Differential Equations with Linear Algebra (M427J)

Some answers to practice problems # 10

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§3.8 # 2. The general solution is of the form

$$\mathbf{x}(t) = C_1\,\mathbf{x}^1(t) + C_2\,\mathbf{x}^2(t).$$

where $\mathbf{x}^1(t) = \begin{bmatrix} e^{2t} \\ 4e^{2t} \end{bmatrix}$, and $\mathbf{x}^2(t) = \begin{bmatrix} e^{-t} \\ e^{-t} \end{bmatrix}$.

§3.8 # 4. The general solution is of the form

$$\mathbf{x}(t) = C_1\,\mathbf{x}^1(t) + C_2\,\mathbf{x}^2(t) + C_3\,\mathbf{x}^3(t).$$

where $\mathbf{x}^1(t) = \begin{bmatrix} -3e^{5t} \\ 6e^{5t} \\ 2e^{5t} \end{bmatrix}$, $\mathbf{x}^2(t) = \begin{bmatrix} -e^{3t} \\ 2e^{3t} \\ e^{3t} \end{bmatrix}$, and $\mathbf{x}^3(t) = \begin{bmatrix} -e^{2t} \\ e^{2t} \\ e^{2t} \end{bmatrix}$.

§3.8 # 8. $\mathbf{x}(t) = \begin{bmatrix} 3e^{-t} - 3e^{4t} \\ 2e^{-t} + 3e^{4t} \end{bmatrix}$.

§3.8 # 10. $\mathbf{x}(t) = \begin{bmatrix} e^{t} - 2e^{-t} \\ -4e^{-t} \\ 14e^{-t} - e^{t} \end{bmatrix}$.

§3.8 # 12. $\mathbf{x}(t) = \dfrac{1}{3}\begin{bmatrix} -28e^{-t} + 27e^{t} + 4e^{2t} \\ 8e^{-t} + 4e^{2t} \\ -52e^{-t} + 27e^{t} + 4e^{2t} \end{bmatrix}$.

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§3.9 # 2. The general solution is $\mathbf{x}(t) = C_1\mathbf{x}_1(t) + C_2\mathbf{x}_2(t) + C_3\mathbf{x}_3(t)$, where

$$\mathbf{x}_1(t) = \begin{bmatrix} 2e^{-t}\cos(t) - e^{-t}\sin(t) \\ e^{-t}\cos(t) \\ 0 \end{bmatrix}, \quad \mathbf{x}_2(t) = \begin{bmatrix} 2e^{-t}\sin(t) + e^{-t}\cos(t) \\ e^{-t}\sin(t) \\ 0 \end{bmatrix}, \quad \mathbf{x}_3(t) = \begin{bmatrix} 0 \\ 0 \\ e^{t} \end{bmatrix}.$$

§3.9 # 4. The general solution is $\mathbf{x}(t) = C_1\mathbf{x}_1(t) + C_2\mathbf{x}_2(t) + C_3\mathbf{x}_3(t)$, where

$$\mathbf{x}_1(t) = \begin{bmatrix} \sin(t) - \cos(t) \\ \cos(t) - \sin(t) \\ 2\cos(t) \end{bmatrix}, \quad \mathbf{x}_2(t) = \begin{bmatrix} -\sin(t) - \cos(t) \\ \sin(t) + \cos(t) \\ 2\sin(t) \end{bmatrix}, \quad \mathbf{x}_3(t) = \begin{bmatrix} 0 \\ e^{t} \\ 0 \end{bmatrix}.$$

§3.9 # 6. $\mathbf{x}(t) = \begin{bmatrix} e^t(\cos(2t) - 4\sin(2t)) \\ e^t(5\cos(2t) - 3\sin(2t)) \end{bmatrix}.$

§3.9 # 8. $\mathbf{x}(t) = \begin{bmatrix} \sin(2t) + \cos(2t) \\ \cos(2t) - \sin(2t) \\ \cos(3t) \\ \sin(3t) \end{bmatrix}.$

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§3.11 # 2. $\frac{1}{10}\begin{bmatrix} -e^{-3t} + 15e^t - 4e^{2t} & 10e^{2t} - 10e^t & e^{-3t} + 5e^t - 6e^{2t} \\ -7e^{-3t} + 15e^t - 8e^{2t} & 20e^{2t} - 10e^t & 7e^{-3t} + 5e^t - 12e^{2t} \\ -11e^{-3t} + 15e^t - 4e^{2t} & 10e^{2t} - 10e^t & 11e^{-3t} + 5e^t - 6e^{2t} \end{bmatrix}.$

§3.11 # 4. $\begin{bmatrix} e^t & 0 & 0 \\ -\frac{3}{2}e^t + e^t\sin(2t) + \frac{3}{2}e^t\cos(2t) & e^t\cos(2t) & -e^t\sin(2t) \\ e^t + \frac{3}{2}e^t\sin(2t) - e^t\cos(2t) & e^t\sin(2t) & e^t\cos(2t) \end{bmatrix}.$