Introduction to Number Theory (M328K)

Homework # 6

Fall 2025

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1. §4.1 # 14. **2.** §4.1 # 30. **3.** §4.1 # 34.

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4. Let $a \in \mathbb{Z}$ and $b \in \mathbb{N}$. Show that the following statements are equivalent.

a) $(a, b) = 1$.

b) There exists an integer z such that $az \equiv 1 \pmod{b}$.

(*Hint:* You have to show **a)** $\Longrightarrow$ **b)** and **b)** $\Longrightarrow$ **a)**.)

5. Let x_1, x_2, y_1, y_2 be four integers such that $x_1 y_1 \equiv 1 \pmod{11}$ and $x_2 y_2 \equiv 1 \pmod{7}$. We define

$$z = 7x_1 - 11x_2$$

and

$$\bar{z} = 33y_2 - 14y_1.$$

Show that $z\bar{z} \equiv 1 \pmod{77}$.

6. Let x_n be a sequence satisfying the recursion $x_{n+1} = 4x_n + 15$, where x_1 is an integer such that $x_1 \equiv 6 \pmod{45}$. Use the PMI to show that

$$x_n \equiv 15n + 9\,(-1)^n \pmod{45},$$

for all $n \in \mathbb{N}$.

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7. §4.2 # 2. Do parts b), c), e), and f).

8. §4.2 # 6.

9. §4.2 # 10. (*Hint:* Use problem 4.)

10. §4.2 # 18.