Differential Equations with Linear Algebra (M427J)

Some answers to practice problems # 3

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§2.1 # 2 (a). 0.

§2.1 # 2 (b). $-3e^{2t}$.

§2.1 # 2 (c). $-4e^{3t}$.

§2.1 # 2 (d). $(r-5)(r-1)e^{rt}$.

§2.1 # 2 (e). $5t-6$.

§2.1 # 2 (f). $5t^2-12t+2$.

§2.1 # 2 (g). $5t^2-2t-10$.

§2.1 # 6 (b). $W[y_1,y_2](t)=e^{-\frac{t^2}{2}}$.

§2.1 # 6 (c). The wronskian is never zero on the interval, so the solutions are linearly independent and form a fundamental set of solutions.

§2.1 # 6 (d). The solution satisfies $y(t)=C_1y_1(t)+C_2y_2(t)$, where C_1 and C_2 are constants that we need to determine. Using the initial conditions, we get the system of equations

$$C_1y_1(0)+C_2y_2(0)=0,\qquad C_1y_1'(0)+C_2y_2'(0)=1.$$

Solving, we find that $C_1=0$ and $C_2=1$. The solution is $y(t)=y_2(t)=e^{-\frac{t^2}{2}}\displaystyle\int_0^t e^{\frac{s^2}{2}}\,ds$.

§2.2 # 2. $y(t)=C_1\,e^{t/6}+C_2\,e^{t}$.

§2.2 # 4. $y(t)=C_1\,e^{r_1t}+C_2\,e^{r_2t}$, where $r_1=-1-\dfrac{1}{\sqrt{3}}$, $\quad r_2=\dfrac{1}{\sqrt{3}}-1$.

§2.2 # 6. $y(t)=\dfrac{16}{9}e^{\frac{5}{2}-\frac{5t}{2}}+\dfrac{29}{9}e^{2t-2}$.

§2.2.1 # 4. $y(t)=C_1\,e^{t/8}\cos\left(\dfrac{\sqrt{15}}{8}t\right)+C_2\,e^{t/8}\sin\left(\dfrac{\sqrt{15}}{8}t\right)$.

§2.2.1 # 6. $y(t)=e^{-t}\sin(2t)$.

§2.2.1 # 8. $y(t)=e^{(t-1)/4}\left[\dfrac{3}{\sqrt{23}}\sin\left(\dfrac{\sqrt{23}}{4}(t-1)\right)+\cos\left(\dfrac{\sqrt{23}}{4}(t-1)\right)\right]$.