THE P U Z Z L I N G BEAUTY OF NUMBERS

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A little known verse of the Bible reads

And he made a molten sea, ten cubits from the one brim to the other: it was round all about, and his height was five cubits: and a line of thirty cubits did compass it about. (I King 7:23)

Bible	**550?BCE**	**1 place**	**3**

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Bellard	**1996**	**400.000.000.000 places**	

Wallis: $$\frac{\pi}{2} = \frac{2 \cdot 2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8 \cdot 8 \cdot 10 \cdot 10 \ldots}{1 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7 \cdot 9 \cdot 9 \cdot 11 \ldots}$$

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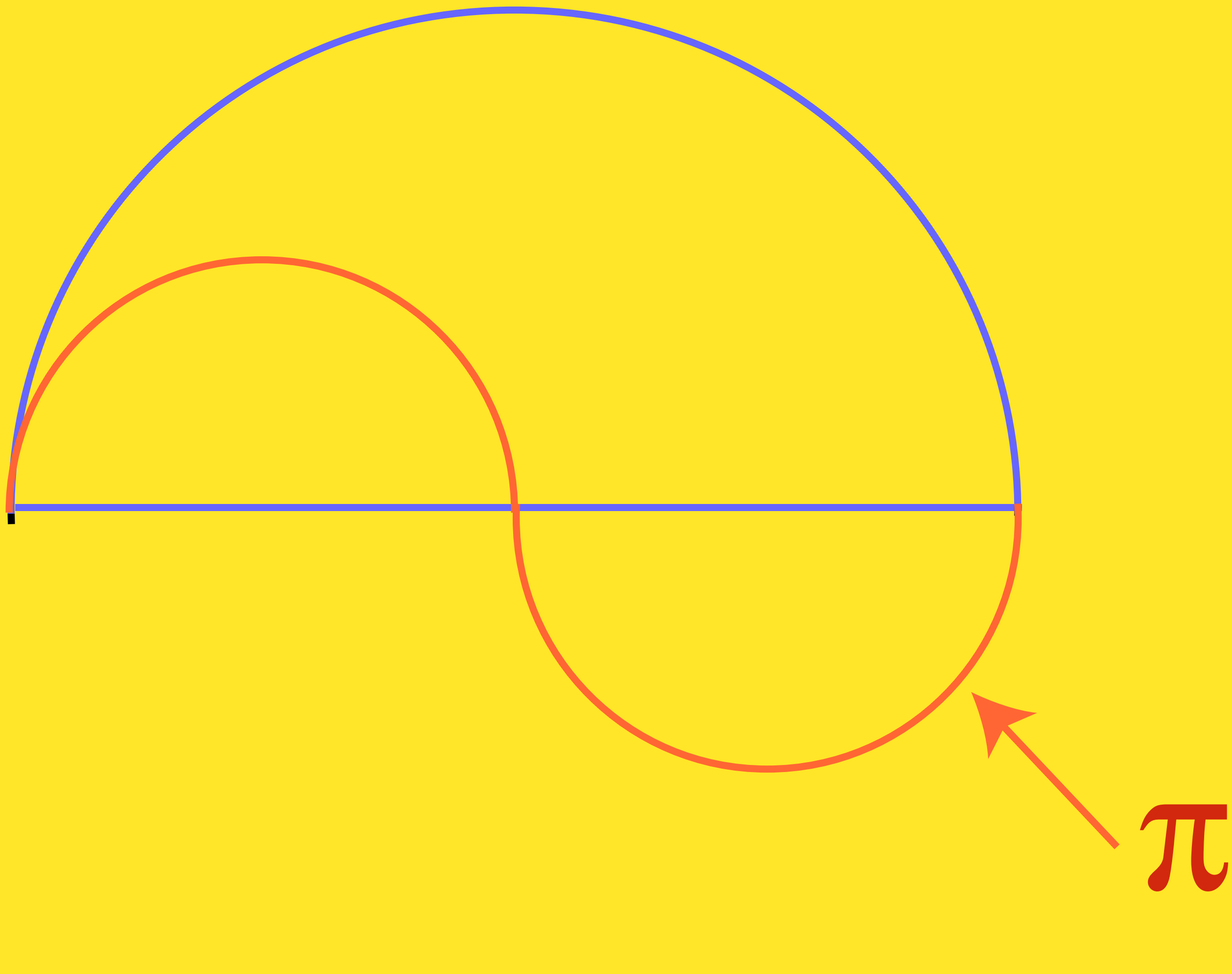
Leibniz: $$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \ldots$$


π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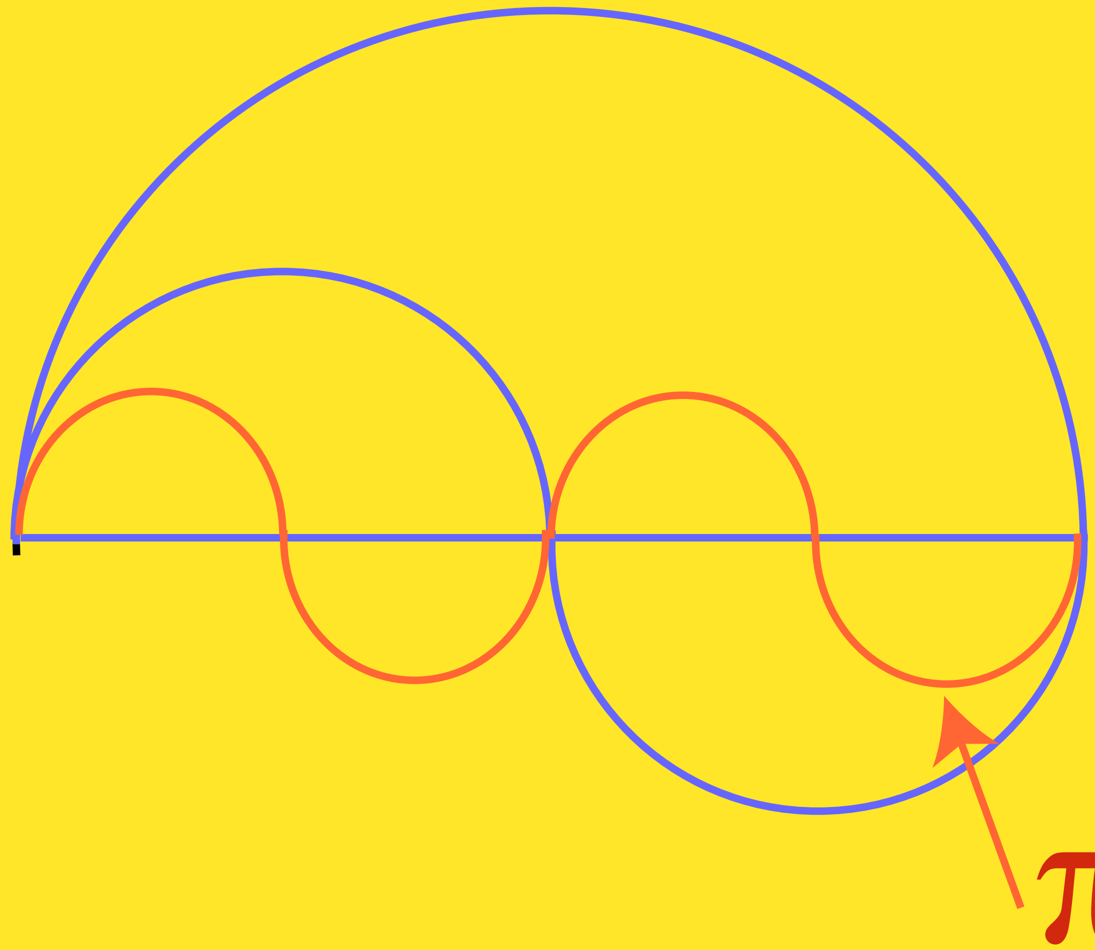
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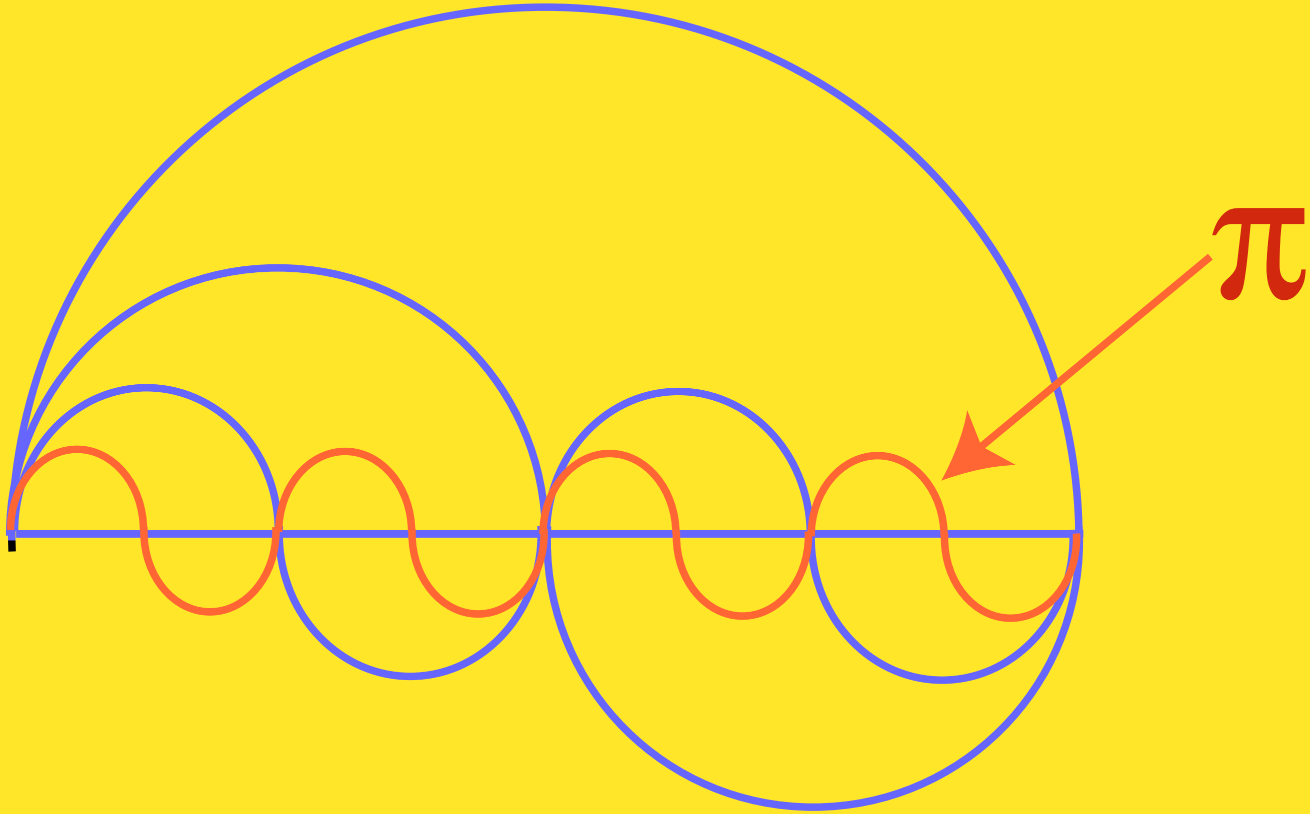
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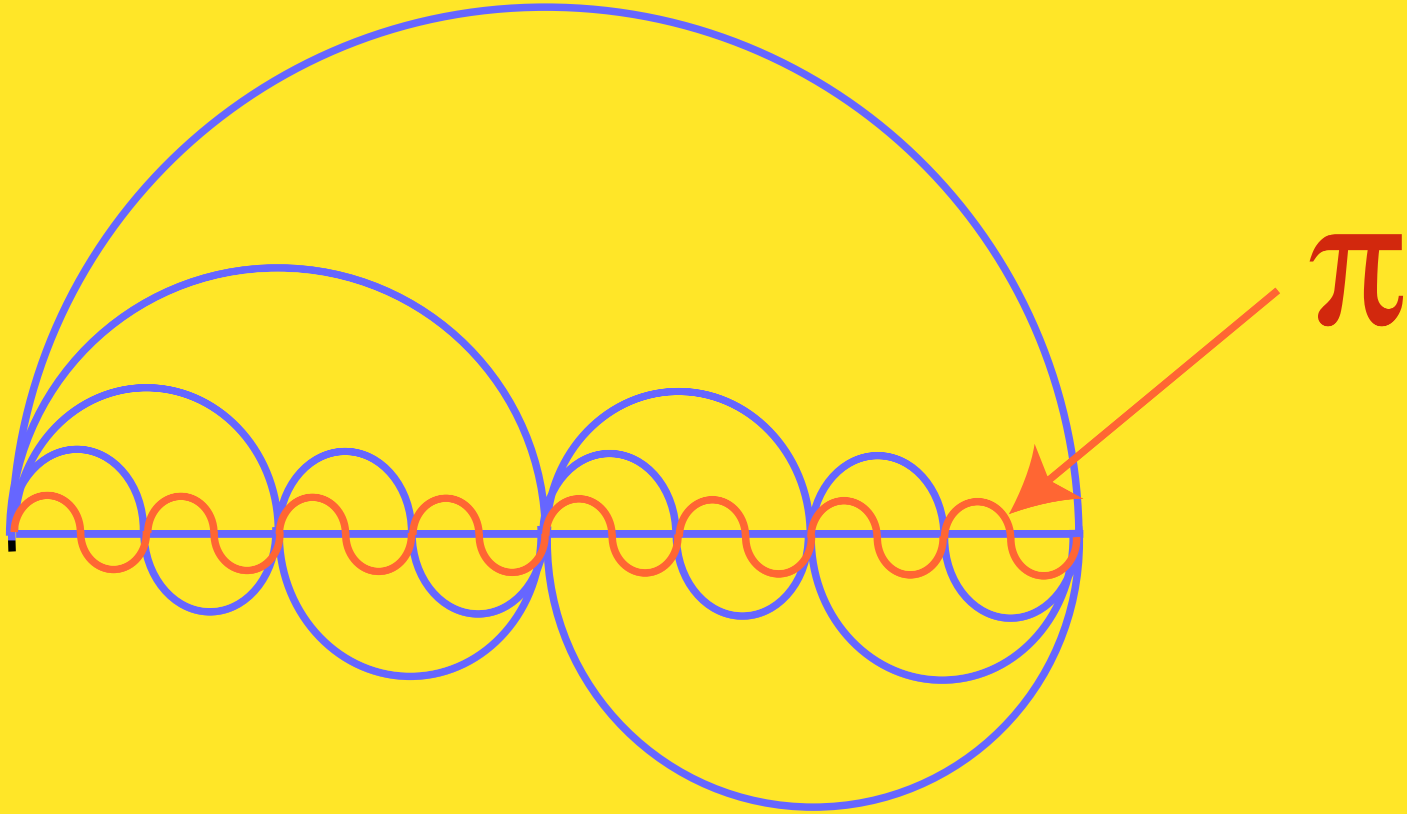
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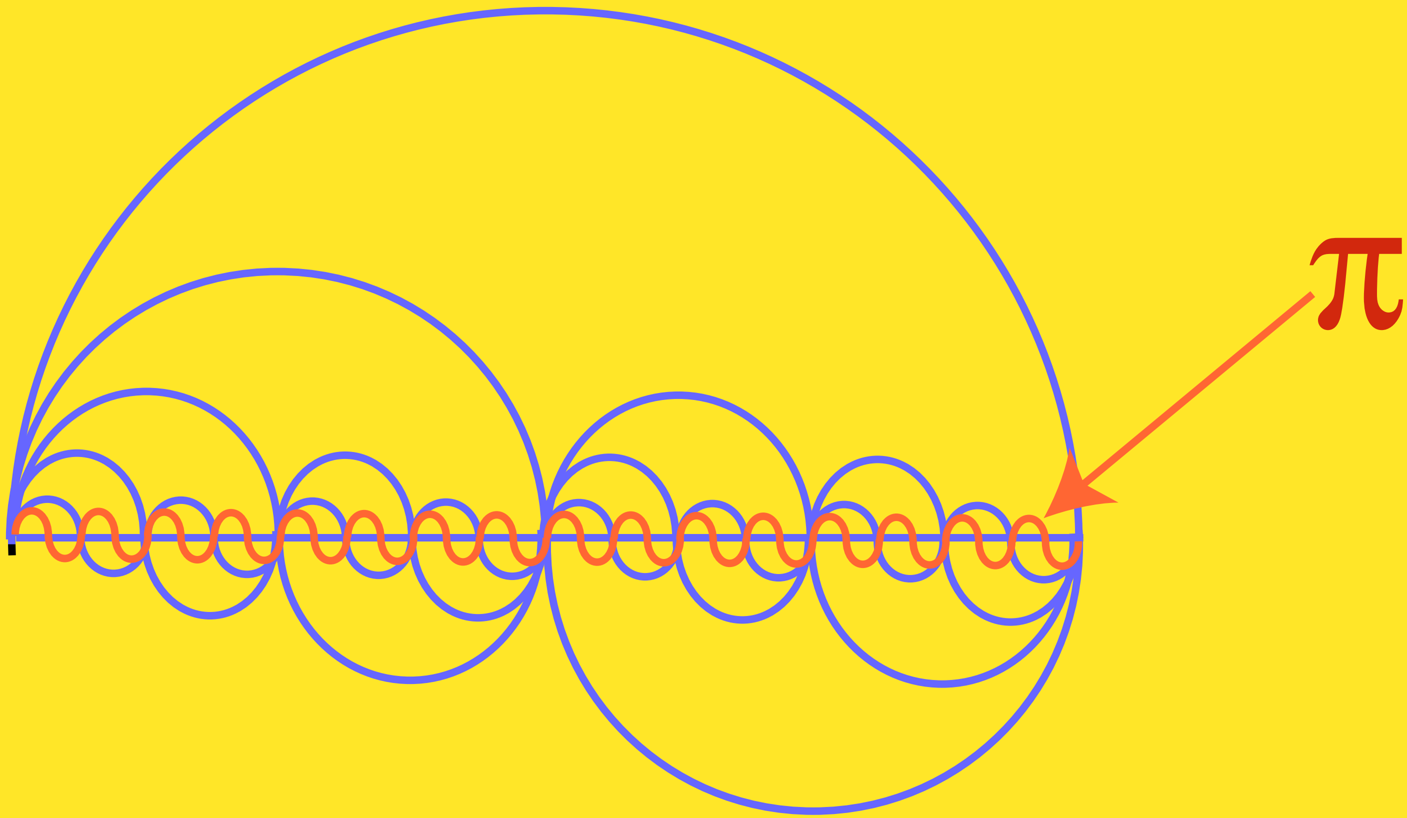
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1. You can't confuse e with a food product.

-1

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$$\begin{aligned} -1 = (-1)^1 &= (-1)^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} \\ &= (-1)^{\frac{1}{6}\cdot 2+\frac{1}{6}\cdot 2+\frac{1}{6}\cdot 2} \\ &= (-1)^{\frac{1}{6}\cdot 2}(-1)^{\frac{1}{6}\cdot 2}(-1)^{\frac{1}{6}\cdot 2} \end{aligned}$$

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DEFINITION:

Suppose that a is a nonzero real number and that m and n are integers such that $n > 0$. Then

$$a^{m/n} = (a^m)^{1/n}\,,$$

provided $a^{1/n}$ exists.

