Math M427J. Fall 2025
Guide for Midterm Exam 1

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Solve the following problems.

1.2 4–6, 16.

1.4 3, 6, 7.

1.9 3–6, 10, 12.

2.1 5–7, 13.

2.2 5–9.

2.2.1 5, 6, 8–10.

2.2.2 (Pag. 149) 1–4.

2.3 (Pag. 152) 1–3.

2.4 (Pag. 156) 2–6.

Concentrate on the following concepts.

- First-order linear equations.
- Integrating factor.
- Separable equations.
- Exact equations.
- Second-order equations.
- Linear operator.
- Wronskian.
- Fundamental set of solutions.
- Linear dependence and independence.
- Differential equation satisfied by the Wronskian.
- Linear equations with constant coefficients.
- Characteristic equation.
- Two real roots case.
- Complex roots case.
- Repeated root case.
- Nonhomogeneous equations.
- Particular solution.
- Variation of parameters method.

Do the following practice questions.

1) Find the general solution of $y' - 2y = t^2 - t$.

2) Find the general solution of

$$y' - \tan(t)y = 3\sec(t).$$

3) Find the solution of the IVP

$$y' + y = 4t\,e^t, \qquad y(0) = 0.$$

4) Find the solution of the IVP

$$t\,y' + (t+1)y = 2t\,e^{-t}, \qquad y(1) = 6, \quad t > 0.$$

5) Solve $(\sin t)y' + (\cos t)y = e^t$, with $y(\pi/2) = a$. Use $0 < t < \pi$.

6) Solve

$$\frac{dy}{dt} = \frac{t^2 - 1}{y^2 + 1}, \qquad y(-1) = 1.$$

7) Find the value of b for which the given equation is exact, and solve it.

$$(t\,y^2 + b\,t^2 y) + (t + y)t^2 y' = 0.$$

8) Suppose that $\{y_1, y_2\}$ is a fundamental set of solutions of the equation $t\,y'' + 2y' + te^t y = 0$, on the interval $I = (0, \infty)$. Find the wronskian $W[y_1, y_2](t)$, if we know that $W[y_1, y_2](1) = 3$.

9) Find the solution of the following IVP.

a) $y'' + 2y' = 3 + 4\sin t$, $y(0) = 0$, $y'(0) = 6$.

b) $y'' + 5y' + 6y = \cos t$, $y(0) = 5$, $y'(0) = 10$.

c) $y'' + 4y = 3\sin 2t$, $y(0) = 2$, $y'(0) = -1$.

d) $y'' + 2y' + y = 2e^{-t}$, $y(0) = 0$, $y'(0) = 1$.

10) Solve $y' = 2\tan(y)$, $y(0) = 11\pi/3$.