Solve the following problems.

§5.1: 73, 74. §5.5: 30, 31, 32. §7.1: 34, 35, 38.
§5.2: 8, 13–15. §6.1: 3, 5, 30. §7.2: 10, 11, 12.
§5.4: 6, 7, 8, 24. §6.3: 6, 8, 12. Additional problems: All.

Concentrate on the following topics.

- Sequences.
- Sum and Product notation.
- Recursive definition of sequences.
- Principle of Mathematical Induction.
- Strong Induction.
- Well-ordering Principle.
- Sets.
- Element method of proof.
- Union, Intersection, Complement.
- Definition of a function.
- Range.
- Inverse Image.
- Direct Image.
- One-to-one.
- Onto.

Do the following practice questions.

1. Prove by induction that \(5|n^5 - n\), for all \(n \geq 1\).

2. Prove by induction that
\[
\sum_{k=1}^{n} \frac{k}{2^k} = 2 - \frac{n + 2}{2^n},
\]
for all \(n \geq 1\).

3. Let \(p_n\) be a sequence of natural numbers, defined for all \(n \geq 1\). Assume that \(p_1 = 2\) and, for all \(n \in \mathbb{N}\),
\[
p_{n+1} \leq z_n + 1,
\]
where \(z_n = \prod_{k=1}^{n} p_k\). Show that
\[
z_n \leq 2^{2^n - 1},
\]
for all \(n \geq 1\). (Hint: In the induction step, prove that \(p_{n+1} \leq 2^{2^n}\).)
4. A sequence \( x_n \) is defined recursively as follows: \( x_0 = 100 \) and 
\[
x_{n+1} = 11x_n - 10,
\]
for all \( n \geq 0 \). Show that the sequence satisfies that \( x_n = 9 \cdot 11^n + 1 \), for all \( n \geq 0 \).

5. A sequence \( x_n \) is defined recursively as follows: \( x_0 = 1 \) and 
\[
x_{n+1} = 4x_n^2 + 2,
\]
for all \( n \geq 0 \). Show that the sequence satisfies that \( 5|x_n - 1 \), for all \( n \geq 0 \).

6. A sequence is defined, recursively, as follows: \( L_1 = 0 \), \( L_2 = 4 \), \( L_3 = 24 \) and 
\[
L_{n+1} = -3L_{n-2} + L_{n-1} + 3L_n,
\]
for all \( n \geq 3 \). For each \( n \in \mathbb{N} \), let \( G_n \) be \( G_n = 3^n - (-1)^n - 4 \). Show that these sequences satisfy: \( L_n = G_n \), for all \( n \in \mathbb{N} \).

7. Show that \( 2^n/n! \leq 4/n \), for all \( n \in \mathbb{N} \). (Hint: use SPMI.)

8. Let \( m \in \mathbb{N} \) and \( a, b \in \mathbb{Z} \). Assume that \( a \mod m = b \mod m \). Prove that \( a^n \mod m = b^n \mod m \), for all \( n \in \mathbb{N} \).

9. Let \( x, y \in \mathbb{N} \). Prove that, if \( 4|x - y \) then \( 8|3^x - 3^y \).

10. Let \( x, y \in \mathbb{N} \). Prove that, if \( 8|x - y \) then \( 40|3^x - 3^y \).