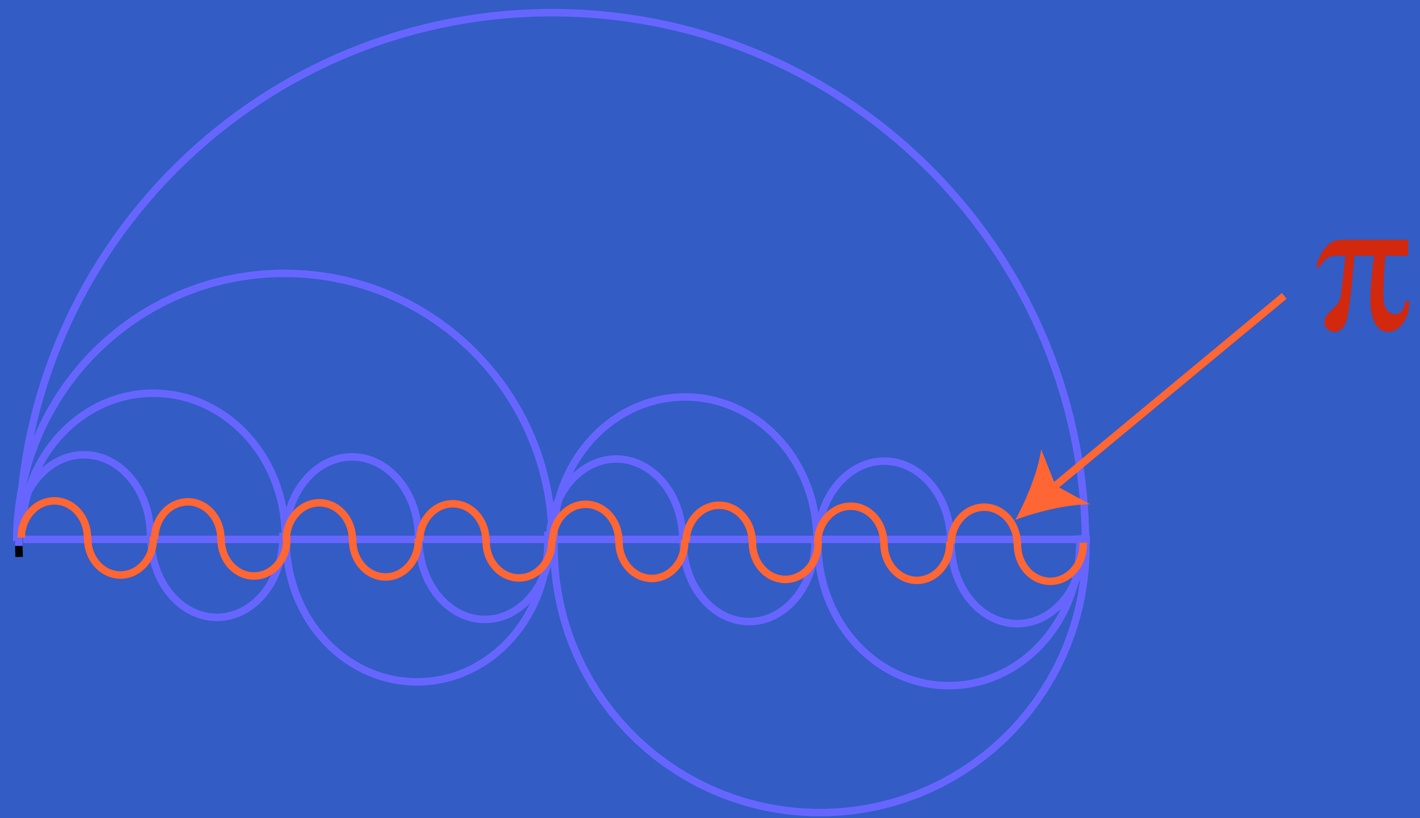
Calculus III



Calculus III



Calculus III



Calculus III