

# Differential Equations with Linear Algebra (M427J)

## Some answers to practice problems # 2

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**§1.9 # 4.**  $\phi(t, y) = te^{ty} + y + t = C$ .

**§1.9 # 8.** First, we compute  $\phi(t, y) = t^3 y + t^2 \cos(y) - \frac{y^2}{2}$ . The solution  $y(t)$  satisfies the implicit equation  $\phi(t, y(t)) = C$ , where  $C$  is a constant. If we use  $y(0) = 2$ , we find that  $\phi(0, 2) = -2$ . We conclude that  $y(t)$  satisfies

$$t^3 y(t) + t^2 \cos(y(t)) - \frac{y(t)^2}{2} = -2.$$

**§1.9 # 12.** In this example, we have  $M(t, y) = t + ye^{2ty}$  and  $N(t, y) = ate^{2ty}$ . Both functions are defined everywhere on the plane. We only need to check that we have the equation.

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}.$$

Solving, we find  $a = 1$ . Using the method of integration, we find that  $\phi(t, y) = \frac{t^2}{2} + \frac{1}{2}e^{2ty} = C$ .

**§1.9 # 16.** In this example, we have  $M(t, y) = y^2 \sin(t)$  and  $N(t, y) = y \cdot f(t)$ . We need to verify the equation

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}.$$

Solving, we find  $2y \sin(t) = yf'(t)$ . We find the equation  $f'(t) = 2 \sin(t)$  and get  $f(t) = C_0 - 2 \cos(t)$ , where  $C_0$  is a constant. With this choice of  $f(t)$ , the equation is exact. The solution for  $\phi$  is

$$\phi(t, y) = \frac{1}{2}y^2 \cdot f(t) = \frac{C_0 y^2}{2} - y^2 \cos(t) = C_1,$$

where  $C_0, C_1$  are constants.