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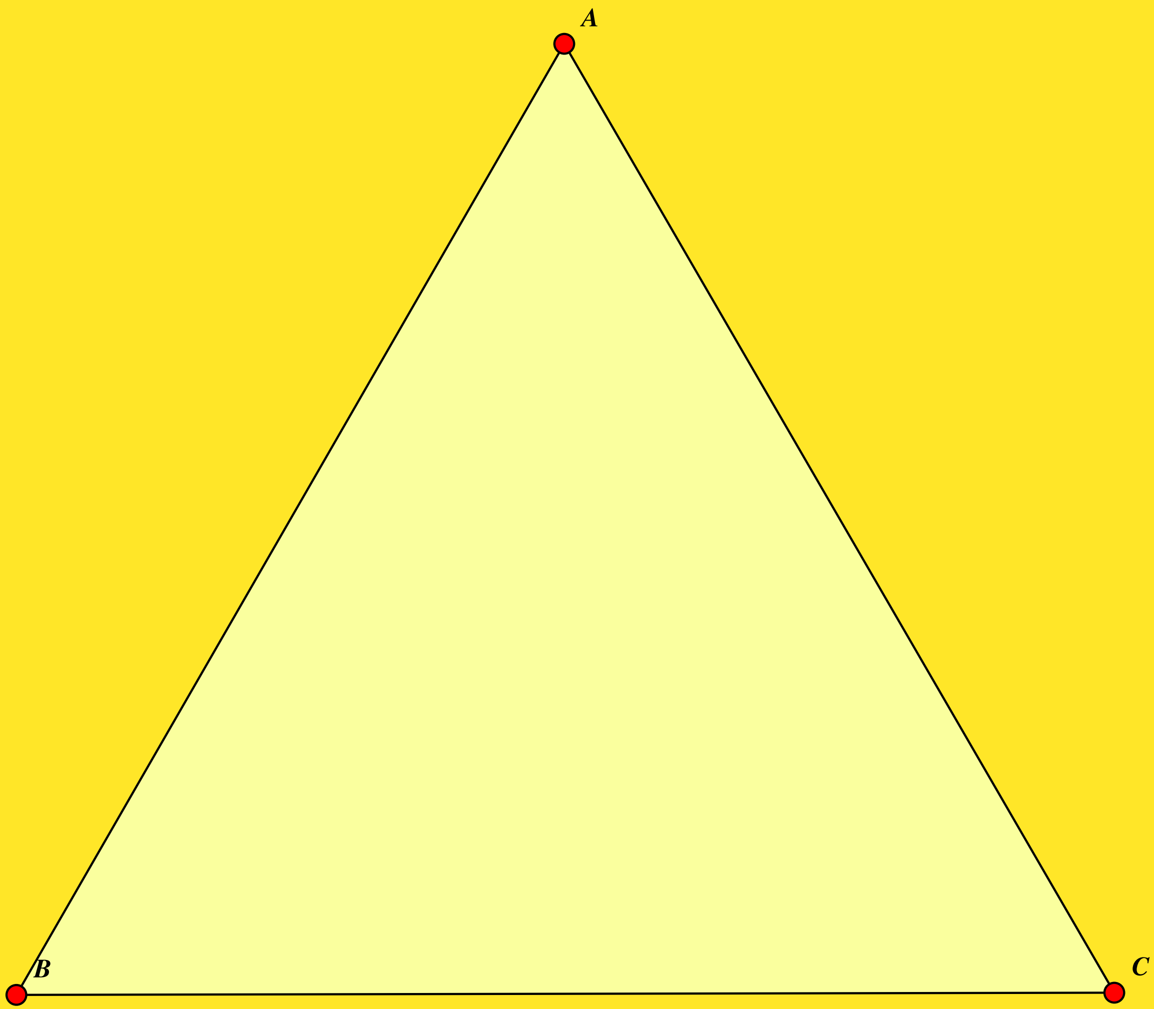
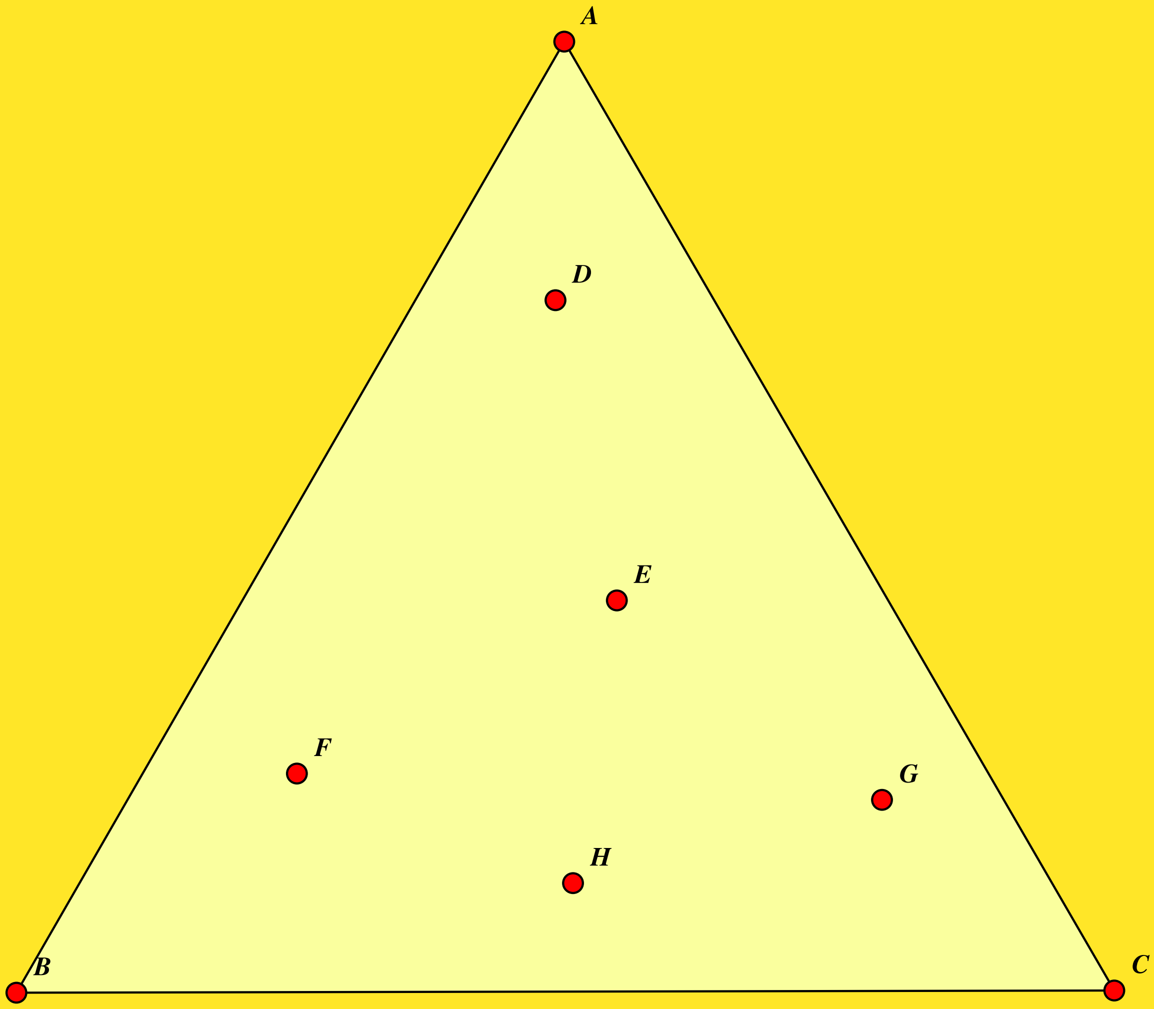
EXAMPLE:

$$(-1)^{2\cdot\frac{1}{6}}(-1)^{2\cdot\frac{1}{6}}(-1)^{2\cdot\frac{1}{6}} \neq \left[(-1)^2\right]^{\frac{1}{6}}\left[(-1)^2\right]^{\frac{1}{6}}\left[(-1)^2\right]^{\frac{1}{6}},$$

since $(-1)^{\frac{1}{6}}$ does not exist.


A
B
C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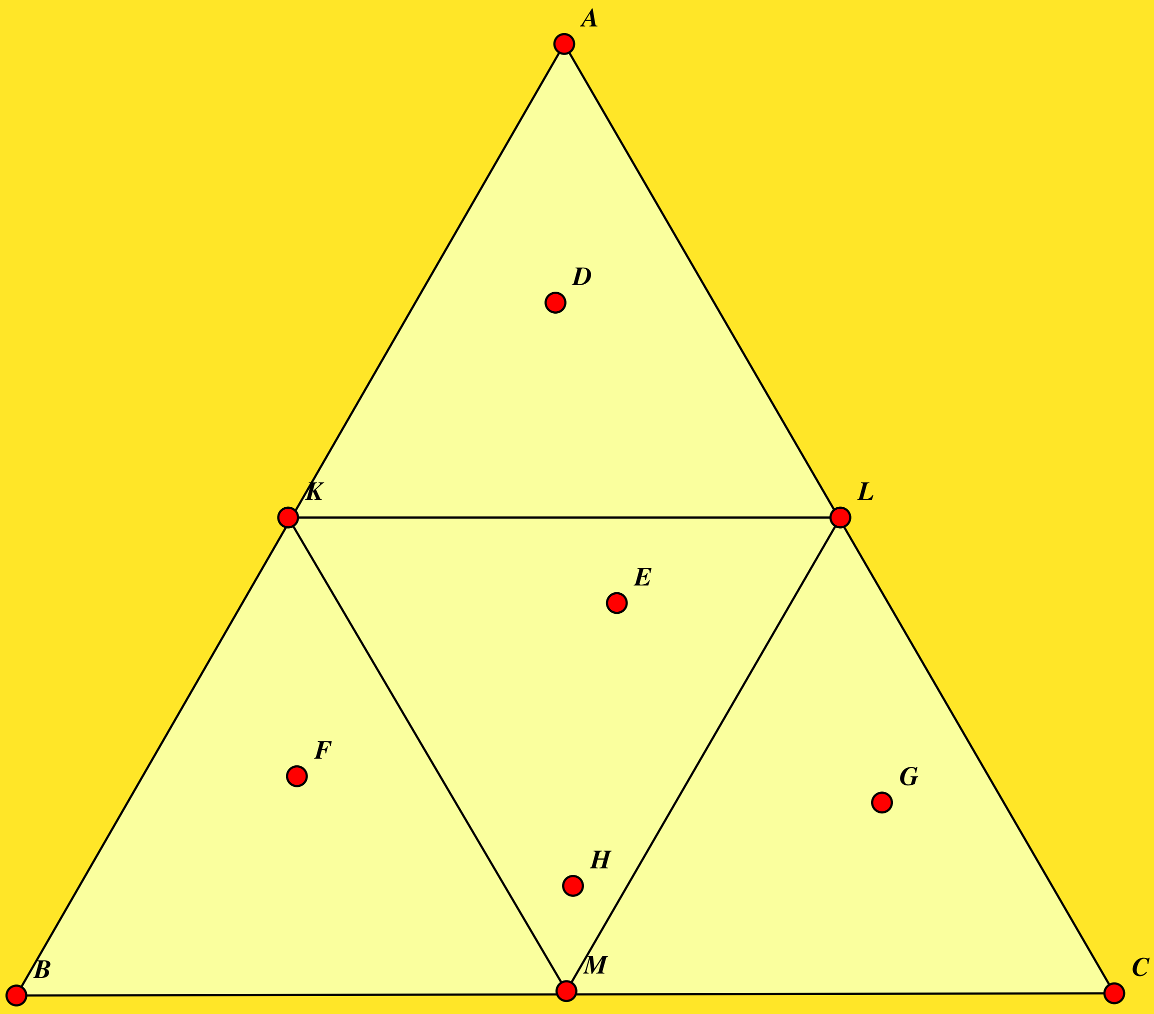
A
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A plane is colored blue and red. Is it always possible to find two points of the same color exactly 1 inch apart?

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At least two of its vertices have to be of the same color.

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SOLUTION:

Think of an equilateral triangle with each side exactly 1 inch long. At least two of its vertices have to be of the same color. This proves that there have to be two points of the same color exactly 1 inch apart.

PROBLEM:

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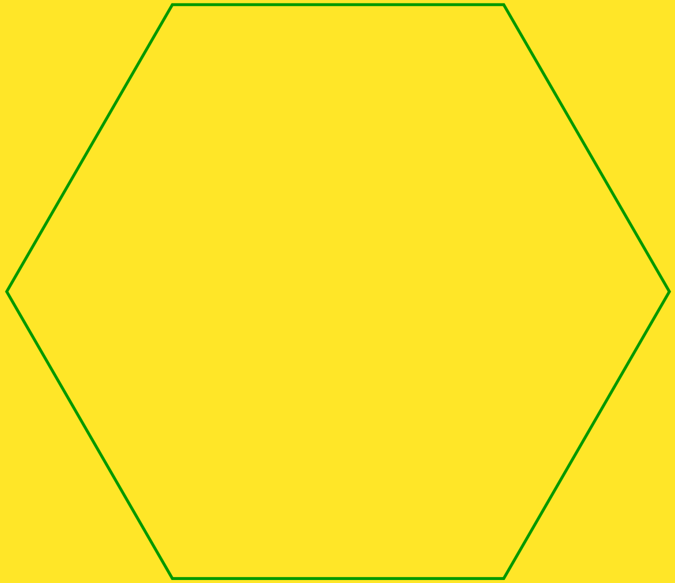
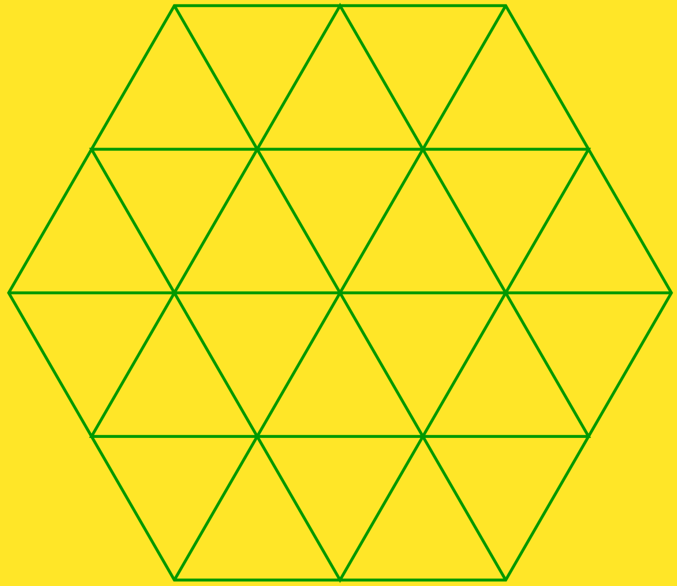
A right hexagon



PROBLEM:

A right hexagon is split into 24 triangles.



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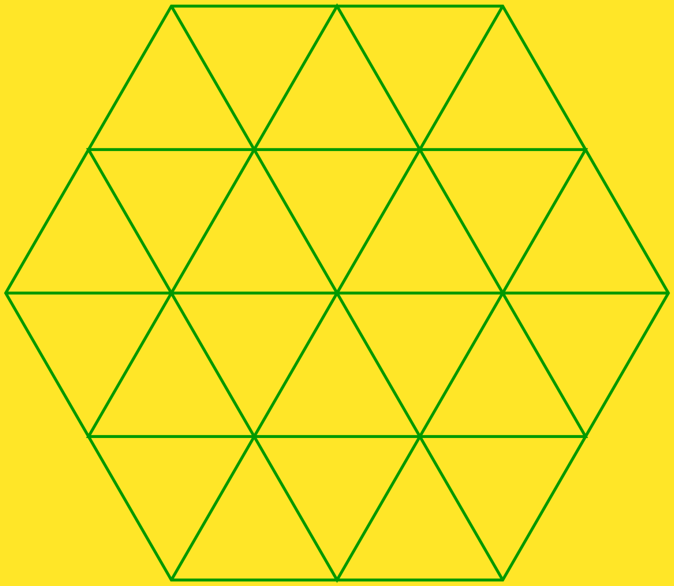

19 different numbers are written in 19 nodes.

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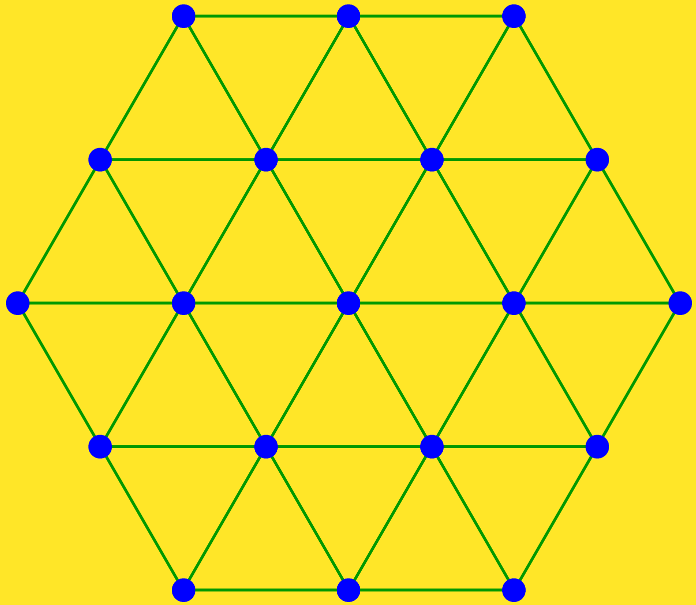

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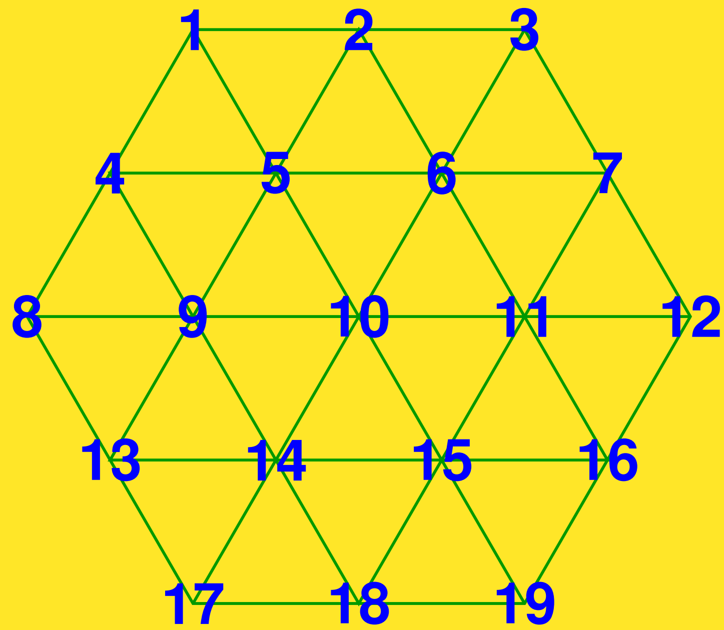

