Differential Equations with Linear Algebra (M427J)

Some answers to homework # 3

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§2.1 # 6 (b). $W[y_1, y_2](t) = e^{-\frac{t^2}{2}}$.

§2.1 # 6 (c). The wronskian is never zero on the interval, so the solutions are linearly independent and form a fundamental set of solutions.

§2.1 # 6 (d). The solution satisfies $y(t) = C_1 y_1(t) + C_2 y_2(t)$, where C_1 and C_2 are constants that we need to determine. Using the initial conditions, we get the system of equations

$$C_1 y_1(0) + C_2 y_2(0) = 0, \qquad C_1 y_1'(0) + C_2 y_2'(0) = 1.$$

Solving, we find that $C_1 = 0$ and $C_2 = 1$. The solution is $y(t) = y_2(t) = e^{-\frac{t^2}{2}} \int_0^t e^{\frac{s^2}{2}}\, ds$.

§2.1 # 14. We know that if y_1, y_2 are two solutions of $y'' + p(t) + q(t)y = 0$, and $W(t) = W[y_1, y_2](t)$ then

$$W'(t) = -p(t)\, W(t).$$

We will exclude the case in which both solutions are dependent. Assuming that $\{y_1, y_2\}$ are independent, we get that $W(t)$ is never zero. If $W(t)$ is a constant, then $W'(t)$ is zero and hence $p(t) = -W'(t)/W(t) = 0$.

§2.2 # 2. $y(t) = C_1\, e^{t/6} + C_2\, e^{t}$.

§2.2 # 4. $y(t) = C_1\, e^{r_1 t} + C_2\, e^{r_2 t}$, where $r_1 = -1 - \dfrac{1}{\sqrt{3}}, \quad r_2 = \dfrac{1}{\sqrt{3}} - 1$.

§2.2 # 6. $y(t) = \dfrac{16}{9} e^{\frac{5}{2} - \frac{5t}{2}} + \dfrac{29}{9} e^{2t-2}$.

§2.2.1 # 4. $y(t) = C_1\, e^{t/8} \cos\left(\dfrac{\sqrt{15}}{8} t\right) + C_2\, e^{t/8} \sin\left(\dfrac{\sqrt{15}}{8} t\right)$.

§2.2.1 # 6. $y(t) = e^{-t} \sin(2t)$.

§2.2.1 # 8. $y(t) = e^{(t-1)/4} \left[\dfrac{3}{\sqrt{23}} \sin\left(\dfrac{\sqrt{23}}{4}(t-1)\right) + \cos\left(\dfrac{\sqrt{23}}{4}(t-1)\right)\right]$.