Math M427J. Fall 2025
Answers to Guide 1.

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§2.2.2 # 2. $y(t) = C_1\, e^{3t/2} + C_2\; t\, e^{3t/2}$.

§2.2.2 # 4. $y(t) = 3\, t\, e^{t/2}$.

§2.3 # 1. $y(t) = C_1 + C_2 e^{2t} + t^2$.

§2.3 # 2. $y(t) = C_1 e^{t^2} + C_2 t e^{t^2} + 1$.

§2.3 # 3. $y_1(t) = e^t$ is a solution of the homogeneous equation and $\Psi(t) = e^{t^2}$ is a particular solution of the non-homogeneous. Then $y_2(t) = e^{-t^3}$ is a solution of the homogeneous. The general solution is

$$y(t) = C_1 e^t + C_2 e^{-t^3} + e^{t^2}.$$

This implies that $y(0) = C_1 + C_2 + 1 = 1$ and $y'(0) = C_1 = 2$. We conclude that $C_2 = -2$.

§2.4 # 2. $y(t) = C_1 e^{2t} + C_2 t e^{2t} + \dfrac{1}{6} e^{2t} t^3$.

§2.4 # 4. $y(t) = C_1 e^t + C_2 e^{2t} + \dfrac{1}{2} + \dfrac{1}{2} e^{3t} t - \dfrac{3e^{3t}}{4}$.

§2.4 # 6. $y(t) = \dfrac{4}{63} e^{-2t} t^{9/2}$.

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1) $y(t) = -\dfrac{t^2}{2} + C e^{2t}$.

2) $y(t) = 3t \sec(t) + C \sec(t)$.

3) $y(t) = e^t (2t - 1) + e^{-t}$.

4) $y(t) = e^{-t} \left(t^2 + 6e - 1\right) t^{-1}$.

5) $y(t) = \left(a + e^t - e^{\pi/2}\right) \csc(t)$.

6) $y^3/3 + y = t^3/3 - t + 2/3$.

7) $b = 3$ and $y(t)$ satisfies the implicit equation $\phi(t, y) = t^2 y^2/2 + t^3 y = C$.

8) In this problem we have $p(t) = 2/t$. We know that the wronskian satisfies the equation $W'(t) + p(t)W(t) = 0$. The integrating factor is

$$\mu(t) = \exp\left(\int p(t)\, dt\right) = \exp\left(\int \frac{2}{t}\, dt\right) = t^2.$$

We conclude that W satisfies $\dfrac{d}{dt}\left(\mu(t)W(t)\right) = 0$. From this, we find that W is of the form $W(t) = C/\mu(t) = C/t^2$, where C is a constant. Using the initial condition, we find that $C = 3$. We conclude that $W(t) = 3/t^2$.

9)

a) $y(t) = \dfrac{3t}{2} - \dfrac{53}{20} e^{-2t} - \dfrac{4\sin(t)}{5} - \dfrac{8\cos(t)}{5} + \dfrac{17}{4}$.

b) $y(t) = \dfrac{123}{5} e^{-2t} - \dfrac{197}{10} e^{-3t} + \dfrac{1}{10}\sin(t) + \dfrac{1}{10}\cos(t)$.

c) $y(t) = -\dfrac{5}{16}\sin(2t) - \dfrac{3}{4} t\cos(2t) + 2\cos(2t) + \dfrac{3}{16}\sin(4t)\cos(2t) - \dfrac{3}{16}\sin(2t)\cos(4t)$.

We can simplify this and get $y(t) = \left(2 - \dfrac{3t}{4}\right)\cos(2t) - \dfrac{1}{8}\sin(2t)$.

d) $y(t) = e^{-t} t^2 + e^{-t} t$.

10) $y(t) = 4\pi - \arcsin\left(\dfrac{\sqrt{3}}{2}\, e^{2t}\right)$.