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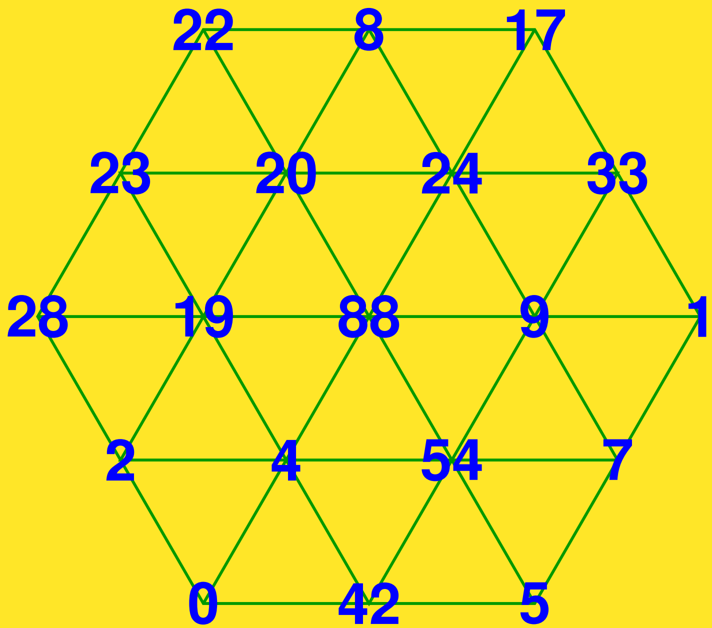

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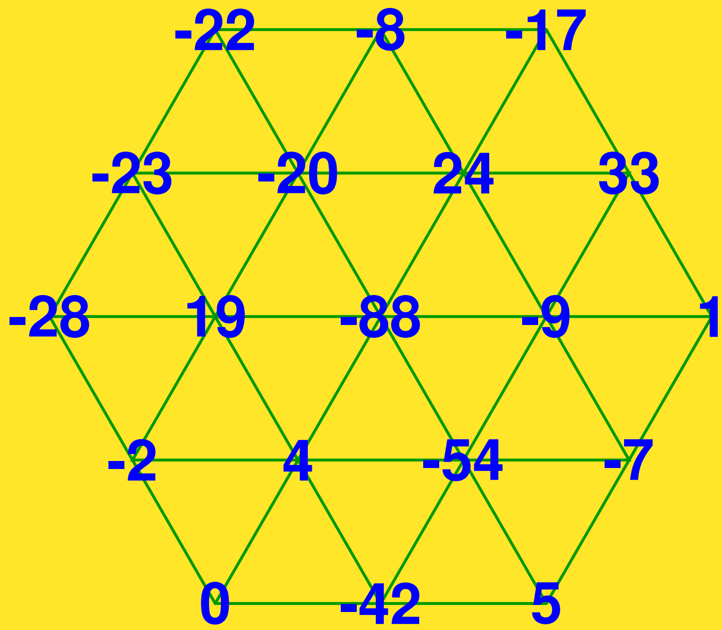

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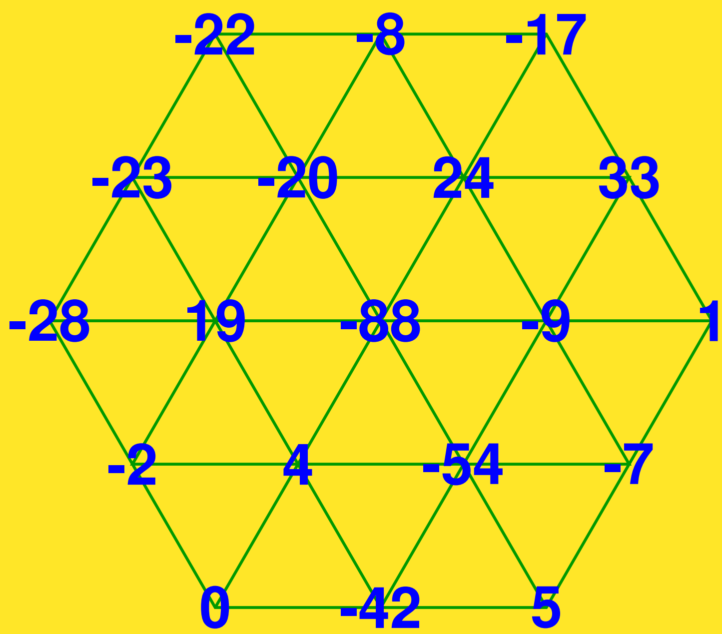

19 different numbers are written in 19 nodes. Prove that at least 7 of 24 triangles have the following property:

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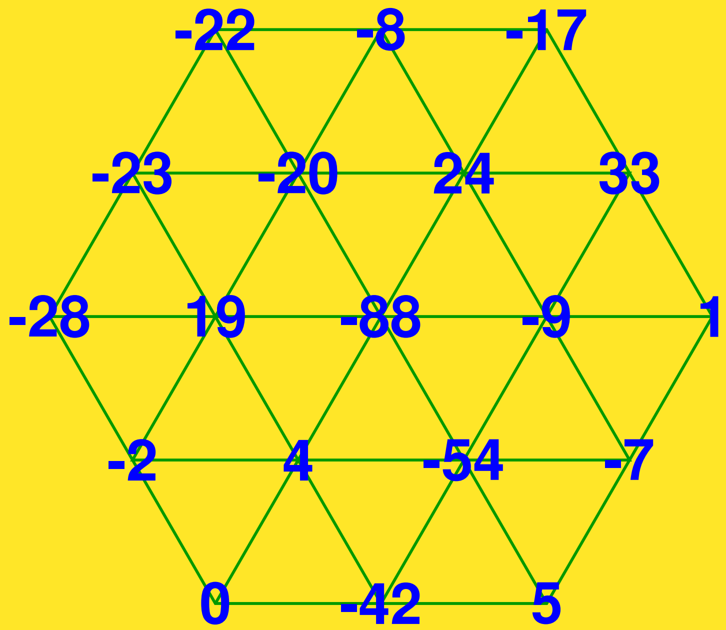

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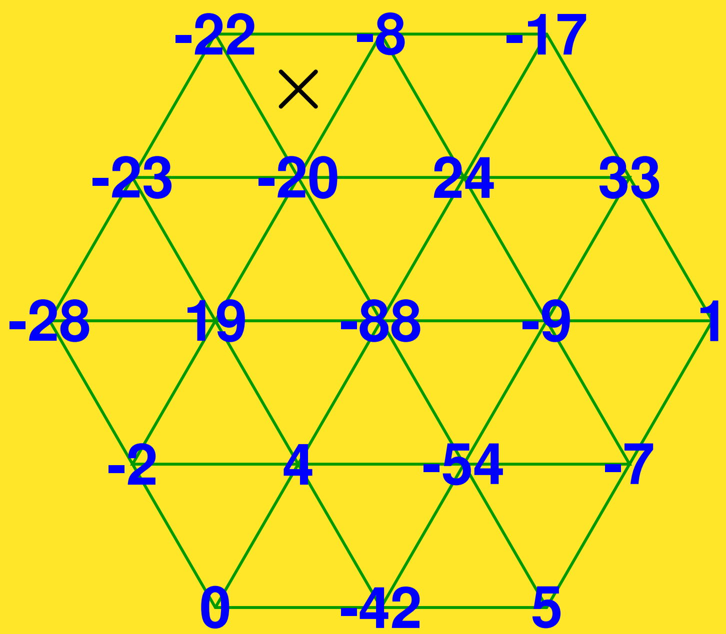

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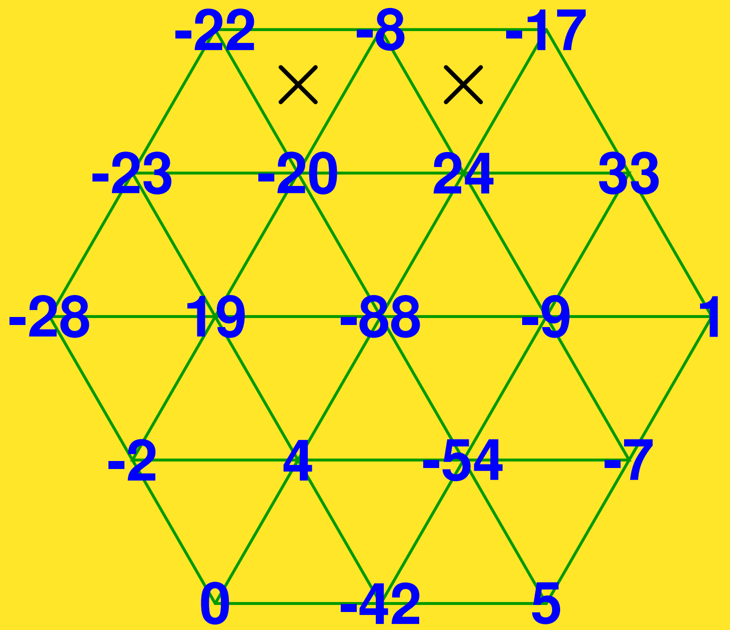

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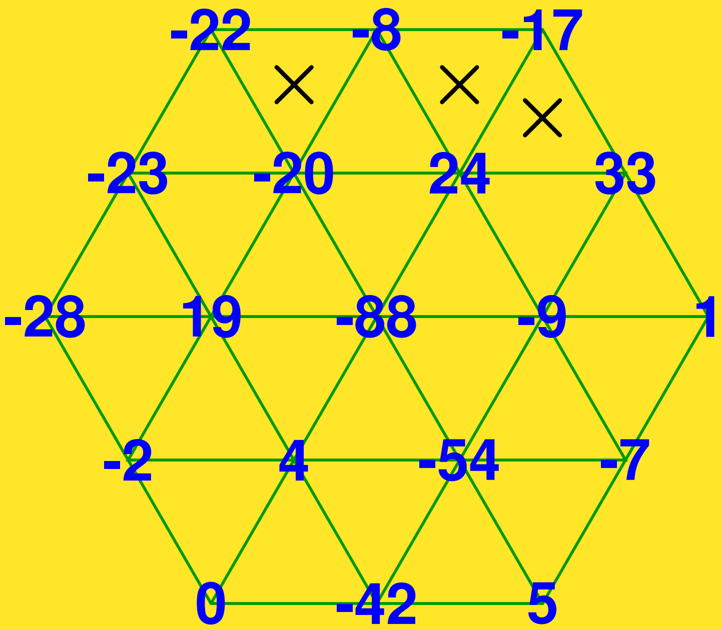

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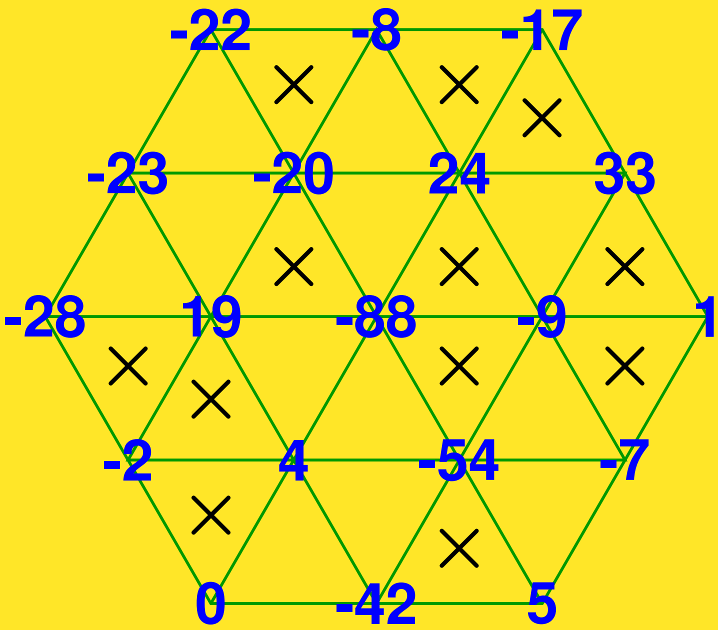

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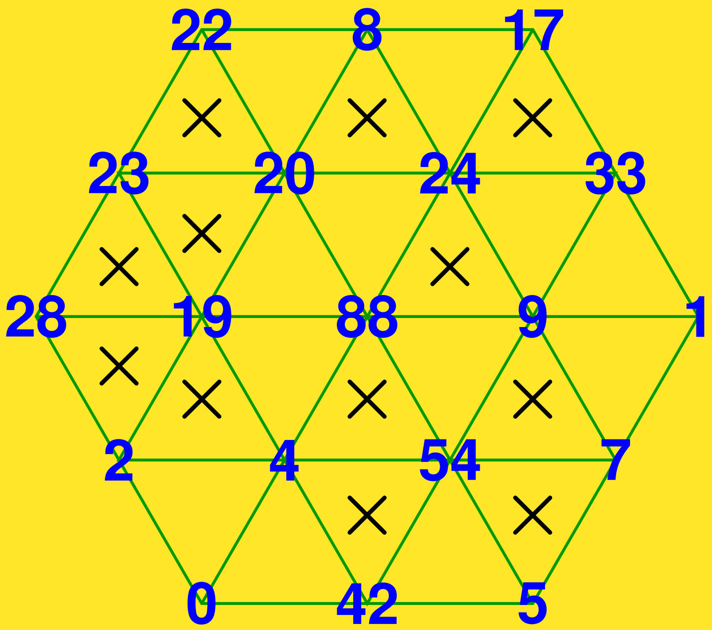

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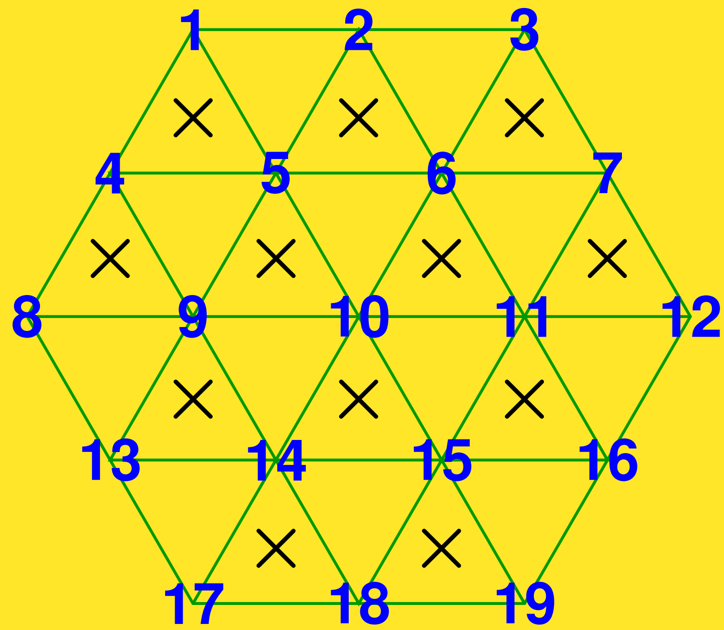

19 different numbers are written in 19 nodes. Prove that at least 7 of 24 triangles have the following property: The numbers in its vertices increase (from the least to the greatest) counterclockwise.

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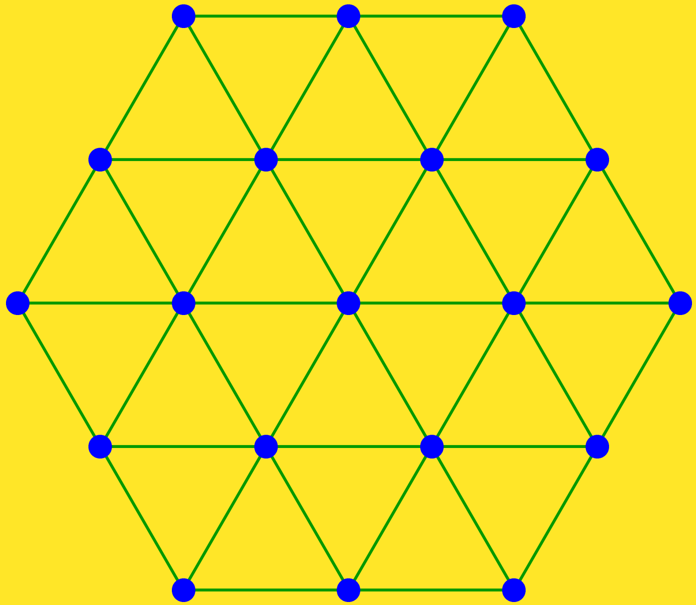

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Math problems?

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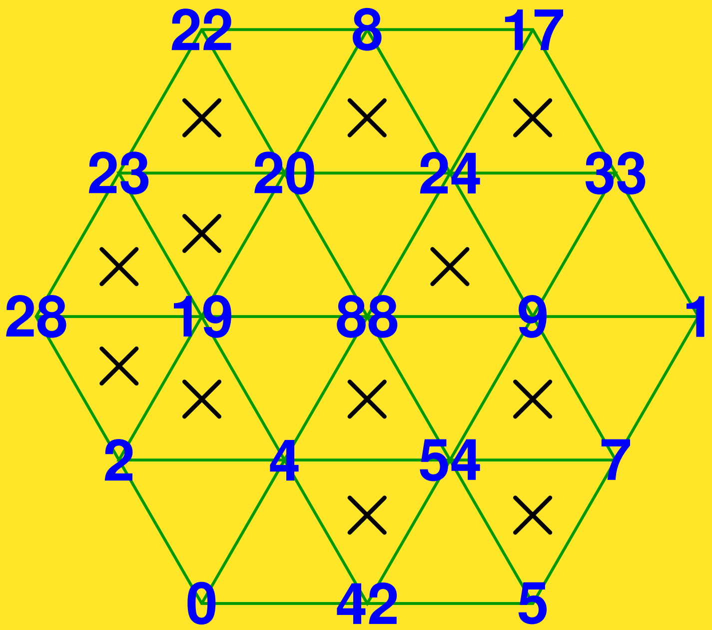

