Solve the following problems.

\$4.1:\ 18, 36, 37, 50. \quad \$4.4:\ 19, 21–25, 37, 41, 43.
\$4.3:\ 13, 20, 23, 28–30, 48. \quad \$4.5:\ 13, 17, 28, 29.

Concentrate on the following topics.

- Proving existential statements.
- Counterexample.
- Method of Generalizing from the Generic Particular
- Proof of a universal statement.
- Proof of a universal conditional.
- Contrapositive alternative.
- Proof of a universal conditional that involves cases.
- Divisibility.
- Quotient Remainder Theorem.
- Proof by way of contradiction.

Solve the following practice problems.

1) Let $d > 1$. Assume that $d|z + 1$, where $z$ is an integer. What is the value of $z \mod d$?

2) Prove the following. In each case, identify all the predicates, domains, and quantifiers.

   a) If $k$ is an integer, then $k^2 + k$ is even.
   b) $8|(n^2 - 1)$, for all odd integers $n$.
   c) The square of any odd integer is of the form $8\ell + 1$, for some integer $\ell$.

3) Let $m$ be a positive integer. Let $a, b, c, d \in \mathbb{Z}$. Assume that $m|a - b$ and $m|c - d$. Prove that $m|ac - bd$.

4) Let $m$ be a positive integer and $a \in \mathbb{Z}$.

   a) Show that, if $r = a \mod m$, then $a^2 \mod m = r^2 \mod m$.
   b) Find all possible values of $a^2 \mod 8$.
      (\textit{Hint:} Consider the eight cases: $r = 0, \ldots, 7$.)

5) Let $n \in \mathbb{Z}$ be a number such that $5 \nmid n$. Let $r = n \mod 5$. Show the following.

   a) $r \in \{1, 2, 3, 4\}$.
   b) $n^4 \mod 5 = r^4 \mod 5$.
   c) $n^4 \mod 5 = 1$. (\textit{Hint:} Consider four cases.)