Math M427J. Fall 2025

Guide for the Final exam

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The final will have **5** multiple-choice problems, **3** free-response and **1** bonus question. The exam is cumulative, study all the previous guides. In addition, solve the following problems.

3.10 1–8. **5.3** 1, 2. **5.6** 1, 2 (d).

Concentrate on the following concepts.

- Exponential matrix.
- Fundamental matrix of solutions.
- Exponential matrix from a fundamental matrix .
- Repeated roots.
- Generalized eigenvector.
- Solutions generated by repeated roots.
- Heat equation.
- Solutions of the heat equation.
- Fourier coefficients
- Initial condition in the heat equation.

PRACTICE PROBLEMS

Do the following practice questions.

1) Find e^{tA}, if $A = \begin{bmatrix} -2 & 2 & 4 \\ -2 & 3 & 2 \\ 1 & -2 & 1 \end{bmatrix}$.
(*Hint:* The matrix has three real eigenvalues.)

2) Solve the following initial value problem

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 0 & 2 \\ 2 & -2 & 4 \\ 1 & -1 & 1 \end{bmatrix}\mathbf{x}, \qquad \mathbf{x}(0) = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}.$$

(*Hint:* The eigenvalue 0 has multiplicity 2.)

3) Find e^{tB}, if $B = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 0 & 1 \\ -6 & 1 & 0 \end{bmatrix}$.
(*Hint:* The matrix has eigenvalues 1, -1. The eigenvalue 1 has multiplicity 2.)

4) Find e^{tM}, if $M = \begin{bmatrix} -1 & 0 & 0 \\ -3 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$.
(*Hint:* The eigenvalue -1 has multiplicity 3. You will get $(M+I)^3 = 0$. We can choose any three linearly independent vectors. For instance, we could choose the standard basis.)

5) Let f be the function given by

$$f(x) = \begin{cases} x, & \text{if } 0 \le x \le 1, \\ 2 - x, & \text{if } 1 \le x \le 2. \end{cases}$$

Solve the heat equation.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},$$

with boundary conditions $u(0,t) = u(2,t) = 0$, and initial condition $u(x,0) = f(x)$.

6) Solve the following initial value problem

$$\dot{\mathbf{x}} = \begin{bmatrix} 3-b & b \\ 1-b & b+2 \end{bmatrix}\mathbf{x}, \qquad \mathbf{x}(0) = \begin{bmatrix} b-5 \\ b-6 \end{bmatrix}.$$

The number b is real number different from 0 and 1.