

Math M427J. Fall 2025

Guide for Midterm Exam 3

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Solve the following problems.

3.4 10.

3.6 9, 10, 12, 14.

3.8 2, 4, 7–12, 14.

3.11 1–4.

3.5 3–8.

3.7 4, 10.

3.9 2–7.

Concentrate on the following concepts.

- Determinant.
- Submatrices, minors and cofactors.
- Column and row expansion.
- Lower and upper diagonal matrices.
- Properties of determinants.
- Adjugate matrix.
- Inverse matrix. Row reduction method.
- Properties of singular and non-singular matrices.
- Homogeneous systems.
- Eigenvalues and eigenvectors.
- Characteristic Polynomial.
- Method of eigenvectors.
- Complex roots case.
- Exponential matrix.
- Fundamental matrix of solutions.
- Exponential matrix from a fundamental matrix .

Do the following practice questions.

1) Find the determinant $\begin{vmatrix} 1 & -2 & 3 & 5 \\ x & y & z & w \\ 1 & 2 & -8 & 0 \\ 2 & 0 & -5 & 2 \end{vmatrix}$.

2) Let $\mathbf{x}(t)$ be the solution of the following initial value problem. What is $\mathbf{x}(\pi/8)$?

$$\dot{\mathbf{x}} = \begin{bmatrix} -3 & -5 \\ 4 & 1 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

3) Solve the following initial value problem

$$\dot{\mathbf{x}} = \begin{bmatrix} 1 & -2 & -1 \\ -1 & 0 & 3 \\ 3 & -2 & -3 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

4) Solve the following initial value problem

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 2 & 1 & 0 & 2 \\ -2 & -3 & 2 & -5 \\ -1 & 0 & 0 & 0 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 0 \end{bmatrix}.$$

5) Find the exponential e^{tA} , if $A = \begin{bmatrix} -1 & 1 \\ 3 & -3 \end{bmatrix}$.

6) Find e^{tA} , if $A = \begin{bmatrix} -2 & 2 & 4 \\ -2 & 3 & 2 \\ 1 & -2 & 1 \end{bmatrix}$.

7) Solve the following initial value problem

$$\dot{\mathbf{x}} = \begin{bmatrix} 1 & -4 & 4 \\ -2 & 3 & 2 \\ 4 & -8 & 1 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} -1 \\ 2 \\ -5 \end{bmatrix}.$$