Differential Equations with Linear Algebra (M427J)

Some answers to homework # 2

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§1.9 # 4. $\phi(t, y) = te^{ty} + y + t = C$.

§1.9 # 8. First, we compute $\phi(t, y) = t^3 y + t^2 \cos(y) - \dfrac{y^2}{2}$. The solution $y(t)$ satisfies the implicit equation $\phi(t, y(t)) = C$, where C is a constant. If we use $y(0) = 2$, we find that $\phi(0, 2) = -2$. We conclude that $y(t)$ satisfies

$$t^3 y(t) + t^2 \cos(y(t)) - \frac{y(t)^2}{2} = -2.$$

§1.9 # 12. In this example, we have $M(t, y) = t + ye^{2ty}$ and $N(t, y) = ate^{2ty}$. Both functions are defined everywhere on the plane. We only need to check that we have the equation.

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}.$$

Solving, we find $a = 1$. Using the method of integration, we find that $\phi(t, y) = \dfrac{t^2}{2} + \dfrac{1}{2}e^{2ty} = C$.

§1.9 # 16. In this example, we have $M(t, y) = y^2 \sin(t)$ and $N(t, y) = yf(t)$. We need to verify the equation

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}.$$

Solving, we find $2y \sin(t) = yf'(t)$. We find the equation $f'(t) = 2\sin(t)$ and get $f(t) = C_0 - 2\cos(t)$, where C_0 is a constant. With this choice of $f(t)$, the equation is exact. The solution is

$$\frac{C_0 y^2}{2} - y^2 \cos(t) = C_1,$$

where C_0, C_1 are constants.

§2.1 # 2 (a). 0.

§2.1 # 2 (b). $-3e^{2t}$.

§2.1 # 2 (c). $-4e^{3t}$.

§2.1 # 2 (d). $(r-5)(r-1)e^{rt}$.

§2.1 # 2 (e). $5t - 6$.

§2.1 # 2 (f). $5t^2 - 12t + 2$.

§2.1 # 2 (g). $5t^2 - 2t - 10$.

§2.1 # 4. In this case, $L[y]$ is a linear operator. This is, $L[c\, y] = c\, L[y]$ and $L[y_1 + y_2] = L[y_1] + L[y_2]$. If we use $y_1(t) = t^2$ and $y_2(t)$ then $L[y_1](t) = t + 1$ and $L[y_2](t) = 2t + 2$. We can write the function $y(t) = t - 2t^2$ as

$$y(t) = -2t^2 + t = -2y_1(t) + y_2(t).$$

Combining these expressions, we find that

$$L[y](t) = L[-2y_1 + y_2](t) = -2L[y_1](t) + L[y_2](t).$$
$$= -2(t+1) + (2t+2) = 0.$$

Given that $L[y](t) = y'' + p(t)y' + q(t)y$, we conclude that $y'' + p(t)y' + q(t)y = 0$.