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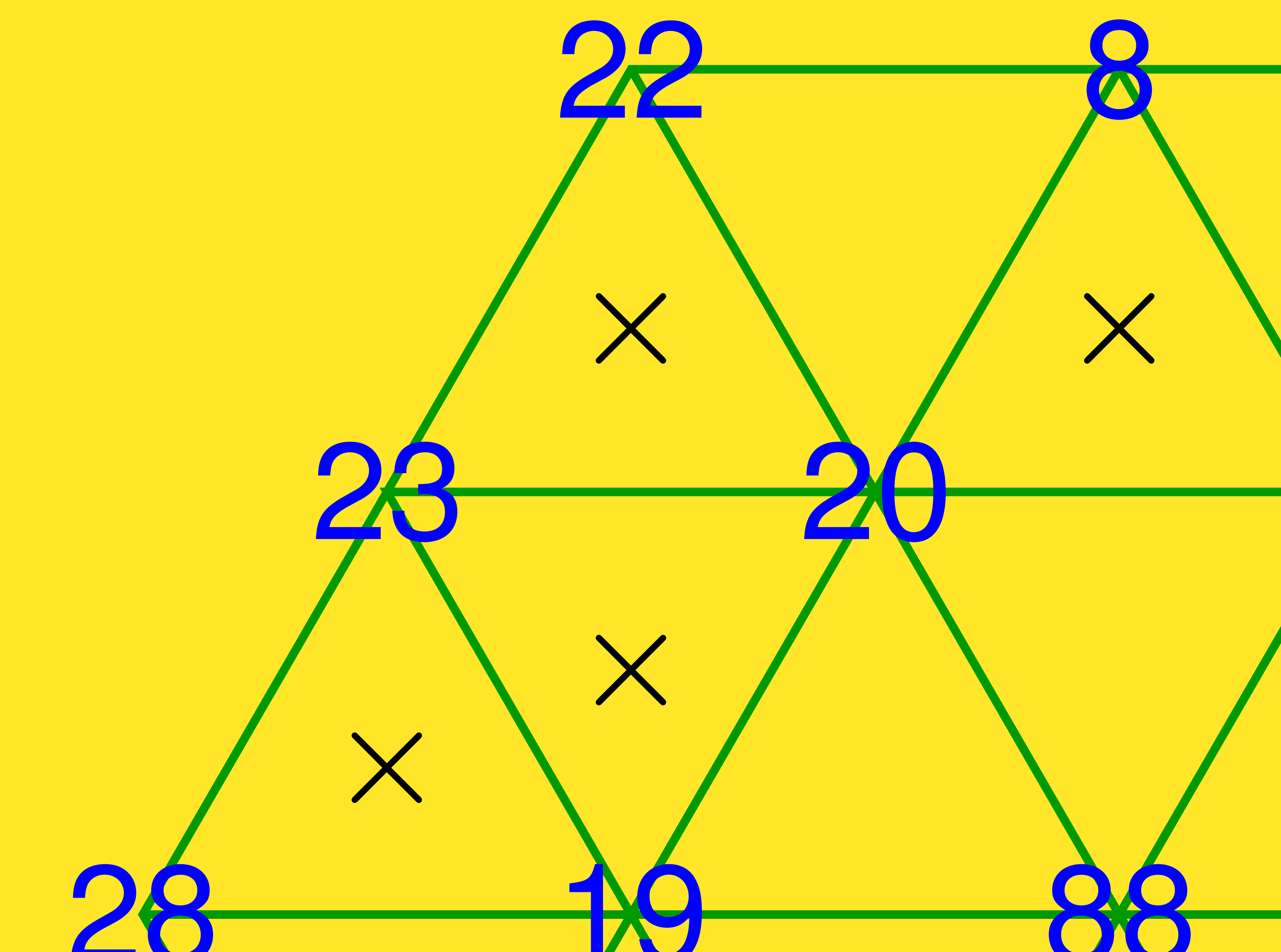
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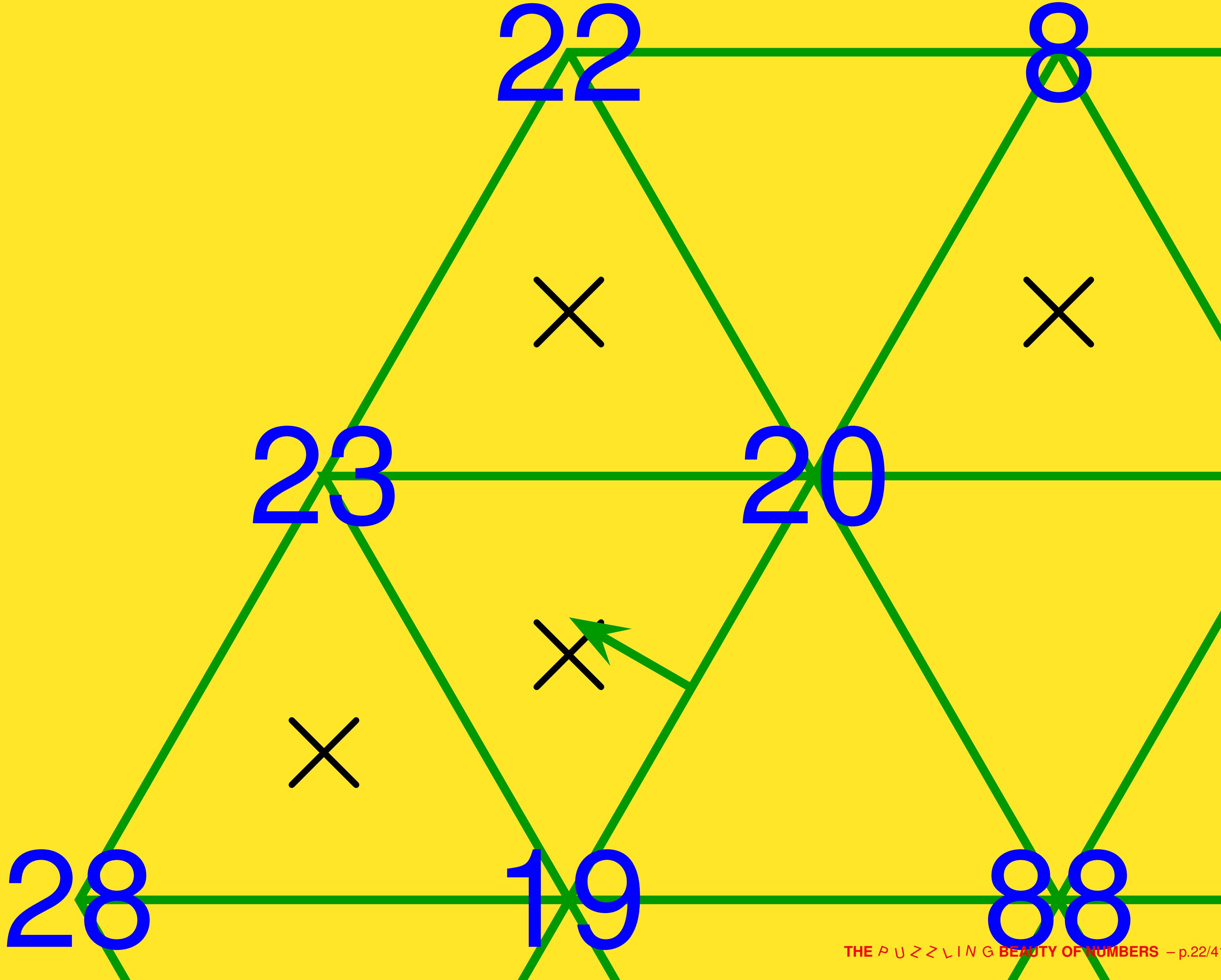
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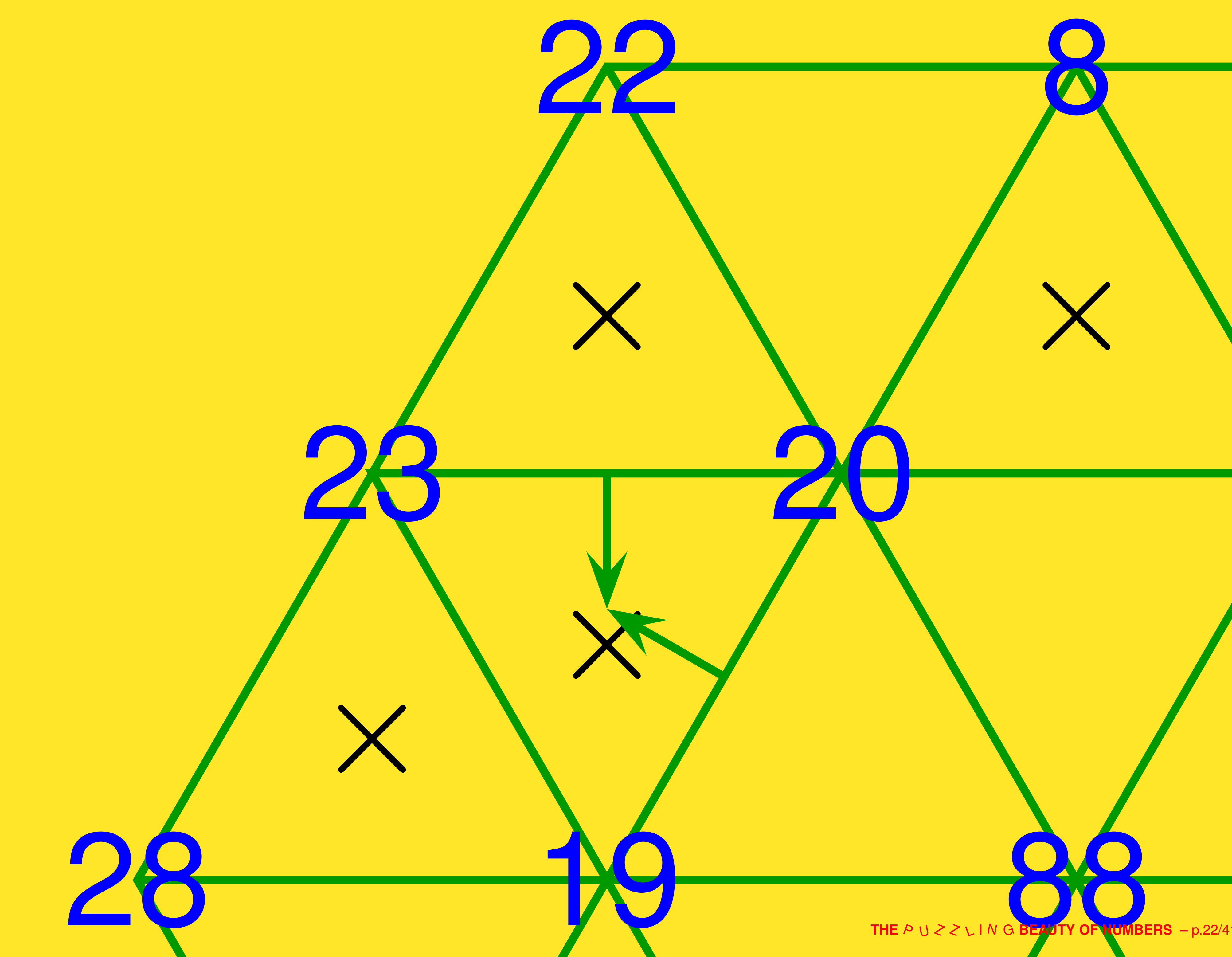
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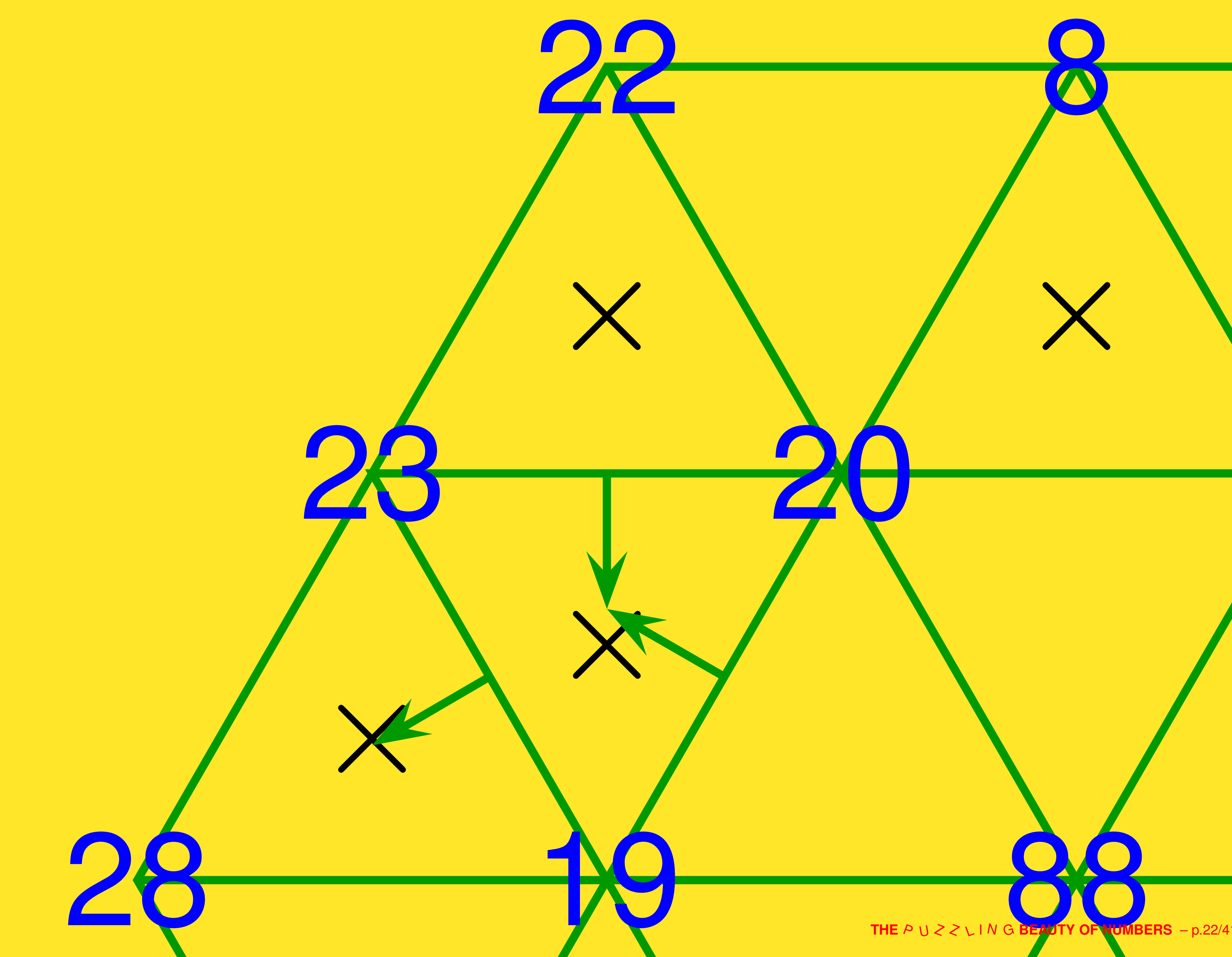
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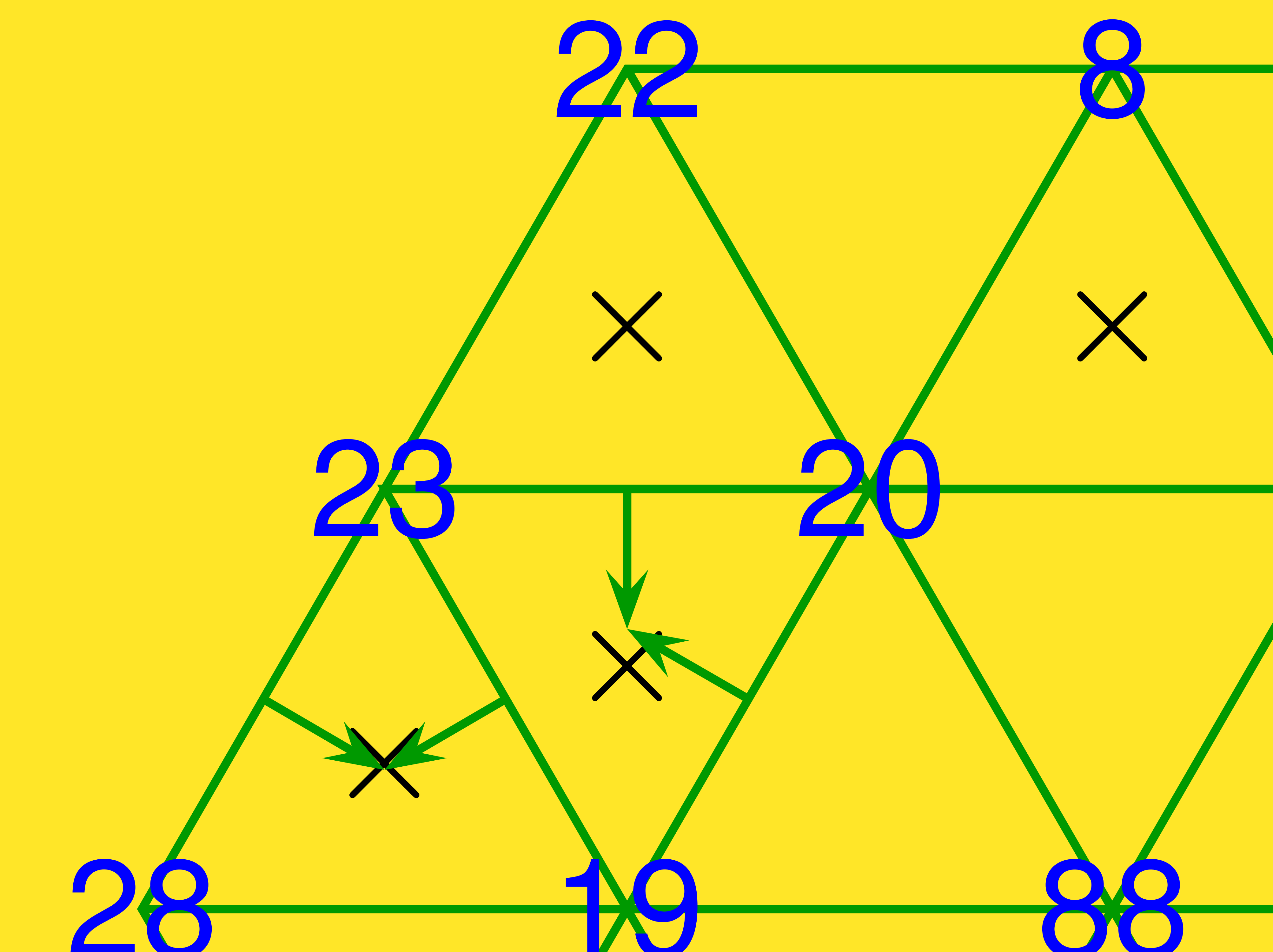
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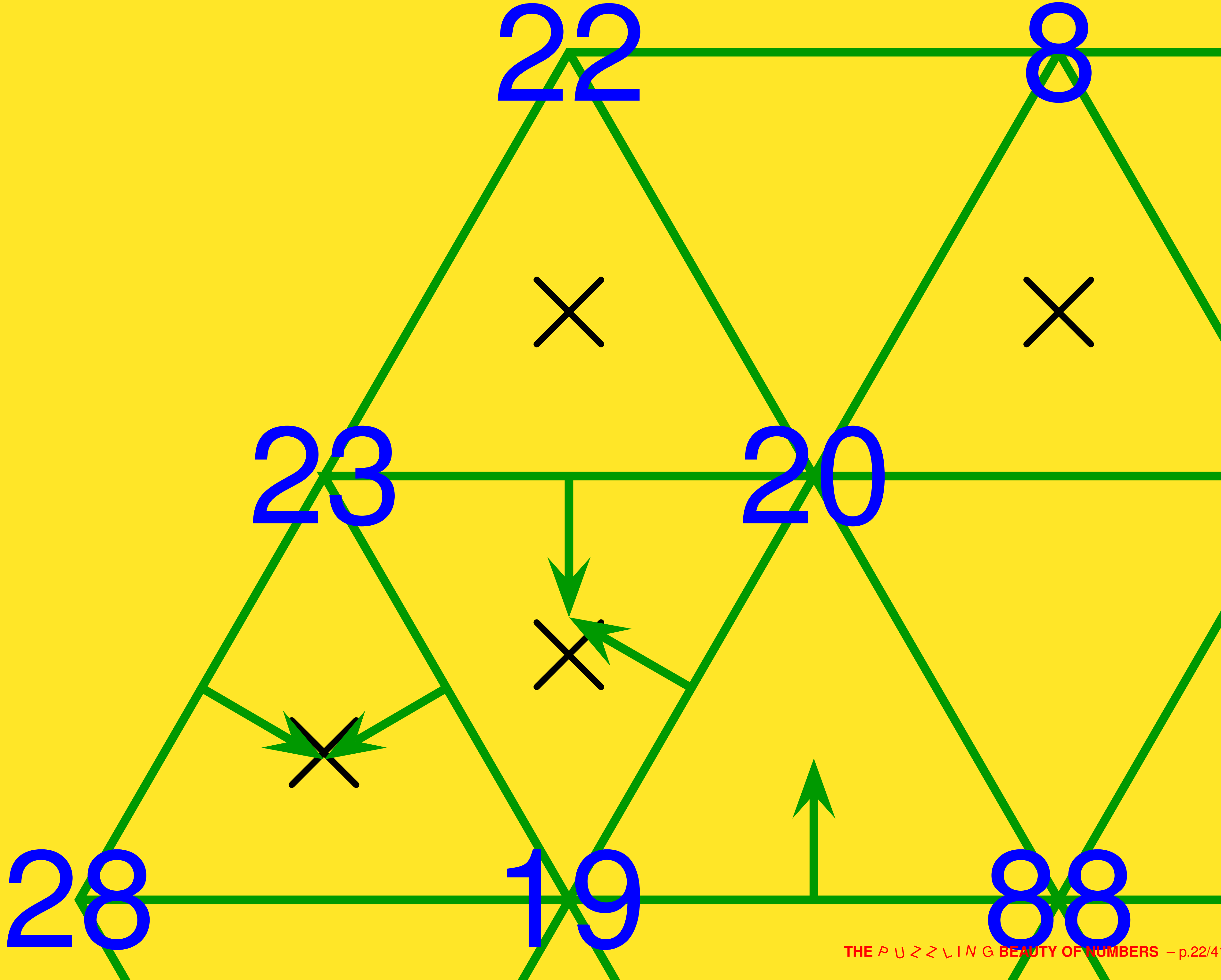
