Introduction to Number Theory (M328K)
Homework # 5
Fall 2025

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1. §3.5 # 36.   **2.** §3.5 # 40.   **3.** §3.5 # 46.

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4. Let a, b be two positive integers with sum $a + b = 798$ and least common multiple $[a, b] = 3465$.

    a) Compute (a, b).

    b) Find a and b.

(*Hint:* Use the previous problem.)

5. Let $a, b \in \mathbb{Z}$ be two integers such that $ab \neq 0$. Let $\beta = \big((a - b)(a + b), 2ab\big)$. Prove the following.

    a) If p is a prime, $p \neq 2$, and p divides β, then $p \mid (a, b)$.
        (*Hint:* Use Euclid's theorem.)

    b) If 2 divides β and $(a, b) = 1$, then $\beta = 2$.

    c) If $(a, b) = 1$ then $\beta = 1$ or $\beta = 2$.

    d) $(a - b)(a + b) \neq 2ab$.

6. Let γ be a real number such that $\gamma^2 = 2\gamma + 1$. Show that γ is irrational.
(*Hint:* Write a proof by contradiction and use the previous problem.)

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7. §3.7 # 2.   **8.** §3.7 # 6.   **9.** §3.7 # 8.