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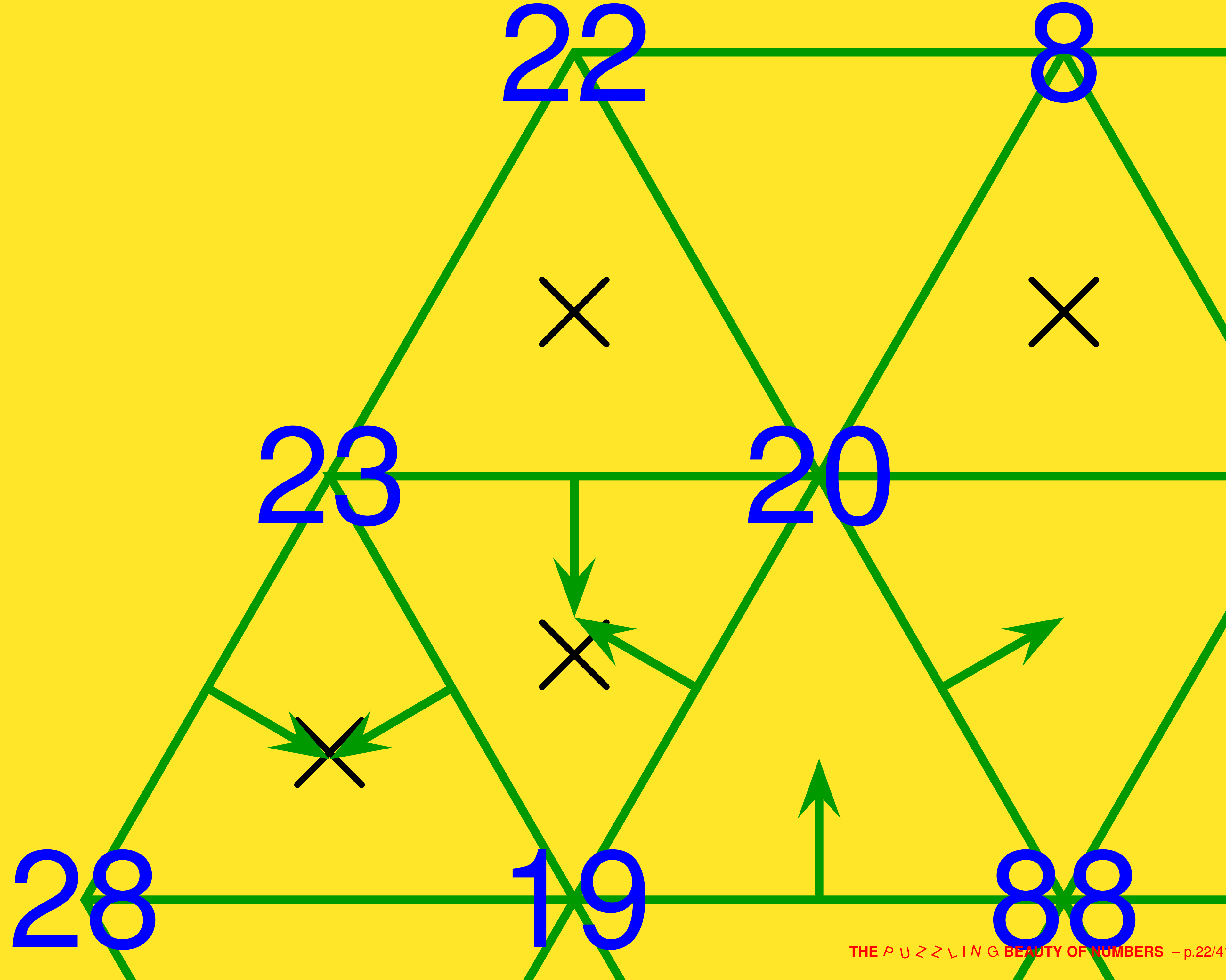
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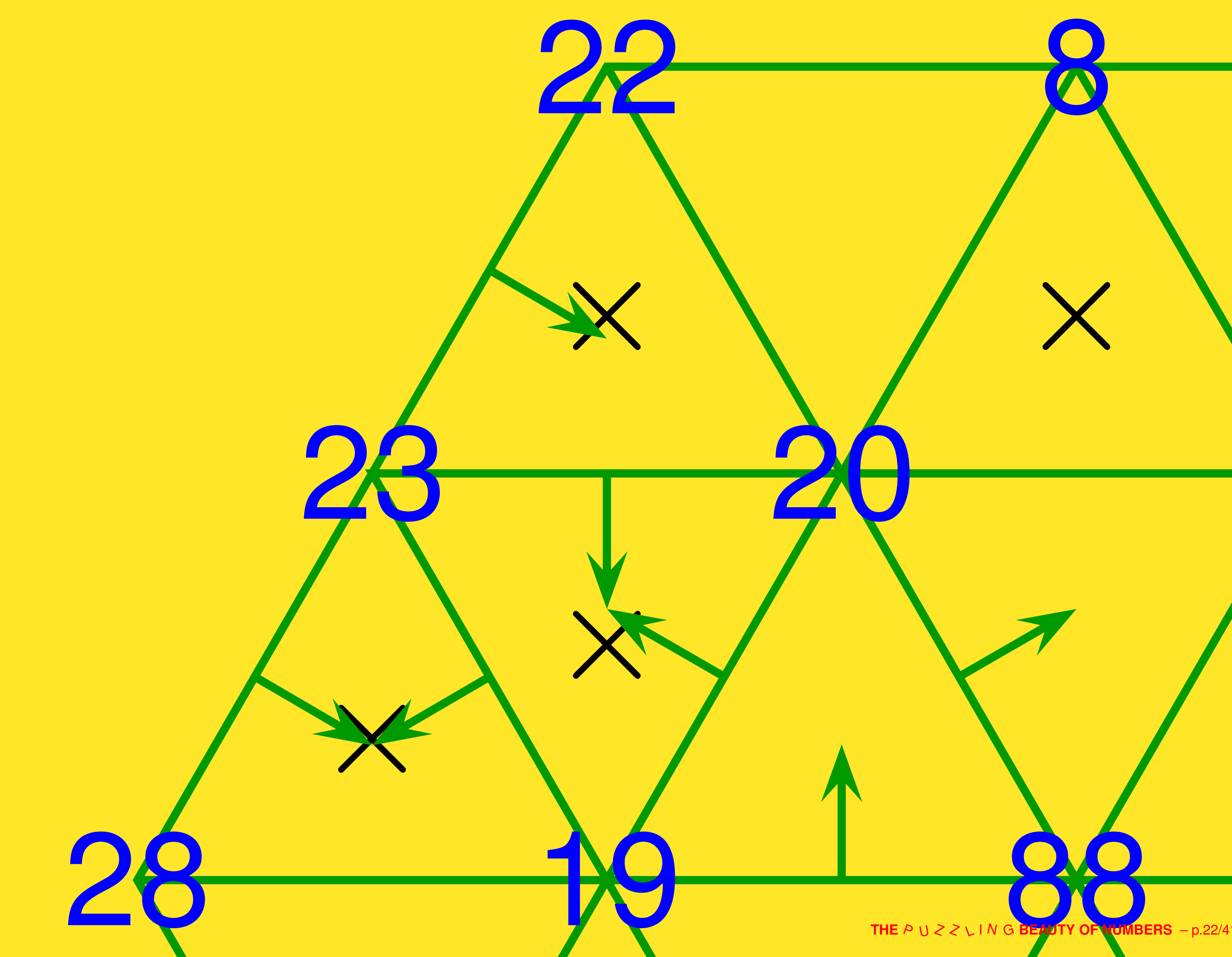
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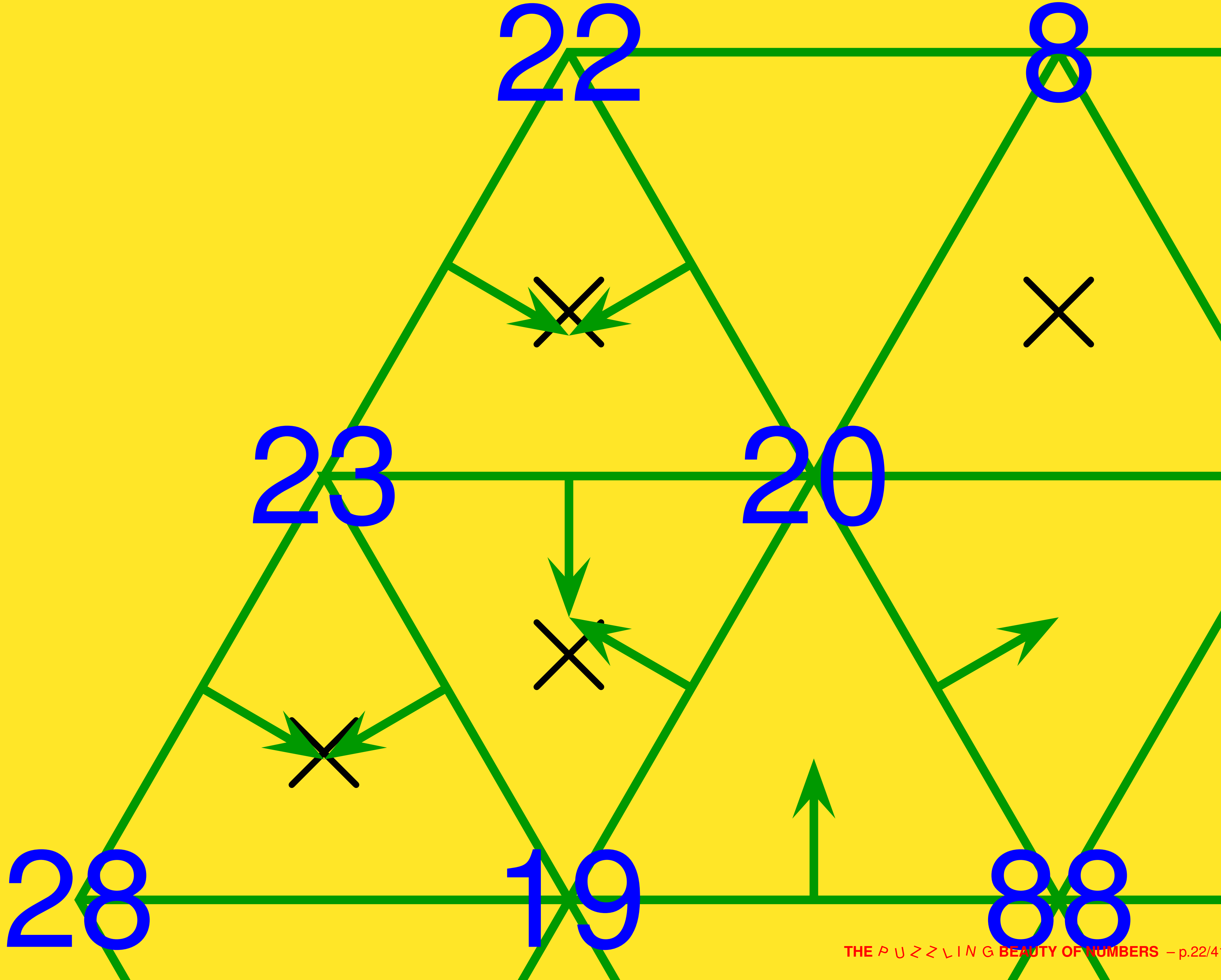
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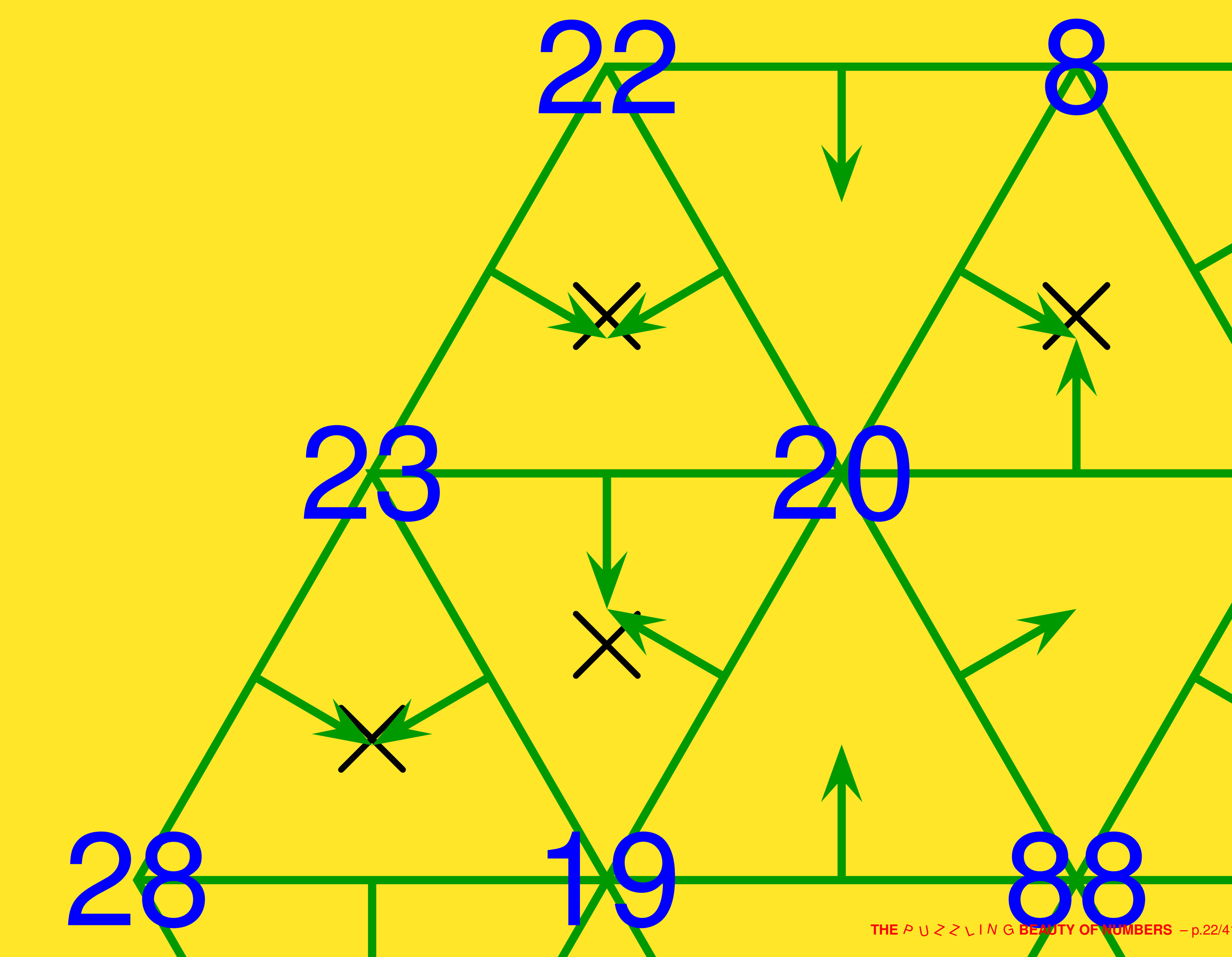
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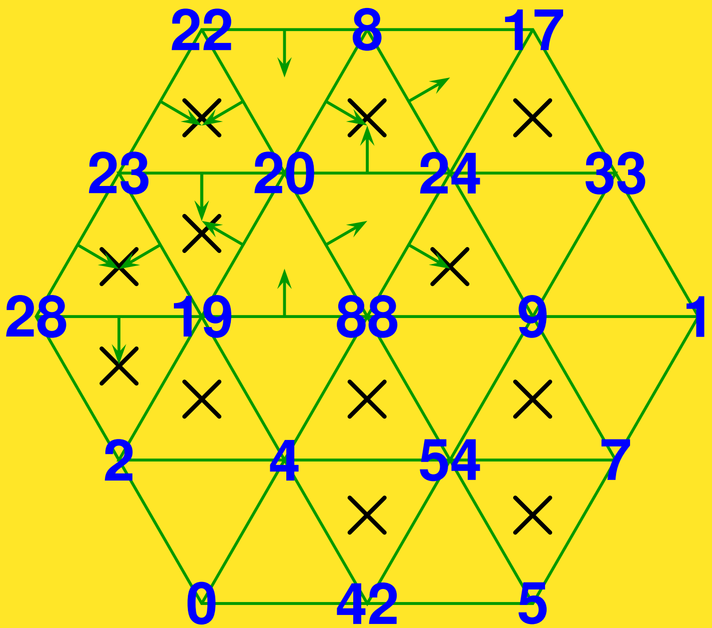

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Goldbach's Conjecture: Every even positive integer greater that 2 can be written as the sum of two primes.

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In fact, one can check that $4294967297 = 641 \cdot 6700417$.

TRUE OR NOT TRUE?

Numbers 8 and 9 are the only consecutive powers.

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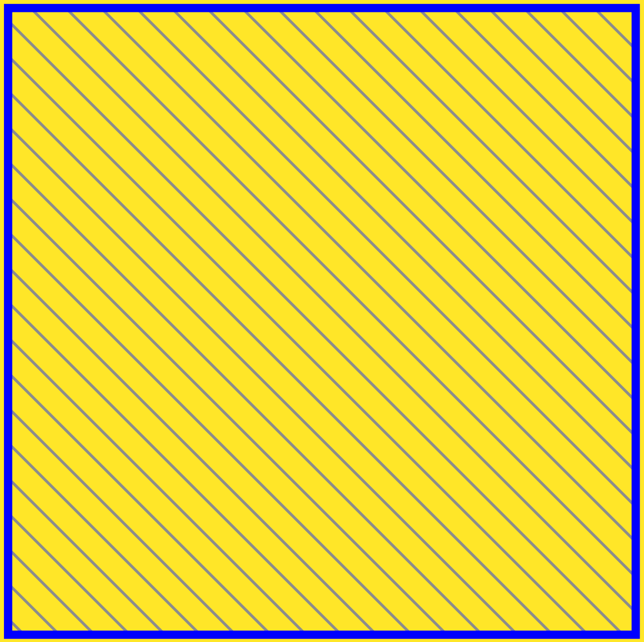
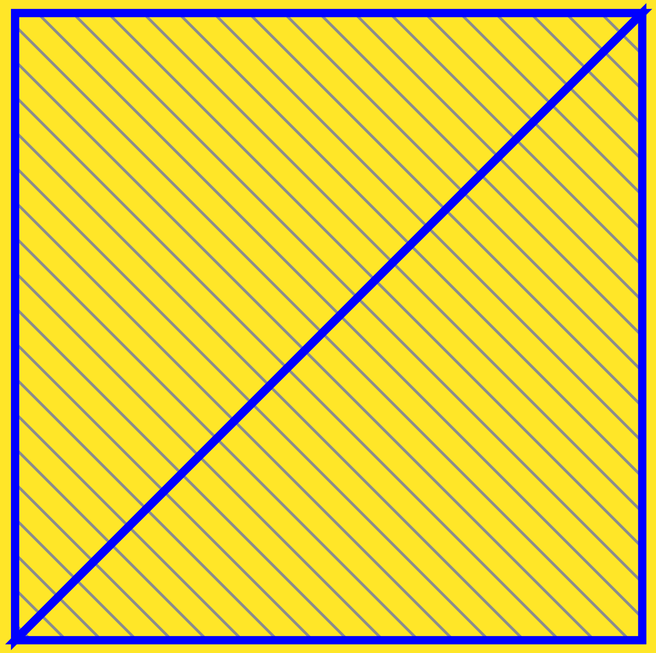
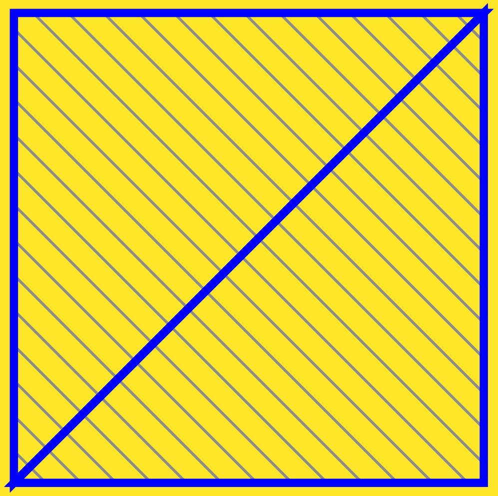
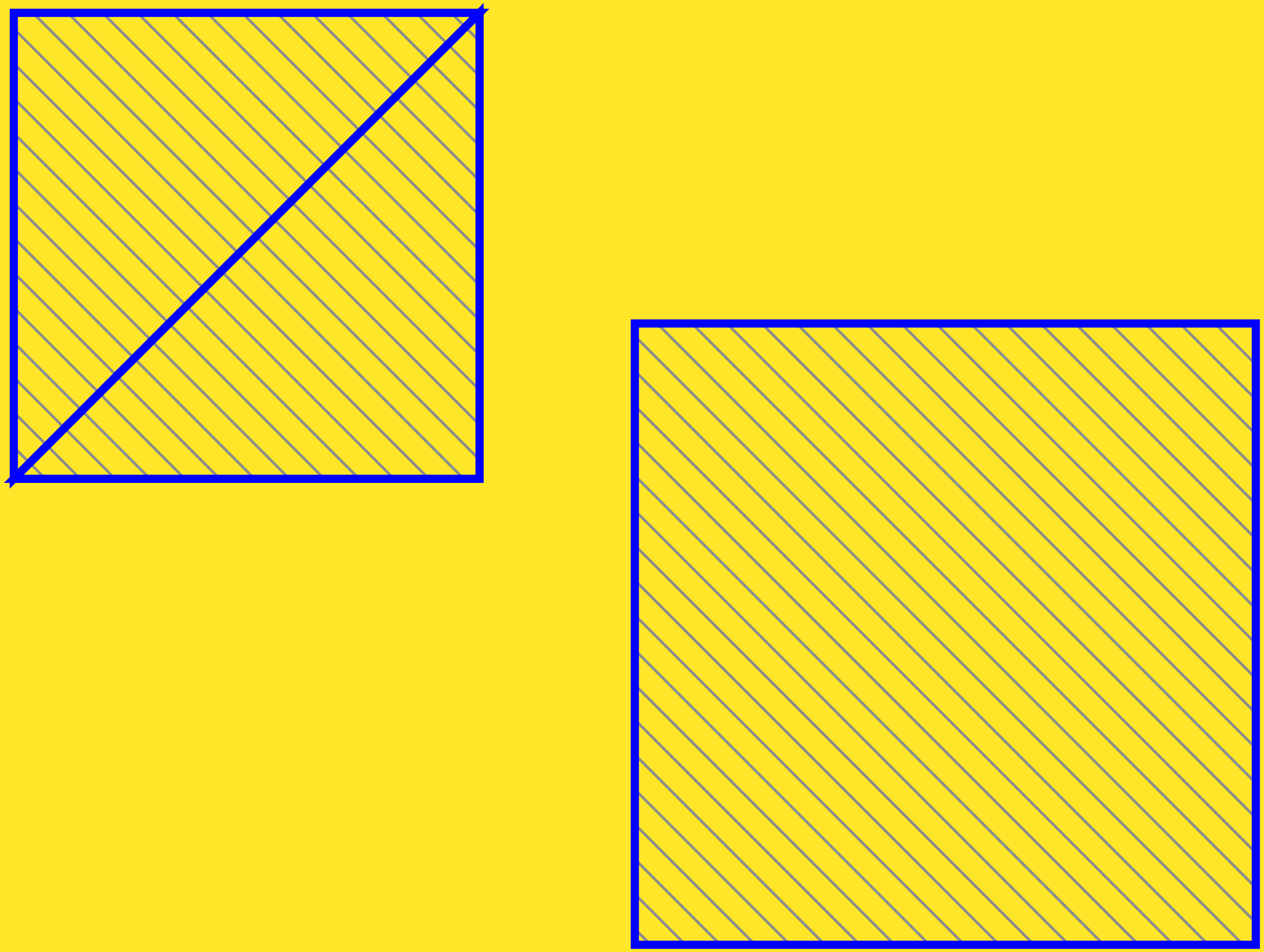
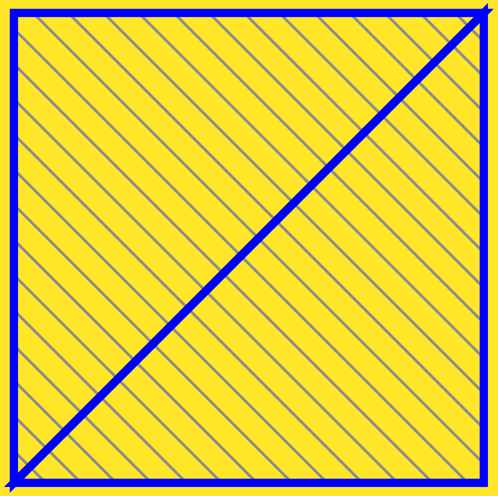
TRUE (Michalesku, 2002).

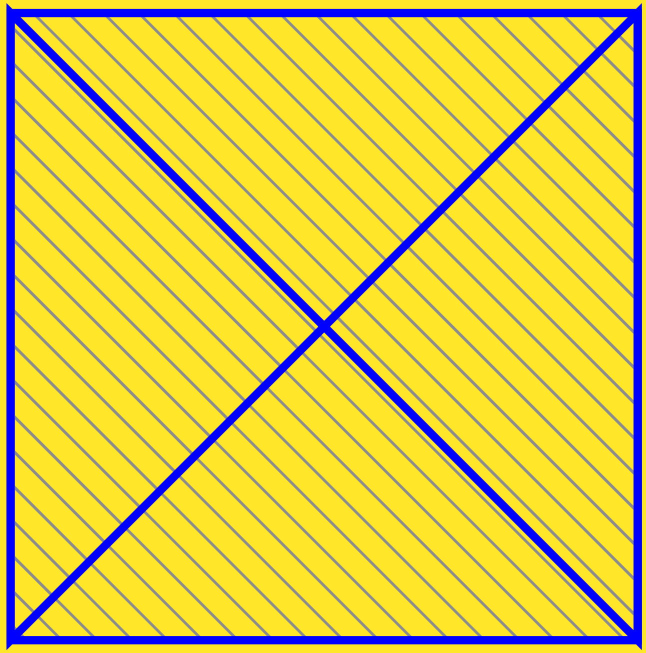


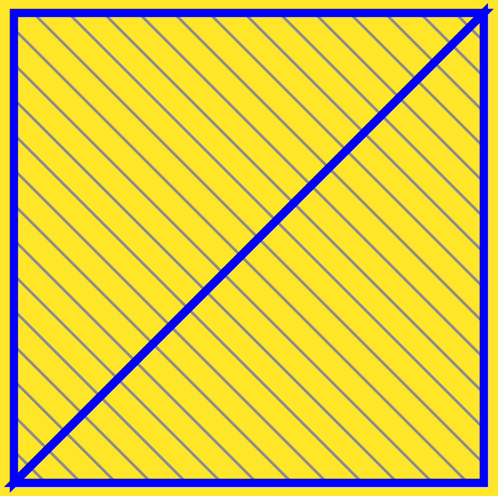


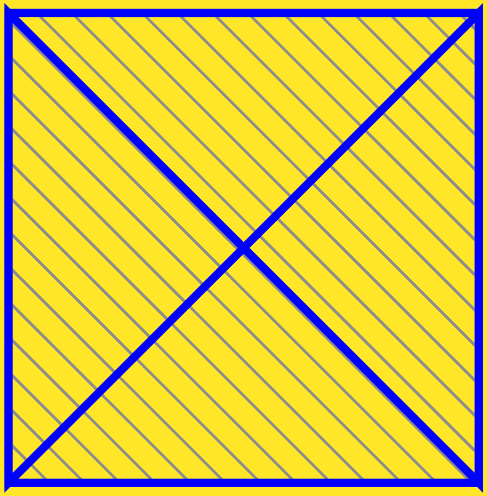


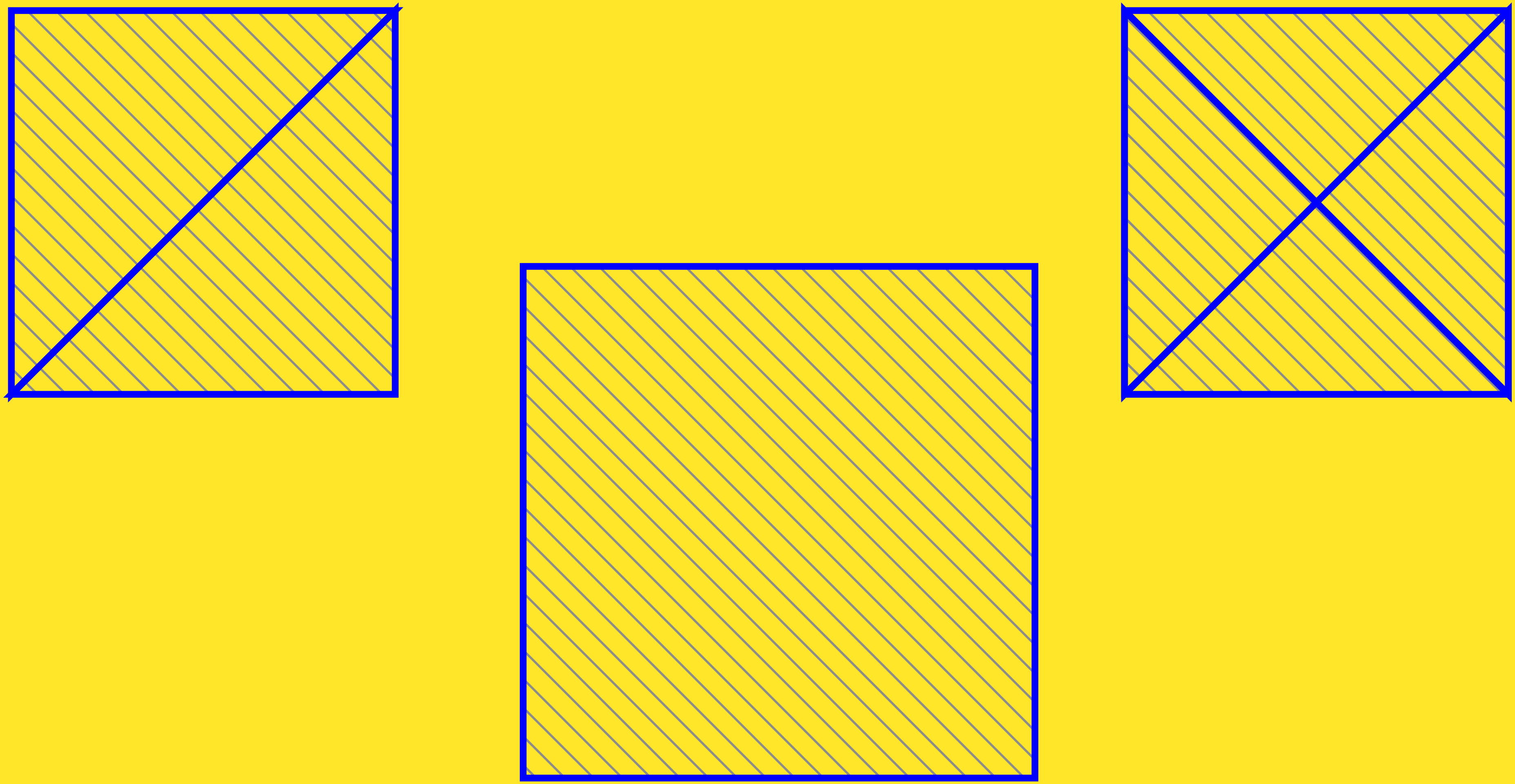


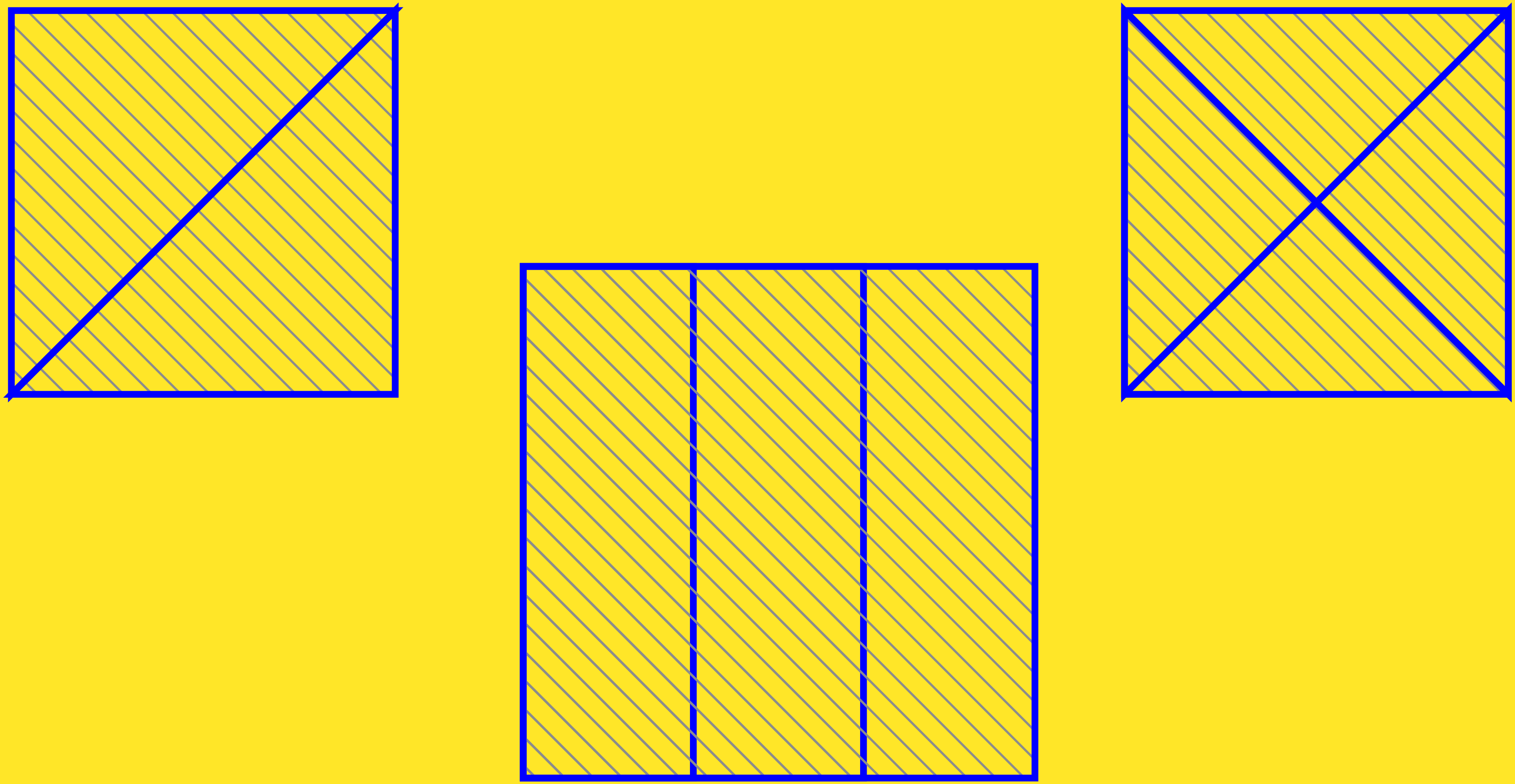












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$$\frac{a_1 + \ldots + a_n}{n} \geq \sqrt[n]{a_1 \ldots a_n}$$

PROOF (ELEMENTARY, SHORT): Put

$$f(x_1, \ldots, x_n) = \frac{x_1 + \ldots + x_n}{n} - \sqrt[n]{x_1 \ldots x_n}$$

Assume to the contrary that $f(x_1, \ldots, x_n) < 0$ for some $x_1, \ldots, x_n \in [A, B]$. Suppose that $f(x_1, \ldots, x_n)$ attains its minimal value on $[A, B]$ at some $a_1, \ldots, a_n \in [A, B]$ with $a_1 \neq a_2$. This gives us a contradiction, since

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THEOREM (Extreme-Value Theorem): If $f(x_1, \ldots, x_n)$ is continuous on a closed and bounded set R, then f has both an absolute maximum and an absolute minimum on R.

We have:

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$$\int \sin 2x \, dx$$

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$$= \left[\begin{array}{c} \sin x = u \\ \\ \\ \end{array} \right]$$

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We have:

$$\begin{aligned}\int \sin 2x\,dx &= \int 2\sin x\cos x\,dx \\ &= \left[\begin{array}{c}\sin x = u \\ d\sin x = du \\ \cos x\,dx = du\end{array}\right] \\ &= \int 2\end{aligned}$$

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$$= u^2 + C$$

$$= \sin^2 x + C$$

We have:

$$\int \sin 2x\,dx = \frac{1}{2}\int 2\sin 2x\,dx = \begin{bmatrix} 2x = u \\ d2x = du \\ 2\,dx = du \end{bmatrix} = \frac{1}{2}\int \sin u\,du = -\frac{1}{2}\cos u + C$$

$$\boxed{= -\frac{1}{2}\cos 2x + C}$$

$$\int \sin 2x\,dx = \int 2\sin x\cos x\,dx = \begin{bmatrix} \sin x = u \\ d\sin x = du \\ \cos x\,dx = du \end{bmatrix} = \int 2u\,du = u^2 + C$$

$$\boxed{= \sin^2 x + C}$$

We have:

$$\int \sin 2x\,dx = \frac{1}{2}\int 2\sin 2x\,dx = \left[\begin{array}{c} 2x = u \\ d2x = du \\ 2\,dx = du \end{array}\right] = \frac{1}{2}\int \sin u\,du = -\frac{1}{2}\cos u + C = -\frac{1}{2}\cos 2x + C$$

$$\int \sin 2x\,dx = \int 2\sin x\cos x\,dx = \left[\begin{array}{c} \sin x = u \\ d\sin x = du \\ \cos x\,dx = du \end{array}\right] = \int 2u\,du = u^2 + C = \sin^2 x + C$$

1. **There is a mistake.**

We have:

$$\int \sin 2x\,dx = \frac{1}{2}\int 2\sin 2x\,dx = \begin{bmatrix} 2x = u \\ d2x = du \\ 2\,dx = du \end{bmatrix} = \frac{1}{2}\int \sin u\,du = -\frac{1}{2}\cos u + C$$

$$= -\frac{1}{2}\cos 2x + C$$

$$\int \sin 2x\,dx = \int 2\sin x\cos x\,dx = \begin{bmatrix} \sin x = u \\ d\sin x = du \\ \cos x\,dx = du \end{bmatrix} = \int 2u\,du = u^2 + C$$

$$= \sin^2 x + C$$

1. **There is a mistake. Where???**

We have:

$$\begin{aligned}
\int \sin 2x\,dx &= \frac{1}{2}\int 2\sin 2x\,dx \\
&= \begin{bmatrix} 2x = u \\ d2x = du \\ 2\,dx = du \end{bmatrix} \\
&= \frac{1}{2}\int \sin u\,du \\
&= -\frac{1}{2}\cos u + C \\
&= -\frac{1}{2}\cos 2x + C
\end{aligned}$$

$$\begin{aligned}
\int \sin 2x\,dx &= \int 2\sin x\cos x\,dx \\
&= \begin{bmatrix} \sin x = u \\ d\sin x = du \\ \cos x\,dx = du \end{bmatrix} \\
&= \int 2u\,du \\
&= u^2 + C \\
&= \sin^2 x + C
\end{aligned}$$

1. There is a mistake. Where???
2. $-\frac{1}{2}\cos 2x = \sin^2 x.$

We have:

$$\int \sin 2x\,dx = \frac{1}{2}\int 2\sin 2x\,dx = \begin{bmatrix} 2x = u \\ d2x = du \\ 2\,dx = du \end{bmatrix} = \frac{1}{2}\int \sin u\,du = -\frac{1}{2}\cos u + C$$

$$= -\frac{1}{2}\cos 2x + C$$

$$\int \sin 2x\,dx = \int 2\sin x\cos x\,dx = \begin{bmatrix} \sin x = u \\ d\sin x = du \\ \cos x\,dx = du \end{bmatrix} = \int 2u\,du = u^2 + C$$

$$= \sin^2 x + C$$

1. **There is a mistake. Where???**
2. ~~$-\frac{1}{2}\cos 2x = \sin^2 x$.~~ **Wrong!**

We have:

$$\int \sin 2x\,dx = \frac{1}{2}\int 2\sin 2x\,dx = \begin{bmatrix} 2x = u \\ d2x = du \\ 2\,dx = du \end{bmatrix} = \frac{1}{2}\int \sin u\,du = -\frac{1}{2}\cos u + C = -\frac{1}{2}\cos 2x + C$$

$$\int \sin 2x\,dx = \int 2\sin x\cos x\,dx = \begin{bmatrix} \sin x = u \\ d\sin x = du \\ \cos x\,dx = du \end{bmatrix} = \int 2u\,du = u^2 + C = \sin^2 x + C$$

1. **There is a mistake. Where???**
2. ~~$-\frac{1}{2}\cos 2x = \sin^2 x$.~~ **Wrong!**
3. $-\frac{1}{2}\cos 2x + \frac{1}{2} = \sin^2 x$.

We have:

$$\int \sin 2x\,dx = \frac{1}{2}\int 2\sin 2x\,dx = \begin{bmatrix} 2x = u \\ d2x = du \\ 2\,dx = du \end{bmatrix} = \frac{1}{2}\int \sin u\,du = -\frac{1}{2}\cos u + C = -\frac{1}{2}\cos 2x + C$$

$$\int \sin 2x\,dx = \int 2\sin x\cos x\,dx = \begin{bmatrix} \sin x = u \\ d\sin x = du \\ \cos x\,dx = du \end{bmatrix} = \int 2u\,du = u^2 + C = \sin^2 x + C$$

1. There is a mistake. Where???
2. ~~$-\frac{1}{2}\cos 2x = \sin^2 x$.~~ Wrong!
3. $-\frac{1}{2}\cos 2x + \frac{1}{2} = \sin^2 x$. Correct.

We have:

$$\int \sin 2x\,dx = \frac{1}{2}\int 2\sin 2x\,dx = \begin{bmatrix} 2x = u \\ d2x = du \\ 2\,dx = du \end{bmatrix} = \frac{1}{2}\int \sin u\,du = -\frac{1}{2}\cos u + C$$

$$= -\frac{1}{2}\cos 2x + C$$

$$\int \sin 2x\,dx = \int 2\sin x\cos x\,dx = \begin{bmatrix} \sin x = u \\ d\sin x = du \\ \cos x\,dx = du \end{bmatrix} = \int 2u\,du = u^2 + C$$

$$= \sin^2 x + C$$

1. There is a mistake. Where???
2. ~~$-\frac{1}{2}\cos 2x = \sin^2 x$~~. Wrong!
3. $-\frac{1}{2}\cos 2x + \frac{1}{2} = \sin^2 x$. Correct.

$$\lim_{x \to 8^+} \frac{1}{x - 8}$$

$$\lim_{x \to 8^+} \frac{1}{x - 8} = +\infty$$

$$\lim_{x\to 8^+} \frac{1}{x-8} = +\infty$$

$$\lim_{x\to 4^+} \frac{1}{x-4}$$

$$\lim_{x \to 8^+} \frac{1}{x - 8} = +\infty$$

$$\lim_{x \to 4^+} \frac{1}{x - 4} =$$

$$\lim_{x\to 8^+} \frac{\mathbf{1}}{\boldsymbol{x} - \mathbf{8}} = +\infty$$

$$\lim_{x\to 4^+} \frac{\mathbf{1}}{\boldsymbol{x} - \mathbf{4}} = +\rotatebox{90}{$\mathbf{4}$}$$

$$\lim_{x\to 8^+} \frac{1}{x-8} = +\infty$$

$$\lim_{x\to 4^+} \frac{1}{x-4} = +4$$

$$\lim_{x \to 8^+} \frac{1}{x-8} = +\infty$$

$$\lim_{x \to 4^+} \frac{1}{x-4} = +\rotatebox{90}{4}$$

PROBLEM:

PROBLEM:

Consider the following weights:

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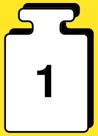
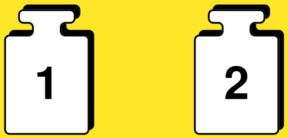
Consider the following weights:



PROBLEM:

Consider the following weights:



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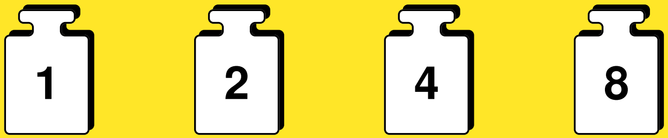

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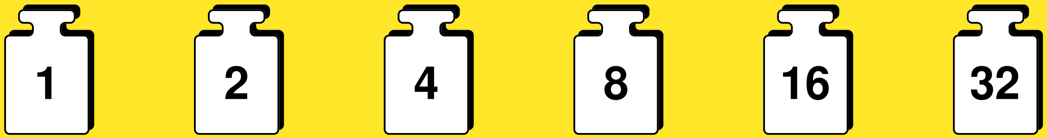

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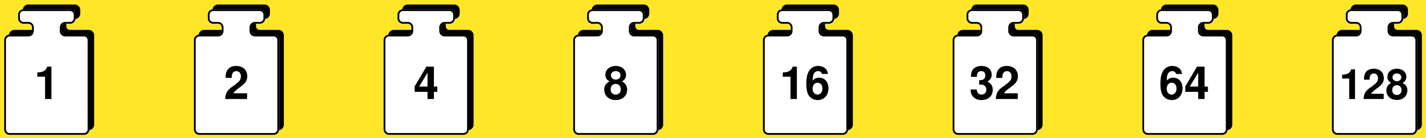
Consider the following weights:



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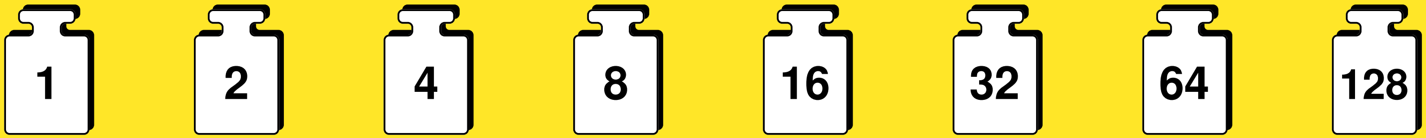
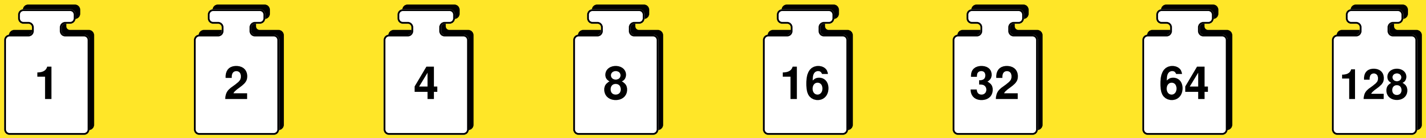
Consider the following weights:



etc.

PROBLEM:

Consider the following weights:

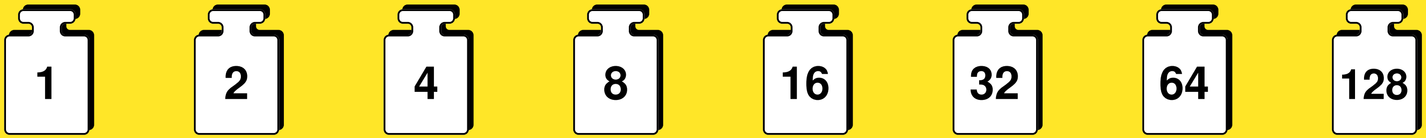


etc.

Which weight can be weighed by them?

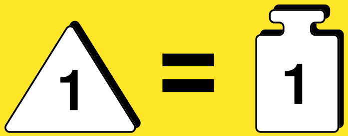
PROBLEM:

Consider the following weights:



etc.

Which weight can be weighed by them?



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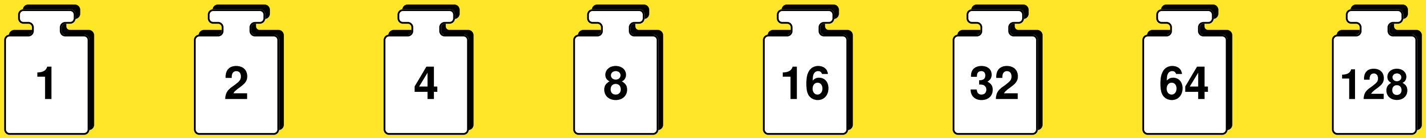
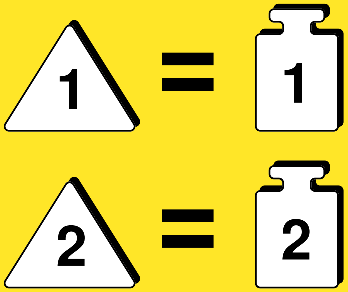
Consider the following weights:



etc.

Which weight can be weighed by them?



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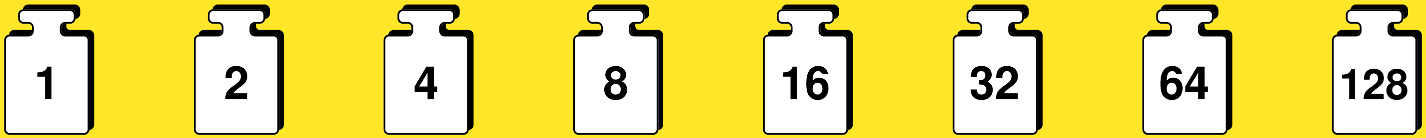
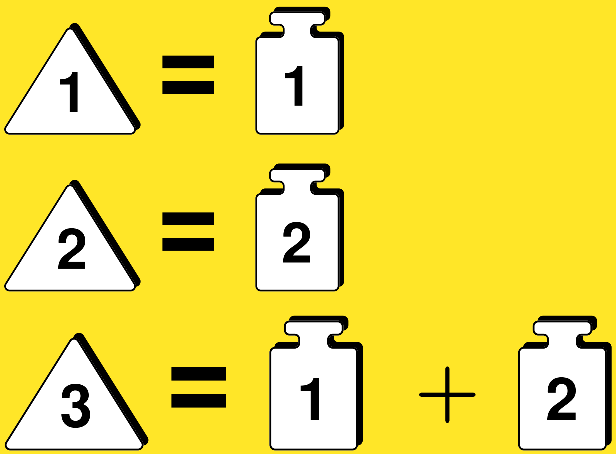
Consider the following weights:



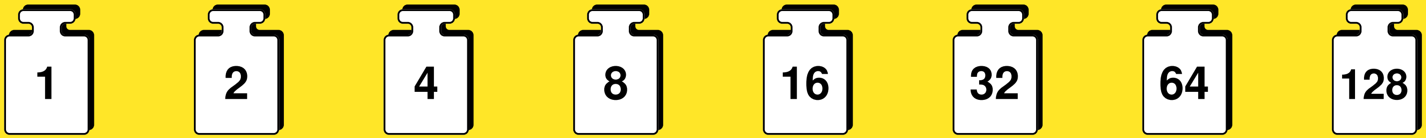
etc.

Which weight can be weighed by them?



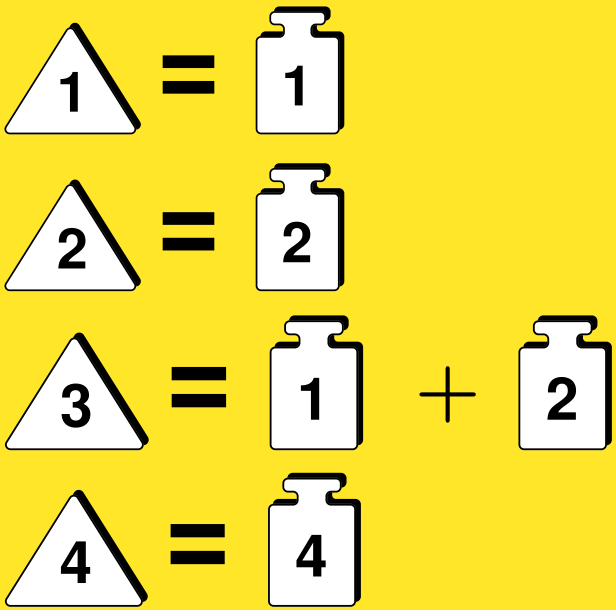
PROBLEM:

Consider the following weights:



etc.

Which weight can be weighed by them?



PROBLEM:

Consider the following weights:

1 2 4 8 16 32 64 128 **etc.**

Which weight can be weighed by them?

1 = 1

2 = 2

3 = 1 + 2

4 = 4

5 = 1 + 4

PROBLEM:

Consider the following weights:

1 2 4 8 16 32 64 128 **etc.**

Which weight can be weighed by them?

1 = 1

2 = 2

3 = 1 + 2

4 = 4

5 = 1 + 4

6 = 2 + 4

PROBLEM:

Consider the following weights:

[1] [2] [4] [8] [16] [32] [64] [128] **etc.**

Which weight can be weighed by them?

Triangle 1 = [1]

Triangle 2 = [2]

Triangle 3 = [1] + [2]

Triangle 4 = [4]

Triangle 5 = [1] + [4]

Triangle 6 = [2] + [4]

Triangle 7 = [1] + [2] + [4]

PROBLEM:

Consider the following weights:

1 2 4 8 16 32 64 128 **etc.**

Which weight can be weighed by them?

1 = 1

2 = 2

3 = 1 + 2

4 = 4

5 = 1 + 4

6 = 2 + 4

7 = 1 + 2 + 4

8 = 8

PROBLEM:

Consider the following weights:

1 2 4 8 16 32 64 128 **etc.**

Which weight can be weighed by them?

1 = 1

2 = 2

3 = 1 + 2

4 = 4

5 = 1 + 4

6 = 2 + 4

7 = 1 + 2 + 4

8 = 8

9 = 1 + 8

PROBLEM:

Consider the following weights:

1 2 4 8 16 32 64 128 **etc.**

Which weight can be weighed by them?

$1 = 1$

$2 = 2$

$3 = 1 + 2$

$4 = 4$

$5 = 1 + 4$

$6 = 2 + 4$

$7 = 1 + 2 + 4$

$8 = 8$

$9 = 1 + 8$

$10 = 2 + 8$

PROBLEM:

Consider the following weights:

1 2 4 8 16 32 64 128 **etc.**

Which weight can be weighed by them?

1 = 1

2 = 2

3 = 1 + 2

4 = 4

5 = 1 + 4

6 = 2 + 4

7 = 1 + 2 + 4

8 = 8

9 = 1 + 8

10 = 2 + 8

11 = 1 + 2 + 8

PROBLEM:

Consider the following weights:

1 2 4 8 16 32 64 128 **etc.**

Which weight can be weighed by them?

1 = 1	7 = 1 + 2 + 4
2 = 2	8 = 8
3 = 1 + 2	9 = 1 + 8
4 = 4	10 = 2 + 8
5 = 1 + 4	11 = 1 + 2 + 8
6 = 2 + 4	12 = 4 + 8

$1=1$
$2=2$
$3=1+2$
$4=4$
$5=1+4$
$6=2+4$
$7=1+2+4$
$8=8$
$9=1+8$
$10=2+8$
$11=1+2+8$
$12=4+8$
$13=1+4+8$
$14=2+4+8$
$15=1+2+4+8$

$16=16$
$17=1+16$
$18=2+16$
$19=1+2+16$
$20=4+16$
$21=1+4+16$
$22=2+4+16$
$23=1+2+4+16$
$24=8+16$
$25=1+8+16$
$26=2+8+16$
$27=1+2+8+16$
$28=4+8+16$
$29=1+4+8+16$
$30=2+4+8+16$

$31=1+2+4+8+16$
$32=32$
$33=1+32$
$34=2+32$
$35=1+2+32$
$36=4+32$
$37=1+4+32$
$38=2+4+32$
$39=1+2+4+32$
$40=8+32$
$41=1+8+32$
$42=2+8+32$
$43=1+2+8+32$
$44=4+8+32$
$45=1+4+8+32$

$$a^{32} - 1$$

$$a^{32} - 1 \equiv (a-1)(a^{31} + a^{30} + a^{29} + \ldots + a^2 + a + 1)$$

$$a^{32} - 1 \equiv (a-1)(a^{31} + a^{30} + a^{29} + \ldots + a^2 + a + 1)$$

$$a^{32} - 1$$

$$a^{32} - 1 \equiv (a - 1)(a^{31} + a^{30} + a^{29} + \ldots + a^2 + a + 1)$$

$$a^{32} - 1 \equiv (a^{16} - 1)(a^{16} + 1)$$

$$a^{32} - 1 \equiv (a-1)(a^{31} + a^{30} + a^{29} + \ldots + a^2 + a + 1)$$

$$a^{32} - 1 \equiv (a^8 - 1)(a^8 + 1)(a^{16} + 1)$$

$$a^{32} - 1 \equiv (a - 1)(a^{31} + a^{30} + a^{29} + \ldots + a^2 + a + 1)$$

$$a^{32} - 1 \equiv (a^4 - 1)(a^4 + 1)(a^8 + 1)(a^{16} + 1)$$

$$a^{32} - 1 \equiv (a-1)(a^{31} + a^{30} + a^{29} + \ldots + a^2 + a + 1)$$

$$a^{32} - 1 \equiv (a^2-1)(a^2+1)(a^4+1)(a^8+1)(a^{16}+1)$$

$$a^{32} - 1 \equiv (a-1)(a^{31} + a^{30} + a^{29} + \ldots + a^2 + a + 1)$$

$$a^{32} - 1 \equiv (a-1)(a+1)(a^2+1)(a^4+1)(a^8+1)(a^{16}+1)$$

$$(a-1)(a^{31}+a^{30}+a^{29}+\ldots+a^{2}+a+1)$$

$$(a-1)(a+1)(a^{2}+1)(a^{4}+1)(a^{8}+1)(a^{16}+1)$$

$$(a-1)(a^{31}+a^{30}+a^{29}+\ldots+a^2+a+1)$$

$$(a-1)(a+1)(a^2+1)(a^4+1)(a^8+1)(a^{16}+1)$$

$$a^{31} + a^{30} + a^{29} + \ldots + a^2 + a + 1$$

$$\equiv (a+1)(a^2+1)(a^4+1)(a^8+1)(a^{16}+1)$$

$$a^{31} + a^{30} + a^{29} + \ldots + a^2 + a + 1$$

$$\equiv (a+1)(a^2+1)(a^4+1)(a^8+1)(a^{16}+1)$$

$$\equiv 1 + a + a^2 + a^{1+2} + a^4 + a^{1+4} + a^{2+4}$$

$$+ a^{1+2+4} + a^8 + a^{1+8} + a^{2+8} + a^{1+2+8}$$

$$+ a^{4+8} + a^{1+4+8} + a^{2+4+8} + a^{2+4+8+16}$$

$$+ a^{16} + a^{1+16} + a^{2+16} + a^{1+2+16} + a^{4+16}$$

$$+ a^{1+4+16} + a^{2+4+16} + a^{1+2+8+16} + a^{8+16}$$

$$+ a^{1+4+8+16} + a^{2+8+16} + a^{1+8+16} + a^{4+8+16}$$

$$+ a^{1+2+4+8} + a^{1+2+4+16} + a^{1+2+4+8+16}$$

$$a^{31} + a^{30} + a^{29} + \ldots + a^2 + a + 1$$

$$\equiv (a+1)(a^{2}+1)(a^{4}+1)(a^{8}+1)(a^{16}+1)$$

$$\equiv 1 + a + a^2 + a^{1+2} + a^4 + a^{1+4} + a^{2+4}$$
$$+ a^{1+2+4} + a^8 + a^{1+8} + a^{2+8} + a^{1+2+8}$$
$$+ a^{4+8} + a^{1+4+8} + a^{2+4+8} + a^{2+4+8+16}$$
$$+ a^{16} + a^{1+16} + a^{2+16} + a^{1+2+16} + a^{4+16}$$
$$+ a^{1+4+16} + a^{2+4+16} + a^{1+2+8+16} + a^{8+16}$$
$$+ a^{1+4+8+16} + a^{2+8+16} + a^{1+8+16} + a^{4+8+16}$$
$$+ a^{1+2+4+8} + a^{1+2+4+16} + a^{1+2+4+8+16}$$

$$a^{31} + a^{30} + a^{29} + \ldots + a^2 + a + 1$$

$$\equiv (a+1)(a^{2}+1)(a^{4}+1)(a^{8}+1)(a^{16}+1)$$

$$\equiv 1 + a + a^{2} + a^{1+2} + a^{4} + a^{1+4} + a^{2+4}$$

$$+ a^{1+2+4} + a^{8} + a^{1+8} + a^{2+8} + a^{1+2+8}$$

$$+ a^{4+8} + a^{1+4+8} + a^{2+4+8} + a^{2+4+8+16}$$

$$+ a^{16} + a^{1+16} + a^{2+16} + a^{1+2+16} + a^{4+16}$$

$$+ a^{1+4+16} + a^{2+4+16} + a^{1+2+8+16} + a^{8+16}$$

$$+ a^{1+4+8+16} + a^{2+8+16} + a^{1+8+16} + a^{4+8+16}$$

$$+ a^{1+2+4+8} + a^{1+2+4+16} + a^{1+2+4+8+16}$$

$$\boxed{a^{31} + a^{30} + a^{29} + \ldots + a^2 + a + 1}$$

$$\equiv (a+1)(a^2+1)(a^4+1)(a^8+1)(a^{16}+1)$$

$$\equiv 1 + a + a^2 + a^{1+2} + a^4 + a^{1+4} + a^{2+4}$$

$$+ a^{1+2+4} + a^8 + a^{1+8} + a^{2+8} + a^{1+2+8}$$

$$+ a^{4+8} + a^{1+4+8} + a^{2+4+8} + a^{2+4+8+16}$$

$$+ a^{16} + a^{1+16} + a^{2+16} + a^{1+2+16} + a^{4+16}$$

$$+ a^{1+4+16} + a^{2+4+16} + a^{1+2+8+16} + a^{8+16}$$

$$+ a^{1+4+8+16} + a^{2+8+16} + a^{1+8+16} + a^{4+8+16}$$

$$+ a^{1+2+4+8} + a^{1+2+4+16} + a^{1+2+4+8+16}$$

$$a^{31} + a^{30} + a^{29} + \ldots + a^2 + a + 1$$

$$\equiv (a+1)(a^2+1)(a^4+1)(a^8+1)(a^{16}+1)$$

$$\equiv 1 + a + a^2 + a^{1+2} + a^4 + a^{1+4} + a^{2+4}$$
$$+ a^{1+2+4} + a^8 + a^{1+8} + a^{2+8} + a^{1+2+8}$$
$$+ a^{4+8} + a^{1+4+8} + a^{2+4+8} + a^{2+4+8+16}$$
$$+ a^{16} + a^{1+16} + a^{2+16} + a^{1+2+16} + a^{4+16}$$
$$+ a^{1+4+16} + a^{2+4+16} + a^{1+2+8+16} + a^{8+16}$$
$$+ a^{1+4+8+16} + a^{2+8+16} + a^{1+8+16} + a^{4+8+16}$$
$$+ a^{1+2+4+8} + a^{1+2+4+16} + a^{1+2+4+8+16}$$

$$a^{31} + a^{30} + a^{29} + \ldots + a^2 + a + 1$$

$$\equiv (a+1)(a^2+1)(a^4+1)(a^8+1)(a^{16}+1)$$

$$\equiv 1 + a + a^2 + a^{1+2} + a^4 + a^{1+4} + a^{2+4}$$
$$+ a^{1+2+4} + a^8 + a^{1+8} + a^{2+8} + a^{1+2+8}$$
$$+ a^{4+8} + a^{1+4+8} + a^{2+4+8} + a^{2+4+8+16}$$
$$+ a^{16} + a^{1+16} + a^{2+16} + a^{1+2+16} + a^{4+16}$$
$$+ a^{1+4+16} + a^{2+4+16} + a^{1+2+8+16} + a^{8+16}$$
$$+ a^{1+4+8+16} + a^{2+8+16} + a^{1+8+16} + a^{4+8+16}$$
$$+ a^{1+2+4+8} + a^{1+2+4+16} + a^{1+2+4+8+16}$$

$$a^{31} + a^{30} + a^{29} + \ldots + a^2 + a + 1$$

$$\equiv (a+1)(a^2+1)(a^4+1)(a^8+1)(a^{16}+1)$$

$$\begin{aligned}
\equiv\ & 1 + a + a^2 + a^{1+2} + a^4 + a^{1+4} + a^{2+4} \\
& + a^{1+2+4} + a^8 + a^{1+8} + a^{2+8} + a^{1+2+8} \\
& + a^{4+8} + a^{1+4+8} + a^{2+4+8} + a^{2+4+8+16} \\
& + a^{16} + a^{1+16} + a^{2+16} + a^{1+2+16} + a^{4+16} \\
& + a^{1+4+16} + a^{2+4+16} + a^{1+2+8+16} + a^{8+16} \\
& + a^{1+4+8+16} + a^{2+8+16} + a^{1+8+16} + a^{4+8+16} \\
& + a^{1+2+4+8} + a^{1+2+4+16} + a^{1+2+4+8+16}
\end{aligned}$$

$$a^{31} + a^{30} + a^{29} + \ldots + a^2 + a + 1$$

$$\equiv (a+1)(a^2+1)(a^4+1)(a^8+1)(a^{16}+1)$$

$$\equiv 1 + a + a^2 + a^{1+2} + a^4 + a^{1+4} + a^{2+4}$$
$$+ a^{1+2+4} + a^8 + a^{1+8} + a^{2+8} + a^{1+2+8}$$
$$+ a^{4+8} + a^{1+4+8} + a^{2+4+8} + a^{2+4+8+16}$$
$$+ a^{16} + a^{1+16} + a^{2+16} + a^{1+2+16} + a^{4+16}$$
$$+ a^{1+4+16} + a^{2+4+16} + a^{1+2+8+16} + a^{8+16}$$
$$+ a^{1+4+8+16} + a^{2+8+16} + a^{1+8+16} + a^{4+8+16}$$
$$+ a^{1+2+4+8} + a^{1+2+4+16} + a^{1+2+4+8+16}$$

$$a^{31} + a^{30} + a^{29} + \ldots + a^2 + a + 1$$

$$\equiv (a+1)(a^2+1)(a^4+1)(a^8+1)(a^{16}+1)$$

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$$+ a^{1+2+4+8} + a^{1+2+4+16} + a^{1+2+4+8+16}$$

$$a^{31} + a^{30} + a^{29} + \ldots + a^2 + a + 1$$

$$\equiv (a+1)(a^2+1)(a^4+1)(a^8+1)(a^{16}+1)$$

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$$+ a^{1+2+4} + a^8 + a^{1+8} + a^{2+8} + a^{1+2+8}$$
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$$+ a^{16} + a^{1+16} + a^{2+16} + a^{1+2+16} + a^{4+16}$$
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$$a^{31} + a^{30} + a^{29} + \ldots + a^2 + a + 1$$

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$$+ a^{1+2+4} + a^8 + a^{1+8} + a^{2+8} + a^{1+2+8}$$

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$$+ a^{16} + a^{1+16} + a^{2+16} + a^{1+2+16} + a^{4+16}$$
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$$+ a^{1+2+4+8} + a^{1+2+4+16} + a^{1+2+4+8+16}$$

$$a^{2^n-1} + a^{2^n-2} + a^{2^n-3} + \ldots + a^2 + a + 1$$

$$\equiv (a+1)(a^2+1)(a^4+1)\ldots(a^{2^{n-1}}+1)$$

$$\frac{1}{1-a}$$

$$\frac{1}{1-a} = 1 + a + a^2 + a^3 + \ldots$$

$$\frac{1}{1-a} = 1 + a + a^2 + a^3 + \dots$$

$$= (1+a)(1+a^2)(1+a^4)\dots$$

$$\frac{1}{1-a} = 1 + a + a^2 + a^3 + \dots \qquad 1,\ 2,\ 2^2, \dots \Rightarrow N$$

$$= (1+a)(1+a^2)(1+a^4)\dots$$

$$\frac{1}{1-a} = 1 + a + a^2 + a^3 + \ldots \qquad 1,\ 2,\ 2^2, \ldots \Rightarrow N$$

$$= (1+a)(1+a^2)(1+a^4)\ldots$$

$$\frac{1}{(1-a)^2}$$

$$\frac{1}{1-a} = 1 + a + a^2 + a^3 + \dots \qquad 1,\ 2,\ 2^2, \dots \Rightarrow N$$

$$= (1+a)(1+a^2)(1+a^4)\dots$$

$$\frac{1}{(1-a)^2} = 1 + 2a + 3a^2 + 4a^4 + \dots$$

$$\frac{1}{1-a} = 1 + a + a^2 + a^3 + \ldots \qquad 1,\ 2,\ 2^2, \ldots \Rightarrow N$$

$$= (1+a)(1+a^2)(1+a^4)\ldots$$

$$\frac{1}{(1-a)^2} = 1 + 2a + 3a^2 + 4a^4 + \ldots$$

$$= (1 + a + a^2 + a^3 + \ldots)^2$$

$$\frac{1}{1-a} = 1 + a + a^2 + a^3 + \ldots \qquad 1,\ 2,\ 2^2, \ldots \Rightarrow N$$

$$= (1+a)(1+a^2)(1+a^4)\ldots$$

$$\frac{1}{(1-a)^2} = 1 + 2a + 3a^2 + 4a^4 + \ldots \qquad x_1 + x_2 = N$$

$$= (1 + a + a^2 + a^3 + \ldots)^2$$

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$$\frac{1}{(1-a)^2} = 1 + 2a + 3a^2 + 4a^4 + \ldots \qquad x_1 + x_2 = N$$

$$= (1 + a + a^2 + a^3 + \ldots)^2$$

$$\frac{1}{(1-a)^3} = 1 + 3a + 6a^2 + 10a^4 + \ldots$$

$$= (1 + a + a^2 + a^3 + \ldots)^3$$

$$\frac{1}{1-a} = 1 + a + a^2 + a^3 + \ldots \qquad 1,\ 2,\ 2^2, \ldots \Rightarrow N$$

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$$\frac{1}{(1-a)^3} = 1 + 3a + 6a^2 + 10a^4 + \ldots \qquad x_1 + x_2 + x_3 = N$$

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GENERATING FUNCTIONS

$$\frac{1}{1-a} = 1 + a + a^2 + a^3 + \ldots \qquad 1,\ 2,\ 2^2, \ldots \Rightarrow N$$

$$= (1+a)(1+a^2)(1+a^4)\ldots$$

$$\frac{1}{(1-a)^2} = 1 + 2a + 3a^2 + 4a^4 + \ldots \qquad x_1 + x_2 = N$$

$$= (1 + a + a^2 + a^3 + \ldots)^2$$

$$\frac{1}{(1-a)^3} = 1 + 3a + 6a^2 + 10a^4 + \ldots \qquad x_1 + x_2 + x_3 = N$$

$$= (1 + a + a^2 + a^3 + \ldots)^3$$

100

$$\mathbf{100} = \mathbf{64} + \mathbf{32} + \mathbf{4}$$

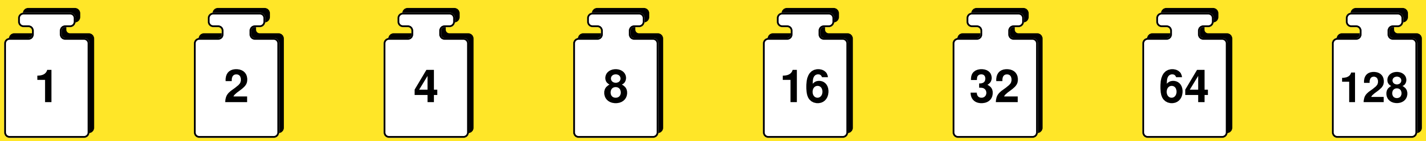
$$\begin{aligned}\mathbf{100} &= \mathbf{64 + 32 + 4} \\ &= \mathbf{1 \cdot 64 + 1 \cdot 32 + 0 \cdot 16 + 0 \cdot 8 + 1 \cdot 4 + 0 \cdot 2 + 0 \cdot 1}\end{aligned}$$

$$
\begin{aligned}
\mathbf{100} &= \mathbf{64 + 32 + 4} \\
&= \mathbf{1 \cdot 64 + 1 \cdot 32 + 0 \cdot 16 + 0 \cdot 8 + 1 \cdot 4 + 0 \cdot 2 + 0 \cdot 1} \\
&= \mathbf{(11001001)_2}
\end{aligned}
$$

$$
\begin{aligned}
100 &= 64 + 32 + 4 \\
&= 1 \cdot 64 + 1 \cdot 32 + 0 \cdot 16 + 0 \cdot 8 + 1 \cdot 4 + 0 \cdot 2 + 0 \cdot 1 \\
&= (11001001)_2
\end{aligned}
$$

PROBLEM:

Consider the following weights:



etc.

Which weight can be weighed by them?

$$\begin{aligned} 100 &= 64 + 32 + 4 \\ &= 1 \cdot 64 + 1 \cdot 32 + 0 \cdot 16 + 0 \cdot 8 + 1 \cdot 4 + 0 \cdot 2 + 0 \cdot 1 \\ &= (11001001)_2 \end{aligned}$$

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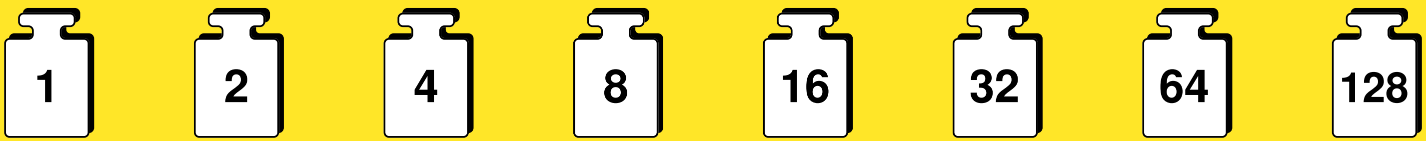
Consider the following weights:



etc.

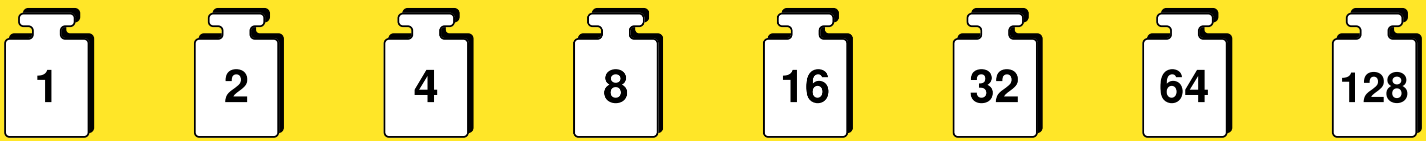
Which weight can be weighed by them?

ANSWER:

$$
\begin{aligned}
100 &= 64 + 32 + 4 \\
&= 1 \cdot 64 + 1 \cdot 32 + 0 \cdot 16 + 0 \cdot 8 + 1 \cdot 4 + 0 \cdot 2 + 0 \cdot 1 \\
&= (11001001)_2
\end{aligned}
$$

PROBLEM:

Consider the following weights:



etc.

Which weight can be weighed by them?

ANSWER:

Any, since any decimal number can be converted into the binary system.